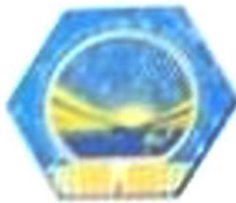


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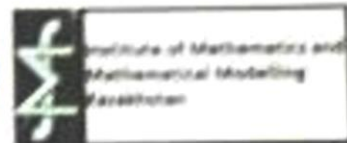
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ABSTRACT BOOK

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The deficiency indices of singular differential operators in vector-valued functions space

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Abstract: In this work, we consider the minimal differential operator L_0 in the space $H = L^2(0, \infty) \oplus L^2(0, \infty)$ which generated by the following differential expression:

$$L_0(y) = y^{(4)} + Q(x)y. \quad (1)$$

Here $y(x) = (y_1(x), y_2(x))$, $0 < x < \infty$ and $Q(x) = \|q_{ij}\|_{i,j=1}^2$ – is a real symmetric matrix, whose eigenvalues $\mu_1(x) \rightarrow +\infty$, $\mu_2(x) \rightarrow -\infty$, for $x \rightarrow \infty$.

Introduce $\varphi(x) = \frac{1}{2} \operatorname{arctg} \frac{q_{22} - q_{11}}{2q_{12}}$. The function $\varphi(x)$ is called as the speed of the rotation of eigenvectors of the matrix $Q(x)$.

Theorem 1. Assume that for sufficiently large x_0 and $x > x_0$ the following inequalities

1) $|\dot{\varphi}(x)| < \text{const}$,

2) $0 < A \leq \frac{|\mu_i(x)|}{|\mu_j(x)|} \leq B$, $i, j = 1, 2$,

3) $\int_{x_0}^{\infty} \left| \mu_i^{-\frac{1}{4}}(x) \right| dx < \infty$, $\int_{x_0}^{\infty} \left| \frac{\mu_i'^2(x)}{\mu_i^{\frac{9}{4}}(x)} + \frac{\mu_i''(x)}{\mu_i^{\frac{5}{4}}(x)} \right| dx < \infty$, $\int_{x_0}^{\infty} \left| \frac{\varphi''(x)}{\mu_i^{\frac{1}{4}}(x)} \right| dx < \infty$, $i = 1, 2$

4) $|\mu_i'(x)| \leq C |\mu_i(x)|^\alpha$, $C = \text{const}$, $i = 1, 2$, $0 < \alpha < \frac{5}{4}$

is satisfied. Then, system (1) has eight linearly independent solutions $y_j(x, \lambda)$ for $x \rightarrow \infty$, such that

$$y_1 = \varphi_1(x, \lambda) \exp \left\{ \int_0^x (\lambda - \mu_1(t))^{1/4} dt \right\} (1 + o(1)), y_2 = \varphi_1(x, \lambda) \exp \left\{ - \int_0^x (\lambda - \mu_1(t))^{1/4} dt \right\} (1 + o(1)),$$

$$y_3 = \varphi_1(x, \lambda) \exp \left\{ i \int_0^x (\lambda - \mu_1(t))^{1/4} dt \right\} (1 + o(1)), y_4 = \varphi_1(x, \lambda) \exp \left\{ -i \int_0^x (\lambda - \mu_1(t))^{1/4} dt \right\} (1 + o(1)),$$

$$y_5 = \varphi_2(x, \lambda) \exp \left\{ \int_0^x (\lambda - \mu_2(t))^{1/4} dt \right\} (1 + o(1)), y_6 = \varphi_2(x, \lambda) \exp \left\{ - \int_0^x (\lambda - \mu_2(t))^{1/4} dt \right\} (1 + o(1)),$$

$$y_7 = \varphi_2(x, \lambda) \exp \left\{ i \int_0^x (\lambda - \mu_2(t))^{1/4} dt \right\} (1 + o(1)), y_8 = \varphi_2(x, \lambda) \exp \left\{ -i \int_0^x (\lambda - \mu_2(t))^{1/4} dt \right\} (1 + o(1)),$$

where

$$\varphi_1(x, \lambda) = \frac{1}{\sqrt[8]{(\lambda - \mu_1(x))^3}} \begin{pmatrix} \cos \varphi(x) \\ -\sin \varphi(x) \end{pmatrix}, \varphi_2(x, \lambda) = \frac{1}{\sqrt[8]{(\lambda - \mu_2(x))^3}} \begin{pmatrix} \sin \varphi(x) \\ \cos \varphi(x) \end{pmatrix}.$$

Theorem 2. Let holds all conditions of theorem 1. Then the deficiency indices of operator L_0 equal to (6,6).

Keywords: differential operator, distribution of eigenvalues, indices of deficiency, asymptotics of the spectrum of the differential operator in the vector functions space.

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