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# Establishment of Computing Centers and Curriculum Development in Mathematical Engineering Master programme
# ECCUM

## MASTER PROGRAMME

## MATHEMATICAL ENGINEERING

University of Santiago de Compostela, Spain

Politechnico di Torino, Italy

University of Primorska, Slovenia

Urgench State University, Uzbekistan

Turin Polytechnic University in Tashkent, Uzbekistan

Bukhara Engineering and Technological Institute, Uzbekistan

A.Baitursynov Kostanay State University, Kazakhstan

International IT University, Almaty, Kazakhstan

# QUALIFICATION DESCRIPTION FOR MASTER PROGRAM IN MATHEMATICAL ENGINEERING

**Requirements for awarding master degree in Mathematical Engineering**:

- Completion of 120 ECTS of graduate course work
- Successful completion of a thesis
- Successful completion of the final examination

**Program duration is 2 years.**

**Language of study:** English, Russian, Uzbek, Kazakh.

**The Goal of the Master Programme:**

The master program will train a professional with strong engineering skill and solid mathematical knowledge; especially suitable for work within research and design groups that need in-depth design studies, advanced mathematical procedures, complex mathematical models and require heavy and complex simulations.

The degree is designed to enable student:

- to identify and deduce the mathematical model to adopt; the model based on a trade-off between the desired accuracy and the affordable complexity;
- to search for a satisfactory consistency with the reality and optimizing the computational costs, in terms of time and resource usage while solving the problem;
- to use the most recent numerical methods, visualization strategies to report results to colleagues in other disciplines.

**The master programme specific objectives:**

- to train a professional who can integrate engineering technologies and mathematical knowledge to define and solve complex problems that require all the modern design tools: mathematical modelling, computer simulations and statistical investigation.

- to empower a learner with the competence to address problems from various engineering fields relating to (both) artificial systems, constructed or designed by humans, and to natural systems and phenomena.

The program aims to provide graduates with the skills needed for the following activities:

- choosing the best mathematical model to use, based on a trade-off between desired accuracy and accepted complexity;
- performing qualitative and quantitative analysis of output generated by the model and evaluating the conformity with the phenomenon to be analyzed;
- performing numerical simulations of natural phenomena, industrial processes, behaviors of materials and structural design;
- performing analyses of statistical data, synthesizing them, adapting them to stochastic models of interest in the applications, their implementation for forecasting purposes in reliability and decisional analyses;

The curriculum is intended to ensure all (the) cognitive tools necessary to practice the career of a mathematical engineer, such as

- Mathematical modelling, aimed at deducing the most suitable mathematical model to describe applicative problems and to analyze their solutions from both a qualitative and a quantitative point of view;
- Numerical simulation, aimed at describing the most suitable methods for approximating the solution as well as the methods for graphically representing numerical solutions;
- Probability and statistics applied to solving non-deterministic problems, managing, and interpreting data stemming from experiments or from probabilistic models;
- Broad engineering skills for application in various sectors.

With the preparation and discussion of the thesis, students have an opportunity to put into practice all knowledge acquired, mixing the theoretical and the applicative and/or experimental skills as well as providing an original input.

**Knowledge:**

Knowledge of the mathematical fundamentals of engineering, effective statistical methods to analyze and investigate data, comprehensive knowledge of the engineering fundamentals, engineering for business and economy.

Knowledge and understanding of the mathematical bases as well of the fundamentals of the hard sciences underlying all engineers training.

Detailed understanding of the natural and physical sciences as well of the engineering fundamentals applicable to the practice area.

Deep understanding at the procedural level, of the mathematics, numerical analysis, statistics, computer and information sciences, which underpin the practice area.

**Competences:**

Ability to apply knowledge to the resolution (identification, formulation, resolution) of engineering problems (products, processes and methods).

Ability to solve problems in satisfying the constraints; (economic, environmental, social, political, ethical, health and safety).
Mastery of the theoretical and experimental tools for solving problems, be aware of their limitations and their implications for non-technical, understand their ethical and professional consequences.

**Transferable Skills:**

1. (to) interact effectively as an individual and as a member of a team;
2. (to) use an ample variety of different methods to communicate effectively with the engineering community and with society at large;
3. (to) demonstrate awareness of the health, safety and legal issues and responsibilities of engineering practice;
4. (to) demonstrate awareness of project management and business practices;
5. (to) engage in independent life-long learning;

**Job opportunities**

MSc graduates will have a good basic engineering background and sound mathematical training, enabling them to work as part of design teams to develop mathematical models and to perform simulations, analyze data, perform risk analysis and solve optimization problems.

(Likely) Employment opportunities for graduates are with manufacturing industries, consulting firms, banks and insurance companies, computer companies, engineering design studios specialized in simulation, and research laboratories.

**Typical employers**

- Industrial goods manufacturing companies
- IT Companies
- Environmental agencies
- Biomedical industries
- Design studios that manage with complex civil engineering, structural design and mechanical sysems.
- Engineering companies specialized in simulations
- Research centres and laboratories.

**Final Project/Master Thesis:**

The Final master project consists of:

1. Establishing strategies to formulate industrial problems in terms of mathematical problem.
2. Identifying solution methods, implementing and designing experiments for validation.

The theme of the Master's Thesis will be chosen from proposals submitted by the partner companies of the Master (that will be in addition to specific session's mathematical modeling, as described above) and the bids submitted by program instructors and endorsed by the Academic Committee of the Master.

Under the development of Master's Thesis (outside the personal work supervised by a Professor of the Master) students will participate in (some of) the following activities:

1. Seminars and briefing to develop mathematical models of industrial problems.
2. Seminars and (project methodology, concerning) design meeting to develop projects in the framework of industrial mathematics and in specific domains (for example, software development projects)
3. Practices in enterprises

**Tutorial action plan**: a system of personalized tutoring students will be established. The Master's Academic Committee will assign students' academic tutors.

**Internships**: a program of external practices in companies optional for students in the program will be promoted.

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**Mathematical Engineering master program**

| # | Title of the course | ECTS | Semester |
|---|---|---|---|
| | **National standards compulsory disciplines** | **17** | 1, 2 |
| The list of disciplines developed for Mathematical Engineering within ECCUM project | | | |
| 1 | Numerical Methods For PDE | 4 | 3 |
| 2 | Mechanics Of Solids | 5 | 3 |
| 3 | Introduction To Continuum Mechanics | 5 | 1 |
| 4 | Fluid Mechanics | 5 | 3 |
| 5 | Numerical Methods For ODE | 4 | 2 |
| 6 | Numerical Methods For Pde | 4 | 3 |
| 7 | Optimization And Operation Research | 4 | 3 |
| 8 | Programming Languages | 3 | 2 |
| 9 | General Numerical Methods | 6 | 2 |
| 10 | Partial Differential Equations | 4 | 1 |
| 11 | Statistics For Business Management/<br>Statistics And Probability | 3<br>3 | 3<br>3 |
| | **Total** | **47** | |
| | | | |
| | **Compulsory research work** of a master students, teaching practice, writing and defense of a master's thesis, final examination | **56** | Distributed within semesters |
| **TOTAL** | | **120** | |

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**NUMERICAL METHODS FOR PDE**

**Description**

The course is an introduction to some general-purpose methods for the numerical treatment of partial differential equations which model several phenomena of engineering interest. Particular emphasis will be given to the finite element methods, spectral methods and finite volume methods. The tutorials carried out during the course are designed to train the student to the use of mathematical software for solving problems governed by partial differential equations.

**Program**

Classification of PDEs (Elliptic, Parabolic, Hyperbolic)
Finite Difference Methods for the Poisson problem: 1D, 2D
Stability, consistency and convergence of Finite Difference Methods
Finite Element methods for the Poisson problem: 1D, 2D
Stability, consistency and convergence of Finite Element Methods
Stabilization methods for convection and reaction therms
Finite Difference and Finite Element Methods for Parabolic problems
Conservation laws and Finite Volume methods
Training in implementing some of the methods and in using some code with some practical potentiality.

**Skills and learning outcome**

It is expected that the student will acquire the ability to solve problems in new or unfamiliar environments and within broader contexts, including multidisciplinary research and development in the business environment. That he will be able to find the appropriate model and the most effective numerical method to face the specific professional tasks. It is expected that he will be also able to select a set of numerical techniques, languages and tools, appropriate to make practically effective a certain mathematical model. Another key outcome is that the student will acquire the skill to choose, use and handle the most suitable professional software tools (both commercial and free) for the simulation of specific processes in the industrial and business sector.

**MECHANICS OF SOLIDS**

**Description**

The Solid mechanics studies the behavior of solid materials, especially their motion and deformation under the action of forces, temperature changes, phase changes, and other external or internal agents. Solid mechanics is fundamental for civil, aerospace, nuclear, and mechanical engineering, for geology, and for many branches of physics such as materials science. One of the most common practical applications of solid mechanics is the Euler-Bernoulli beam equation.

**Program**

Linear elastodynamic equations.
Stresses and strains.
Strain tensor.
General methods of resolution in linear elasticity.
Plane problems in linear elasticity.
Axially and spherically symmetric problems.
Bending and torsion of cylindrical beams.
One-dimensional beam models.
Plate models.
Vibrations.
Behaviour laws in elasticity.
Viscoelasticity, plasticity, viscoplasticity.
Nonlinear boundary conditions.
An introduction to fracture mechanics.

**Skills and learning outcome**

It is expected that the student will acquire good understanding the theory, concepts, principles and governing equations of solid mechanics; that the student will have the ability to idealize a complicated practical problem. It is expected that he will be also able to use the contemporary analytical, experimental and computational tools needed to solve the idealized problem; to interpret the results of these solutions, and to use these solutions to guide a corresponding design, manufacture, or failure analysis. Another key outcome is that the student will acquire develop interpersonal understanding, teamwork and communication skills working on group assignments, he will be able to learn independently new solutions, principles and methods, read and understand professional articles on the subject.

**INTRODUCTION TO CONTINUUM MECHANICS**

**Description**

The course introduces students to the general analytic machinery of tensor calculus; to variational principles and conservation laws so that the students are able to formulate governing equations. The course teaches exact, approximate and numerical methods to solve the resulting equations. Particular emphasis will be given to thermodynamic systems or bodies of matter and radiation, and the fundamental concepts of the mechanics of deformable media. The course will develop a more in-depth understanding of analytical techniques employed in continuum solid mechanics with particular emphasis on the response of elastic and plastic bodies.

**Program**

Vector space, Cauchy-Schwarz inequality, and Triangle Inequality
Dot product, Cross product, Outer product, Kronecker delta, Permutation symbol
Definition of tensor, Summation convention, Free index, Dummy index
Examples to understand notations, Operations on second-order tensors (SOT)
Cofactor tensor Invariants of SOT, Inverse of SOT
Eigenvalues and Eigenvectors, Geometric interpretation of eigenvectors, Cayley-Hamilton theorem
Skew-symmetric, Orthogonal, and Symmetric tensors
Additive decomposition, Polar decomposition, Square root tensor
Calculus of tensors
Kinematics
Leibniz rule of integration, Transport theorems
Cauchy hypothesis and Cauchy theorem, Equation of motion
Angular momentum balance
Equation of motion in material coordinates, Piola-Kirchhoff stress tensor
Energy balance
Second law of thermodynamics
Principle of material frame-indifference
Constitutive equations
Linear elasticity
Fluid mechanics

**Skills and learning outcome**

It is expected that the student will be able to explain central terms as material volume, particle and deformation tensor; the difference between Eulerian and Lagrangian definition of the equations of motion; that the student will be able to derive conservation laws for mass, momentum, and energy on local and global form. It is expected that he will be also able to give models for simple motion in ideal and viscous fluids and analyse them; to explain the model of linear elasticity. Another key outcome is that the student will be able to use the second law of thermodynamics for thermodynamic systems or bodies of matter

**FLUID MECHANICS**

**Description**

Fluid mechanics studies the mechanics of fluids (liquids, gases, and plasmas) and the forces on them. Fluid mechanics has a wide range of applications, including for mechanical engineering, chemical engineering, geophysics, astrophysics, and biology. Fluid mechanics can be divided into fluid statics, the study of fluids at rest; and fluid dynamics, the study of the effect of forces on fluid motion. The course aims at introducing students to the fundamentals fluid mechanics and its applications in process engineering.

**Program**

Fundamental models of fluid dynamics.
Incompressible inviscid flows.
Incompressible viscous flows.
Turbulent flows.
Convective heat transfer.

**Skills and learning outcome**

It is expected that by the end of the course the student will be able to solve fluid flow problems and design of pipeline and equipment for fluid transport; the student will understand the incompressible inviscid flows, incompressible viscous flows, convective heat transfer, and apply the basic principles and analysis of both static and dynamic fluid systems, and perform design calculations on engineering fluid systems. The student will be able to apply various mathematical methods to solve the resulting problems, and be able to interpret the mathematical results physically.

# NUMERICAL METHODS FOR ODE

## Description

Numerical methods for ordinary differential equations are methods used to find numerical approximations to the solutions of ordinary differential equations (ODEs). Their use is also known as "numerical integration", although this term is sometimes taken to mean the computation of integrals.

## Program

The theory of difference schemes.
Basic concepts of the theory of difference schemes.
Methods of constructing difference schemes.
Investigation of the stability of difference schemes Cauchy Problem, Numerical Methods for Ordinary Differential Equations.
Taylor-expansion method.
Euler's method. Runge-Kutta methods.
Multi-step methods for the solution of the Cauchy problem.
Extrapolation formula. Interpolation formula.
Investigation of the stability of one-step and multi-step methods.
Numerical methods for solving boundary problems for ordinary differential equations. Formulation of the problem. Approximation of boundary value problem.
Methods of solving the grid boundary value problem

## Skills and learning outcome

It is expected that by the end of the course the student should be able to apply several of the most important numerical methods for solving initial value problems and boundary value problems; derive some of the simpler methods from first principles; the student will know the strengths and weaknesses of the various methods and be able to decide which ones are appropriate for a particular problem; he/she will be able to decide whether a numerical method is stable and to give simple error estimates; write a program in C++ to implement various algorithms for the solution of differential equations. The student will be able to solve systems of linear equations by direct methods and use iterative methods to solve systems of non-linear equations.

# NUMERICAL METHODS FOR PDE

## Description

This course is an advanced introduction to applications and theory of numerical methods for solution of differential equations. In particular, the course focuses on physically-arising partial differential equations, with emphasis on the fundamental ideas underlying various methods.

## Program

Introduction to the numerical methods for the resolution of Differential Equations: finite differences, finite elements, finite volumes.
Methods of finite differences and finite elements in one dimensional problems.
Methods of finite differences and finite elements in several dimensions: elliptical, parabolic and hyperbolic problems.
Interactive classes using COMSOL-MULTIPHYSICS.

## Skills and learning outcome

It is expected that by the end of the course the student should be able to apply the acquired knowledge and abilities to solve problems in new or unfamiliar environments within broader contexts, including the ability to integrate multidisciplinary R & D in the business environment. The student will have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way; he/she will have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies. The student will be able to select or use how to handle most suitable professional software tools (both commercial and free) for the simulation of processes in the industrial and business sector.

# OPTIMIZATION AND OPERATION RESEARCH

## Description

The course introduces students to the fundamentals of Linear Programming. Particular emphasis will be given to constructing the linear optimization models and full analysis of model for sensitivity. It teaches to understand the initial and dual problem. The tutorials carried out during the course are designed to train the student to apply various mathematical methods to solve the resulting problems, and be able to interpret the mathematical results physically and economically.

## Program

The Linear Programming problems. The mathematical formulation of the linear programming problem. Properties of solutions of a linear programming problem.
Construction of linear optimization models. The problem of resource allocation. Graphical method for solving the problem.
Sensitivity analysis to a change in the right sides of restrictions. Analysis of resource values.
Simplex method.
Sensitivity analysis to a change in the right sides of restrictions.
A full analysis of model for sensitivity.
Initial and dual problem. The solution of the dual problem.
The compact matrix inversion methods. Penalty method. The area of applicability. Properties of convergence.
Methods for solving Transport problems.
Tasks of production planning and storage.
Construction of the basis plan. Least cost method.
Finding Initial Basic Feasible Solution. North-west corner method. Optimization of the basic plan.
Solving LP by Excel.
Non-linear Problems of the Operation Research.
Integer programming problems. The first algorithm of Gomory.
The problem of convex programming. The Lagrange multiplier rule.
Network problem. Seasonal fluctuations in consumer demand and power suppliers.
Optimization of dynamic systems on the basis of the maximum principle.

## Skills and learning outcome

It is expected that by the end of the course the student will be able to solve problems with the practically acceptable result; the student will develop skills in full analysis of the model for sensitivity; they will develop skills in mathematical analysis of applied management and the ability to independently understand the mathematics.

**PROGRAMMING LANGUAGES**

**Description**

The course covers the fundamental computational concepts underlying most programming languages; a range of problem solving techniques using computers; it emphasizes the role of programming within the overall software development process; it helps the students to acquire attitudes and working practices appropriate for a professional programmer. Special focus is on the most common features in MATLAB; and how to use MATLAB to solve engineering problems.

**Program**

The C++ programming language
Object Oriented Programming in C++.
Standard Template Library (STL).
Introduction to Programming Matlab

**Skills and learning outcome**

It is expected that by the end of the course the students will have knowledge of techniques for solving problems; basic computational concepts and elementary data structures; the edit-compile-link-run cycle from a user point of view; testing strategies; the main activities of software development and their interactions, and some of the major problems of software development. The course will enable the students to hand-execute simple programs, showing how input data is processed, output data is produced, and how the values of internal variables change; explain at various levels the behaviour of fragments of programming language code; amend existing programs to adjust or correct their functionality; translate well-structured plans into working programs; analyse simple problems involving text, numbers and graphics. The students will develop special professional attitude of a programmer to solving problems and manipulating programming language code.

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# GENERAL NUMERICAL METHODS

## Description

The course provides studying the numerical methods for solving problems, mastering of methodological approaches of numerical calculations development; it trains in the methods for solving research and applied tasks; much attention is given to study of the problem solving methods based on the application of special software (MATLAB). The tutorials help the students to develop practical skills in the use of numerical methods, including using software

## Program

The theory of errors.
Numerical methods of linear algebra.
Numerical methods for solving nonlinear equations.
Numerical methods for solving nonlinear equations.
Interpolation and extrapolation
The numerical solution of optimization problems.

## Skills and learning outcome

It is expected that by the end of the course the students will know - the basics of the theory of error and the approximation theory; the fundamental principles of mathematical modeling; the numerical methods for solving problems of algebra; the methods of numerical integration and differentiation. The students will be able to formulate the problem and find the ways to solve it; classify and select the numerical methods; develop the algorithms of numerical methods and implement them in practice by means of software products; analyze and evaluate the problem solutions. The course will empower the students to apply computational methods and software resources to solve different tasks of industry, to evaluate in practice the accuracy of the results.

# PARTIAL DIFFERENTIAL EQUATIONS

## Description

The course provides basic training in the field of partial differential equations, applied in mechanics, physics, engineering, and mastery of analytical and numerical methods for solving boundary value problems. The basic tenets of the theory of partial differential equations will be presented. Special focus will be on developing the students' ability to build a mathematical model of a physical phenomenon, and defining the approach, its application to modelling and solving mechanical problems, crafts and physical content.

## Program

Classification of differential equations in partial derivatives

Basic concepts and definitions in the theory of linear differential equations.

Classification of linear second order differential equations and reducing them to the canonical form. The basic equations of mathematical physics.

The Cauchy problem. Formulation of the Cauchy problem. The correctness of the Cauchy problem. Stability of solutions. The existence and uniqueness of solutions

Methods for solving differential equations in partial derivatives

Fourier method. The general scheme of the method of separation of variables

Fourier method for equations of string vibration and heat equations.

Equations of hyperbolic type. The Cauchy problem for the wave equation. Boundary value problems for the wave equation.

Equations of parabolic type. The solution of the Cauchy problem for the heat equation. Poisson formula.

Equations of elliptic type. Statement of the main boundary value problems. The equation of Laplace and Poisson. The Green's function.

## Skills and learning outcome

It is expected that as a result of studying the course the student should know the basic concepts of the theory of differential equations; methods for solving basic differential equations; setting the basic boundary value problems for elliptic, parabolic and hyperbolic equations. It will enable him/her to determine the type of the equation, find solutions to boundary value problems, apply the equations for modelling physical processes, find solutions to the Cauchy problem for equations of hyperbolic and parabolic, derive wave and heat equations. The student will apply the methods of derivation of the equations on the basis of the laws of physical phenomena; the methods for solving differential equations in partial derivatives of various types**.**

**STATISTICS FOR BUSINESS MANAGEMENT**

**Description**

This course provides an elementary introduction to probability and statistics with applications. Topics include: basic combinatorics, random variables, probability distributions, Bayesian inference, hypothesis testing, confidence intervals, and linear regression.

**Program**

Descriptive statistics: frequency distribution; mean, median and mode; percentiles; variance and standard deviation; introduction to regression.
Probability: definition and basic properties of random variables; probability distributions; expected value and variance; transformations of random variables; normal distribution.
Sampling and Estimation: random sampling; polls; types of samples; distribution of sample
Estimates; central limit theorem; point estimates; accuracy and precision; bias; constructing confidence intervals; hypothesis tests, p-values.
Regression analysis: simple linear regression; estimation of parameters and significance testing; correlation coefficient; interpreting the regression output; multiple linear regression; dummy variables; model selection; examples of non-linear regression; applications.

**Skills and learning outcome**

By the end of the course, the students will know exploratory data analysis, sampling distribution of a statistic and behavior of sample mean, statistical inference involving confidence intervals and hypothesis testing, statistical modeling of data in the context of regression analysis. The students will be able to use statistical methods in collecting data, to summarize data and draw inferences and conclusions on the basis of data, to apply fundamental concepts in exploratory data analysis, to use appropriate software tool for data analysis, and to use statistical models to represent the reality.

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**STATISTICS AND PROBABILITY**

## Description

The discipline deals with the branches of mathematics concerned with the laws governing random events, including the collection, analysis, interpretation, and display of numerical data.

## Program

Descriptive statistics: frequency distribution; mean, median and mode; percentiles; variance and standard deviation; introduction to regression.
Probability: definition and basic properties of random variables; probability distributions; expected value and variance; transformations of random variables; normal distribution.
Sampling and Estimation: random sampling; polls; types of samples; distribution of sample estimates; central limit theorem; point estimates; accuracy and precision; bias; constructing confidence intervals; hypothesis tests, p-values.
Regression analysis: simple linear regression; estimation of parameters and significance testing; correlation coefficient; interpreting the regression output; multiple linear regression; dummy variables; model selection; examples of non-linear regression; applications.

Statistical software package R is used in a class. R is available to download for free for both Macs and PCs (and on Unix) here: http://cran.us.r-project.org/.

## Skills and Learning outcomes

It is expected that by the end of the course students will know: Exploratory data analysis, Sample design, Basic concepts of probability and random variables, Sampling distribution of a statistic and behaviour of sample means, Statistical inference involving confidence intervals and hypothesis testing, Statistical modelling of data in the context of regression analysis. Students will develop the following abilities: ability to use statistical methods in collecting data, summarizing data and drawing inferences and conclusions on the basis of data, ability to apply fundamental concepts in exploratory data analysis, ability to design studies for obtaining data whilst avoiding common design flows, ability to use appropriate software tool for data analysis, ability to use statistical models to represent the reality

**OPTIMIZATION AND OPERATIONS RESEARCH**

**Description**

The discipline aims at iintroducing students to the fundamentals of Linear Programming; students will be able to construct the linear optimization models, to get full analysis of model for sensitivity, understand the initial and dual problems, understand the properties of convergence and computing procedures requirements. The discipline will cover the issues of applying various mathematical methods to solve the resulting problems, and interpreting the mathematical results physically and economically.

**Program**

1. The Linear Programming problems
1.1. The mathematical formulation of the linear programming problem. Properties of solutions of a linear programming problem.
1.2. Construction of linear optimization models. The problem of resource allocation. Graphical method for solving the problem.
1.3. Sensitivity analysis to a change in the right sides of restrictions. Analysis of resource values. (Graphical method)
1.4. Simplex method.
1.5. Sensitivity analysis to a change in the right sides of restrictions. Analysis of resource values. (Simplex method)
1.6. A full analysis of model for sensitivity.
1.7. Initial and dual problem. The solution of the dual problem.
1.8. The compact matrix inversion methods. Penalty method. The area of applicability. Properties of convergence. Computing procedures requirements.
2. Methods for solving Transport problems.
2.1. Tasks of production planning and storage.
2.2. Construction of the basis plan. Least cost method.
2.3. Finding Initial Basic Feasible Solution. North-west corner method. Optimization of the basic plan. Solving LP by Excel.
3. Non-linear Problems of the Operation Research.
3.1. Integer programming problems. The first algorithm of Gomory.
3.2. The problem of convex programming. The Lagrange multiplier rule.
3.3. Network problem. Seasonal fluctuations in consumer demand and power suppliers. Capacity constraints.
3.4. Optimization of dynamic systems on the basis of the maximum principle.

**Skills and Learning outcomes**

It is expected that by the end of the course students will acquire a good knowledge of problem solving with bringing solutions till the practically acceptable result; develop skills in full analysis of the model for sensitivity; develop skills in mathematical analysis of applied management and the ability to independently understand the mathematics, develop skills in mathematical analysis of applied management and the ability to independently understand the mathematics