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Introduction

Physics is one of the most fundamental of the sciences. Scientists of all disciplines use the ideas of physics, including chemists who study the structure of molecules, paleontologists who try to reconstruct how dinosaurs walked, and climatologists who study how human activities affect the atmosphere and oceans. Physics is also the foundation of all engineering and technology. No engineer could design a flat-screen TV, an interplanetary spacecraft, or even a better mousetrap without first understanding the basic laws of physics. The study of physics is also an adventure. You will find it challenging, sometimes frustrating, occasionally painful, and often richly rewarding. If you've ever wondered why the sky is blue, how radio waves can travel through empty space, or how a satellite stays in orbit, you can find the answers by using fundamental physics. You will come to see physics as a towering achievement of the human intellect in its quest to understand our world and ourselves.

We'll discuss the nature of physical theory and the use of idealized models to represent physical systems. We'll introduce the systems of units used to describe physical quantities and discuss ways to describe the accuracy of a number. We'll look at examples of problems for which we can't (or don't want to) find a precise answer, but for which rough estimates can be useful and interesting. Finally, we'll study several aspects of vectors and vector algebra. Vectors will be needed throughout our study of physics to describe and analyze physical quantities, such as velocity and force, that have direction as well as magnitude.

Physics is an *experimental* science. Physicists observe the phenomena of nature and try to find patterns that relate these phenomena. These patterns are called physical theories or, when they are very well established and widely used, physical laws or principles.

To develop a physical theory, a physicist has to learn to ask appropriate questions, design experiments to try to answer the questions, and draw appropriate conclusions from the results.

The development of physical theories often takes an indirect path, with blind alleys, wrong guesses, and the discarding of unsuccessful theories in favor of more promising ones. Physics is not simply a collection of facts and principles; it is also the *process* by which we arrive at general principles that describe how the physical universe behaves.

No theory is ever regarded as the final or ultimate truth. The possibility always exists that new observations will require that a theory be revised or discarded. It is in the nature of physical theory that we can disprove a theory by finding behaviour that is inconsistent with it, but we can never prove that a theory is always correct.

Getting back to Galileo, suppose we drop a feather and a cannonball. They certainly do *not* fall at the same rate. This does not mean that Galileo was wrong; it means that his theory was incomplete. If we drop the feather and the cannonball *in*

a vacuum to eliminate the effects of the air, then they do fall at the same rate. Galileo's theory has a **range of validity:** It applies only to objects for which the force exerted by the air (due to air resistance and buoyancy) is much less than the weight. Objects like feathers or parachutes are clearly outside this range.

Often a new development in physics extends a principle's range of validity. Galileo's analysis of falling bodies was greatly extended half a century later by Newton's laws of motion and law of gravitation.

Topic 1 MECHANICS

1.1 Motion along a straight-line. Motion in a circle

1.1.1 Average velocity and instantaneous velocity

Suppose a drag racer drives her AA-fuel dragster along a straight track (see fig. 1). To study the dragster's motion, we need a coordinate system. We choose the x-axis to lie along the dragster's straight-line path, with the origin O at the starting

line. We also choose a point on the dragster, such as its front end, and represent the entire dragster by that point. Hence we treat the dragster as a **particle**.



Figure 1 – Positions of dragster at two times during its run

A useful way to describe the motion of the particle that represents the dragster is in terms of the change in the particle's coordinate *x* over a time interval. Suppose that 1.0 s after the start the front of the dragster is at point P_1 , 19 m from the origin, and 4.0 s after the start it is at point P_2 , 277 m from the origin. The *displacement* of the particle is a vector that points from P_1 to P_2 . Figure 1 shows that this vector points along the *x*-axis. The *x*-component of the displacement is the change in the value of *x*, (277 m - 19 m) = 258 m, that took place during the time interval of (4.0 - 1.0) = 3.0 s. We define the dragster's **average velocity** during this time interval as a *vector* quantity whose *x*-component is the change in *x* divided by the time interval: (258 m)/(3.0 s) = 86 m/s.

In general, the average velocity depends on the particular time interval chosen. For a 3.0-s time interval *before* the start of the race, the average velocity would be zero because the dragster would be at rest at the starting line and would have zero displacement.

Let's generalize the concept of average velocity. At time t_1 the dragster is at

point P_1 with coordinate x_1 and at t_2 time it is at point P_2 with coordinate x_2 . The displacement of the dragster during the time interval from t_1 to t_2 is the vector from P_1 to P_2 . The x-component of the displacement, denoted Δx , is the change in the coordinate x:

$$\Delta x = x_2 - x_1 \tag{1}$$

The dragster moves along the *x*-axis only, so the *y*- and *z*-components of the displacement are equal to zero.

The x-component of average velocity, or **average x-velocity**, is the xcomponent of displacement, Δx , divided by the time interval Δt during which the displacement occurs. We use the symbol v_{av-x} for average x-velocity (the subscript "av" signifies average value and the subscript x indicates that this is the x-component):

$$v_{av-x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$
(2)

As an example, for the dragster $x_1 = 19 m$, $x_2 = 277 m$, $t_1 = 1.0 s$ and $t_2 = 4.0 s$, so Eq. (2) gives

$$v_{av-x} = \frac{277 \ m - 19 \ m}{4.0 \ s - 1.0 \ s} = \frac{258 \ m}{3.0 \ s} = 86 \ m/s$$

The average x-velocity of the dragster is positive. This means that during the time interval, the coordinate x increased and the dragster moved in the positive x-direction (to the right in Fig. 1).



Figure 2 – Positions of an official's truck at two times during its motion. The points P_1 and P_2 now indicate the positions of the truck, and so are reverse of Fig.1

If a particle moves in the *negative x*-direction during a time interval, its average velocity for that time interval is negative. For example, suppose an official's truck moves to the left along the track (see fig. 2). The truck is $x_1 = 277 m$ at $t_1 = 16.0 st_1$ at and is at $x_2 = 19 m$ at $t_2 = 25.0 s$. Then $\Delta = (19 m - 277m) = -258 m$ and $\Delta t = (25.0 s - 16.0 s) = 9.0 s$. The x-component of average velocity is $v_{av-x} = \frac{\Delta x}{\Delta t} = (-258 m)/(9.0 s) = -29 m/s$.

Sometimes the average velocity is all you need to know about a particle's motion. For example, a race along a straight line is really a competition to see whose average velocity v_{av-x} , has the greatest magnitude. The prize goes to the competitor who can travel the displacement from the start to the finish line in the shortest time interval, Δt (see fig. 3).



Figure 3 – The winner of a 50-m swimming race is the swimmer whose average velocity has the greatest magnitude – that is, the swimmer who traverses a displacement Δx of 50 m in the shortest elapsed time Δt

But the average velocity of a particle during a time interval can't tell us how fast, or in what direction, the particle was moving at any given time during the interval. To do this we need to know the **instantaneous velocity**, or the velocity at a specific instant of time or specific point along the path.

To find the instantaneous velocity of the dragster in Fig. 1 at the point P_1 , we move the second point P_2 closer and closer to the first point P_1 and compute the average velocity v_{av-x} over the ever-shorter displacement and time interval. Both Δx and Δt become very small, but their ratio does not necessarily become small. In the language of calculus, the limit of $\Delta x/\Delta t$ as Δt approaches zero is called the **derivative** of x with respect to t and is written dx/dt. The instantaneous velocity is the limit of the average velocity as the time interval approaches zero; it equals the instantaneous rate of change of position with time. We use the symbol with no "av" subscript, for the instantaneous velocity along the x-axis, or the **instantaneous x-velocity**:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$
(3)

The time interval Δt is always positive, so v_x has the same algebraic sign as Δx . A positive value of v_x means that x is increasing and the motion is in the positive x-direction; a negative value of v_x means that x is decreasing and the motion is in the negative x-direction. A body can have positive x and negative v_x or the reverse; x tells us where the body is, while v_x tells us how it's moving (see fig. 4).



Figure 4 – Even when he's moving forward, this cyclist's instantaneous x-velocity can be negative – if he's travelling in the negative x-direction. In any problem, the choice of which direction is positive and which is negative is entirely up to you

Instantaneous velocity, like average velocity, is a vector quantity; Eq. (3) defines its x-component. In straight-line motion, all other components of instantaneous velocity are zero. In this case we often call v_x simply the instantaneous velocity. When we use the term "velocity," we will always mean instantaneous rather than average velocity.

The terms "velocity" and "speed" are used interchangeably in everyday language, but they have distinct definitions in physics. We use the term **speed** to denote distance traveled divided by time, on either an average or an instantaneous basis. Instantaneous *speed*, for which we use the symbol v with *no* subscripts, measures how fast a particle is moving; instantaneous *velocity* measures how fast *and* in what direction it's moving. Instantaneous speed is the magnitude of instantaneous velocity and so can never be negative. For example, a particle with

instantaneous velocity $v_x = 25 m/s$ and a second particle with $v_x = -25 m/s$ are moving in opposite directions at the same instantaneous speed 25 m/s.

1.1.2 Average and instantaneous acceleration

Just as velocity describes the rate of change of position with time, *acceleration* describes the rate of change of velocity with time. Like velocity, acceleration is a vector quantity. When the motion is along a straight line, its only nonzero component is along that line. As we'll see, acceleration in straight-line motion can refer to either speeding up or slowing down.

Let's consider again a particle moving along the x-axis. Suppose that at time t_1 the particle is at point P_1 and has x-component of (instantaneous) velocity v_{1x} and at a later time t_2 it is at point P_2 and has x-component of velocity v_{2x} . So the x-component of velocity changes by an amount $\Delta v_x = v_{2x} - v_{1x}$ during the time interval $\Delta t = t_2 - t_1$.

We define the **average acceleration** of the particle as it moves from P_1 to P_2 to be a vector quantity whose *x*-component a_{av-x} (called the **average** *x***-acceleration**) equals Δv_x , the change in the *x*-component of velocity, divided by the time interval Δt :

$$a_{av-x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$
(4)

For straight-line motion along the *x*-axis we will often call simply the average acceleration. If we express velocity in meters per second and time in seconds, then average acceleration is in meters per second per second, or This is usually written as and is read "meters per second squared."

The upper part of Fig. 5 is our graph of the x-velocity as a function of time. On this $v_x - t$ graph, the slope of the line connecting the endpoints of each interval is the average x-acceleration $a_{av-x} = \Delta v_x / \Delta t$ for that interval. The four slopes (and thus the *signs* of the average accelerations) are, respectively, positive, negative, negative, and positive. The third and fourth slopes (and thus the average accelerations themselves) have greater magnitude than the first and second.

We can now define **instantaneous acceleration** following the same procedure that we used to define instantaneous velocity. As an example, suppose a race car driver is driving along a straightaway. To define the instantaneous acceleration at point P_1 we take the second point P_2 to be closer and closer to so that the average acceleration is computed over shorter and shorter time intervals. *The instantaneous acceleration is the limit of the average acceleration as the time interval approaches zero.* In the language of calculus, *instantaneous acceleration equals the derivative of velocity with time.* Thus

$$a_x = \lim_{\Delta x \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$
(5)



Figure 5 – Our graphs of *x*-velocity versus time (top) and average *x*-acceleration versus time (bottom)

Note that in Eq. (5) is really the *x*-component of the acceleration vector, or the **instantaneous** *x*-acceleration; in straight-line motion, all other components of this vector are zero. From now on, when we use the term "acceleration," we will always mean instantaneous acceleration, not average acceleration.

1.1.3 Motion with constant acceleration

The simplest kind of accelerated motion is straight-line motion with *constant* acceleration. In this case the velocity changes at the same rate throughout the motion. As an example, a falling body has a constant acceleration if the effects of the air are not important. The same is true for a body sliding on an incline or along a rough horizontal surface, or for an airplane being catapulted from the deck of an aircraft carrier.

Figure 6 is a motion diagram showing the position, velocity, and acceleration for a particle moving with constant acceleration. Figures 7 and 8 depict this same motion in the form of graphs. Since the *x*-acceleration is constant, the a_x -t graph (graph of *x*-acceleration versus time) in Fig. 7 is a horizontal line.

The graph of *x*-velocity versus time, or v_x -*t* graph, has a constant *slope* because the acceleration is constant, so this graph is a straight line (see fig. 8).







Figure 7 – An acceleration-time (a_x-t) for straight-line motion with constant positive *x*-acceleration a_x



Figure 8 – A velocity-time (v_x -t) graph for straight-line motion with constant positive *x*-acceleration a_x . The initial *x*-velocity v_{0x} is also positive in this case

When the x-acceleration a_x is constant, the average x-acceleration a_{av-x} for any time interval is the same as a_x . This makes it easy to derive equations for the position x and the x-velocity v_x as functions of time. To find an expression for v_x we first replace a_{av-x} in Eq. (2.4) by a_x :

$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \tag{6}$$

Now we let $t_1 = 0$ and let t_2 be any later time *t*. We use the symbol v_{0x} for the *x*-velocity at the initial time t = 0; the *x*-velocity at the later time *t* is v_x . Then Eq. (6) becomes

$$a_x = \frac{v_x - v_{0x}}{t - 0} \tag{7}$$

or

$$v_x = v_{0x} + a_x t \tag{8}$$

In Eq. (8) the term $a_x t$ is the product of the constant rate of change of x-velocity, a_x and the time interval t. Therefore it equals the *total* change in x-velocity from the initial time t = 0 to the later time t. The x-velocity at any time t

then equals the initial x-velocity v_{0x} (at t = 0) plus the change in x-velocity $a_x t$ (see fig. 8).

Equation (8) also says that the change in x-velocity $v_x - v_{0x}$ of the particle between t = 0 and any later time t equals the *area* under $a_x - t$ the graph between those two times. You can verify this from Fig. 7: Under this graph is a rectangle of vertical side a_x , horizontal side t, and area $a_x t$. From Eq. (8) this is indeed equal to the change in velocity $v_x - v_{0x}$ Next we'll show that even if the x-acceleration is not constant, the change in x-velocity during a time interval is still equal to the area under the $a_x - t$ curve, although in that case Eq. (8) does not apply.

Next we'll derive an equation for the position x as a function of time when the x-acceleration is constant. To do this, we use two different expressions for the average x-velocity v_{av-x} during the interval from to any later time t. The first expression comes from the definition of v_{av-x} Eq. (2), which is true whether or not the acceleration is constant. We call the position at time t = 0 the *initial position*, denoted by x_0 . The position at the later time t is simply x. Thus for the time interval $\Delta t = t - 0$ the displacement is $\Delta x = x - x_0$ and Eq. (2) gives

$$v_{av-x} = \frac{x - x_0}{t} \tag{9}$$

We can also get a second expression for v_{av-x} that is valid only when the *x*-acceleration is constant, so that the *x*-velocity changes at a constant rate. In this case the average *x*-velocity for the time interval from 0 to *t* is simply the average of the *x*-velocities at the beginning and end of the interval:

$$v_{av-x} = \frac{v_{0x} - v_x}{2}$$
(10)

(This equation is *not* true if the *x*-acceleration varies during the time interval.) We also know that with constant *x*-acceleration, the *x*-velocity v_x at any time *t* is given by Eq. (8). Substituting that expression for v_x into Eq. (10), we find

$$v_{av-x} = \frac{1}{2}(v_{0x} + v_{0x} + a_x t) = v_{0x} + \frac{1}{2}a_x t$$
⁽¹¹⁾

Finally, we set Eqs. (9) and (11) equal to each other and simplify:

$$v_{0x} + \frac{1}{2}a_x t = \frac{x - x_0}{t} \tag{12}$$

or

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
⁽¹³⁾

Here's what Eq. (13) tells us: If at time t = 0 a particle is at position x_0 and has x-velocity v_{0x} , its new position x at any later time t is the sum of three terms -

its initial position x_0 , plus the distance $v_{0x}t$ that it would move if its x-velocity were constant, plus an additional distance $\frac{1}{2}a_x t^2$ caused by the change in x-velocity.

A graph of Eq. (13) - that is, an x-t graph for motion with constant xacceleration (see fig. 9a) - is always a *parabola*. Figure 9b shows such a graph. The curve intercepts the vertical axis (x-axis) at x_0 the position at t = 0. The slope of the tangent at t = 0 equals v_{0x} , the initial x-velocity, and the slope of the tangent at any time t equals the x-velocity v_x at that time. The slope and x-velocity are continuously increasing, so the x-acceleration a_x is positive; you can also see this because the graph in Fig. 9b is concave up (it curves upward). If a_x is negative, the x-t graph is a parabola that is concave down (has a downward curvature).



Figure 9 – (a) Straight-line motion with constant acceleration. (b) A position-time (x-t) graph for this motion. For this motion the position x_0 , the initial velocity v_{0x} , and acceleration a_x are all positive

If there is zero x-acceleration, the x-t graph is a straight line; if there is a constant x-acceleration, the additional $\frac{1}{2}a_x t^2$ term in Eq. (13) for x as a function of t curves the graph into a parabola (see fig. 9a). We can analyze the $v_x - t$ graph in the same way. If there is zero x-acceleration this graph is a horizontal line (the x-velocity is constant); adding a constant x-acceleration gives a slope to the $v_x - t$ graph (see fig. 9b).



Figure 10 – (a) How a constant x-acceleration affects a body's (a) x-t graph and (b) v_x -t graph

Just as the change in x-velocity of the particle equals the area under the $a_x - t$ graph, the displacement—that is, the change in position—equals the area under the v_x -t graph. To be specific, the displacement $x - x_0$ of the particle between t = 0 and any later time t equals the area under the $v_x - t$ graph between those two times. In Fig. 8 we divide the area under the graph into a dark-colored rectangle (vertical side v_{0x} , horizontal side t, and area $v_x t$) and a light-colored right triangle (vertical side $a_x t$, horizontal side t, and area $\frac{1}{2}(a_x t)(t) = \frac{1}{2}a_x t^2$. The total area under the $v_x - t$ graph is

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \tag{14}$$

in agreement with Eq. (13).

The displacement during a time interval is always equal to the area under the $v_x - t$ curve. This is true even if the acceleration is *not* constant, although in that case Eq. (13) does not apply.

It's often useful to have a relationship for position, *x*-velocity, and (constant) x-acceleration that does not involve the time. To obtain this, we first solve Eq. (8) for *t* and then substitute the resulting expression into Eq. (13):

$$t = \frac{v_x - v_{0x}}{a_x} \tag{15}$$

$$x = x_0 + v_{0x} \left(\frac{v_x - v_{0x}}{a_x}\right) + \frac{1}{2} a_x \left(\frac{v_x - v_{0x}}{a_x}\right)^2$$
(16)

We transfer the term to the left side and multiply through by $2a_x$:

$$2a_x(x - x_0) = 2v_{0x}v_x - 2v_{0x}^2 - 2v_{0x}v_x + v_{0x}^2$$
(17)

Finally, simplifying gives us

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \tag{18}$$

We can get one more useful relationship by equating the two expressions for v_{av-x} Eqs. (9) and (10), and multiplying through by *t*. Doing this, we obtain

$$x - x_0 = \left(\frac{v_x - v_{0x}}{2}\right)t$$
(19)

Note that Eq. (19) does not contain the x-acceleration a_x . This equation can be handy when a_x is constant but its value is unknown.

Equations (8), (13), (18), and (19) are the *equations of motion with constant acceleration* (Table 1). By using these equations, we can solve *any* problem involving straight-line motion of a particle with constant acceleration.

For the particular case of motion with constant x-acceleration depicted in Fig. 6 and graphed in Figs. 7, 8, and 9, the values of x_0 , v_{0x} and a_x are all positive. We invite you to redraw these figures for cases in which one, two, or all three of these quantities are negative.

Equation	N⁰	Includes quantities			
$v_x = v_{0x} + a_x t$	(8)	t		v_x	a_x
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	(13)	t	x		a_x
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	(18)		x	v_x	a_x
$x - x_0 = \left(\frac{v_x - v_{0x}}{2}\right)t$	(19)	t	x	v_x	

Table 1 - Equations of motion with constant acceleration

1.1.4 Position, velocity and acceleration vectors

To describe the *motion* of a particle in space, we must first be able to describe the particle's *position*. Consider a particle that is at a point P at a certain instant. The **position vector** \vec{r} of the particle at this instant is a vector that goes from the origin of the coordinate system to the point P (see fig. 11). The Cartesian coordinates x, y, and z of point P are the x-, y-, and z-components of vector \vec{r} . Using the unit vectors, we can write

$$\vec{r} = x\vec{\iota} + y\vec{j} + z\vec{k} \tag{20}$$

During a time interval Δt the particle moves from P_1 , where its position vector is $\vec{r_1}$, to P_2 , where its position vector is $\vec{r_2}$. The change in position (the

displacement) during this interval is $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$. We define the **average velocity** \vec{v}_{av} during this interval, as the displacement divided by the time interval:

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$$
(21)

Dividing a vector by a scalar is really a special case of multiplying a vector by a scalar the average velocity \vec{v}_{av} is equal to the displacement vector $\Delta \vec{r}$ multiplied by $\frac{1}{\Delta t}$, the reciprocal of the time interval. Note that the *x*-component of Eq. (21) is $v_{av-x} = \frac{(x_2-x_1)}{(t_2-t_1)} = \Delta x/\Delta t$. This is just Eq. (2), the expression for average *x*-velocity that we found early for one-dimensional motion.



Figure 11 – The position vector \vec{r} from the origin to point P has x, y and z. The path that the particle follows through space is in general a curve (see fig. 12)

We now define **instantaneous velocity**: It is the limit of the average velocity as the time interval approaches zero, and it equals the instantaneous rate of change of position with time. The key difference is that position \vec{r} and instantaneous velocity \vec{v} are now both vectors:

$$\vec{\nu} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
(22)

The *magnitude* of the vector \vec{v} at any instant is the *speed* v of the particle at that instant. The *direction* \vec{v} of at any instant is the same as the direction in which the particle is moving at that instant.

Note that as $\Delta t \to 0$ points P_1 and P_2 in Fig. 12 move closer and closer together. In this limit, the vector $\Delta \vec{r}$ becomes tangent to the path. The direction of in this limit is also the direction of the instantaneous velocity \vec{v} . This leads to an important conclusion: At every point along the path, the instantaneous velocity vector is tangent to the path at that point (see fig. 13).



Figure 12 – The average velocity \vec{v}_{av} between points P_1 and P_2 has the same direction as the displacement $\Delta \vec{r}$



Figure 13 –The vectors \vec{v}_1 and \vec{v}_2 are the instantaneous velocities at the points P_2 and P_2 shown in Fig. 12

It's often easiest to calculate the instantaneous velocity vector using components. During any displacement $\Delta \vec{r}$, the changes Δx , Δy and Δz in the three coordinates of the particle are the *components* of $\Delta \vec{r}$. It follows that the components v_x , v_y and v_z of the instantaneous velocity \vec{v} are simply the time derivatives of the coordinates x, y, and z. That is,

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt}$$
(23)

Hence Eq. (23) is a direct extension of the idea of instantaneous velocity to motion in three dimensions.

We can also get Eq. (23) by taking the derivative of Eq. (20). The unit vectors \vec{i}, \vec{j} and \vec{z} are constant in magnitude and direction, so their derivatives are zero, and we find

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$
(24)

This shows again that the components of \vec{v} are $\frac{dx}{dt}$, $\frac{dy}{dt}$ and $\frac{dz}{dt}$.

The magnitude of the instantaneous velocity vector \vec{v} —that is, the speed—is given in terms of the components v_x , v_y and v_z by the Pythagorean relation:

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$
(25)

Figure 14 shows the situation when the particle moves in the xy-plane. In this case, z and v_z are zero. Then the speed (the magnitude of \vec{v}) is

$$v = \sqrt{v_x^2 + v_y^2} \tag{26}$$

and the direction of the instantaneous velocity \vec{v} is given by the angle α (the Greek letter alpha) in the figure. We see that

$$\tan \alpha = \frac{v_y}{v_x} \tag{27}$$

(We always use Greek letters for angles. We use for the direction of the instantaneous velocity vector to avoid confusion with the direction θ of the *position* vector of the particle.)

The instantaneous velocity vector is usually more interesting and useful than the average velocity vector. From now on, when we use the word "velocity," we will always mean the instantaneous velocity vector \vec{v} (rather than the average velocity vector). Usually, we won't even bother to call \vec{v} a vector; it's up to you to remember that velocity is a vector quantity with both magnitude and direction.



Figure 14 The two velocity components for motion in the xy-plane

Now let's consider the acceleration of a particle moving in space. Just as for motion in a straight line, acceleration describes how the velocity of the particle changes. But since we now treat velocity as a vector, acceleration will describe changes in the velocity magnitude (that is, the speed) and changes in the direction of velocity (that is, the direction in which the particle is moving).

In Fig. 15a, a car (treated as a particle) is moving along a curved road. The vectors \vec{v}_1 and \vec{v}_2 represent the car's instantaneous velocities at time t_1 , when the car is at point P_1 and at time t_2 when the car is at point P_2 . The two velocities may differ in both magnitude and direction. During the time interval from t_1 to t_2 , the vector change in velocity is $\vec{v}_2 - \vec{v}_1 = \Delta \vec{v}$, so $\vec{v}_2 = \vec{v}_1 + \Delta \vec{v}$ (see fig. 15b). We define the **average acceleration** \vec{a}_{av} of the car during this time interval as the velocity change divided by the time interval $t_2 - t_1 = \Delta t$:

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$
(28)



Figure 15 – (a) A car moving along a curved road from P_1 to P_2 . (b) How to obtain the change in velocity $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$ by vector subtraction. (c) The vector $\vec{a}_{av} = \Delta \vec{v} / \Delta t$ represents the average acceleration between P_1 and P_2

Average acceleration is a vector quantity in the same direction as the vector $\Delta \vec{v}$ (see fig. 15c).

As early we define the **instantaneous acceleration** \vec{a} (a vector quantity) at point P_1 as the limit of the average acceleration vector when point P_2 approaches point P_1 , so $\Delta \vec{v}$ and Δt both approach zero (see fig. 16). The instantaneous acceleration is also equal to the instantaneous rate of change of velocity with time:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{d\vec{v}}{dt} = \frac{d\vec{v}}{dt}$$
(29)

The velocity vector \vec{v} , as we have seen, is tangent to the path of the particle. The instantaneous acceleration vector \vec{a} , however, does not have to be tangent to the path. Figure 16a shows that if the path is curved \vec{a} , points toward the concave side of the path—that is, toward the inside of any turn that the particle is making. The acceleration is tangent to the path only if the particle moves in a straight line (see fig. 16b). (a) Acceleration: curved trajectory



direction of the trajectory.

Figure 16 – (a) Instantaneous acceleration at a point P_1 in fig. 15. (b) Instantaneous acceleration for motion along a straight line

To convince yourself that a particle has a nonzero acceleration when moving on a curved path with constant speed, think of your sensations when you ride in a car. When the car accelerates, you tend to move inside the car in a direction *opposite* to the car's acceleration. Thus you tend to slide toward the back of the car when it accelerates forward (speeds up) and toward the front of the car when it accelerates backward (slows down). If the car makes a turn on a level road, you tend to slide toward the outside of the turn; hence the car has an acceleration toward the inside of the turn.

We will usually be interested in the instantaneous acceleration, not the average acceleration. From now on, we will use the term "acceleration" to mean the instantaneous acceleration vector \vec{a} .

Each component of the acceleration vector is the derivative of the corresponding component of velocity:

$$a_x = \frac{dv_x}{dt}, a_y = \frac{dv_y}{dt}, a_z = \frac{dv_z}{dt}$$
(30)

In terms of unit vectors,

$$\vec{a} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j} + \frac{dv_z}{dt}\vec{k}$$
(31)

Figure 17 shows an example of an acceleration vector that has both x- and y- components.



Figure 17 – When arrow is released, its acceleration vector has both a horizontal component a_x and a vertical component a_y

Since each component of velocity is the derivative of the corresponding coordinate, we can express the components a_x , a_y and a_z of the acceleration vector as

$$a_x = \frac{d^2 x}{dt^2}, a_y = \frac{d^2 y}{dt^2}, a_z = \frac{d^2 z}{dt^2}$$
(32)

The acceleration vector itself is

$$\vec{a} = \frac{d^2 x}{dt^2} \vec{i} + \frac{d^2 y}{dt^2} \vec{j} + \frac{d^2 z}{dt^2} \vec{k}$$
(33)

1.1.5 Motion in a circle

When a particle moves along a curved path, the direction of its velocity changes. Ehis means that the particle *must* have a component of acceleration perpendicular to the path, even if its speed is constant (see Fig. 18b). In this section we'll calculate the acceleration for the important special case of motion in a circle.



Figure 18 – The effect of acceleration directed (a) parallel to and (b) perpendicular to a particle's velocity

When a particle moves in a circle with *constant speed*, the motion is called **uniform circular motion.** A car rounding a curve with constant radius at constant speed, a satellite moving in a circular orbit, and an ice skater skating in a circle with constant speed are all examples of uniform circular motion (see fig. 20c; compare Fig. 19a). There is no component of acceleration parallel (tangent) to the path; otherwise, the speed would change. The acceleration vector is perpendicular (normal) to the path and hence directed inward (never outward!) toward the center of the circular path. This causes the direction of the velocity to change without changing the speed.



Figure 19 – Velocity and acceleration vectors for a particle moving through a point *P* on a curved path with (a) constant speed, (b) increasing speed, (c) decreasing speed

(a) Car speeding up along a circular path

(b) Car slowing down along a circular path

(c) Uniform circular motion: Constant speed along a circular path

Component of acceleration parallel to velocity: Changes car's speed



Figure 20 – A car moving along a circular path. If the car is in uniform circular motion as in (c), the speed is constant and the acceleration is directed towards the center of the circular path (compare Fig. 19)

We can find a simple expression for the magnitude of the acceleration in uniform circular motion. We begin with Fig. 21a, which shows a particle moving with constant speed in a circular path of radius R with center at O. The particle moves from P_1 to P_2 in a time Δt . The vector change in velocity $\Delta \vec{v}$ during this time is shown in Fig. 21b.

The angles labeled $\Delta \varphi$ in Figs. 21a and 21b are the same because \vec{v}_1 is perpendicular to the line OP_1 and \vec{v}_1 is perpendicular to the line Hence the triangles in Figs. 21a and 21b are *similar*. The ratios of corresponding sides of similar triangles are equal, so

$$\frac{|\Delta \vec{v}|}{v_1} = \frac{\Delta s}{R} \tag{34}$$

or

$$|\Delta \vec{v}| = \frac{v_1}{R} \Delta s \tag{35}$$

The magnitude of a_{av} the average acceleration during Δt is therefore

$$a_{av} = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v_1}{R} \frac{\Delta s}{\Delta t}$$
(36)

The magnitude *a* of the *instantaneous* acceleration \vec{a} at point P_1 is the limit of this expression as we take point P_2 closer and closer to point P_1 :

$$a = \lim_{\Delta t \to 0} \frac{v_1}{R} \frac{\Delta s}{\Delta t} = \frac{v_1}{R} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$
(37)

If the time interval Δt is short, Δs is the distance the particle moves along its curved path. So the limit $\Delta s/\Delta t$ of is the speed v_1 at point P_1 . Also, P_1 can be any point on the path, so we can drop the subscript and let represent the speed at any point. Then

$$a_{rad} = \frac{v^2}{R} \tag{38}$$

(a) A particle moves a distance Δs at constant speed along a circular path.



Figure 21 – Finding the velocity change $\Delta \vec{v}$, average acceleration \vec{a}_{av} and instantaneous acceleration \vec{a}_{rad} for a particle moving in a circle with constant speed

We have added the subscript "rad" as a reminder that the direction of the instantaneous acceleration at each point is always along a radius of the circle (toward the center of the circle; see Figs. 20c and 21c). So we have found that *in*

uniform circular motion, the magnitude of the instantaneous acceleration isequal to the square of the speed divided by the radius R of the circle. Its direction is perpendicular to \vec{v} and inward along the radius.

Because the acceleration in uniform circular motion is always directed toward the center of the circle, it is sometimes called **centripetal acceleration**. The word "centripetal" is derived from two Greek words meaning "seeking the center." Figure 22a shows the directions of the velocity and acceleration vectors at several points for a particle moving with uniform circular motion.

We can also express the magnitude of the acceleration in uniform circular motion in terms of the **period** *T* of the motion, the time for one revolution (one complete trip around the circle). In a time *T* the particle travels a distance equal to the circumference $2\pi R$ of the circle, so its speed is

$$v = \frac{2\pi R}{T} \tag{39}$$

When we substitute this into Eq. (38), we obtain the alternative expression

$$a_{rad} = \frac{4\pi^2 R}{T^2} \tag{40}$$



Figure 22 – Acceleration and velocity (a) for a particle in uniform circular motion and (b) for a projectile with no air resistance

1.2 Newton's laws of motion

1.2.1 Force and interactions

In everyday language, a **force** is a push or a pull. Abetter definition is that a force is an *interaction* between two bodies or between a body and its environment

(see fig. 23). That's why we always refer to the force that one body *exerts* on a second body. When you push on a car that is stuck in the snow, you exert a force on the car; a steel cable exerts a force on the beam it is hoisting at a construction site; and so on. As Fig. 23 shows, force is a *vector* quantity; you can push or pull a body in different directions.

- · A force is a push or a pull.
- A force is an interaction between two objects or between an object and its environment.
- A force is a vector quantity, with magnitude and direction.



Figure 23 – Some properties of forces

When a force involves direct contact between two bodies, such as a push or pull that you exert on an object with your hand, we call it a **contact force**. Figures 24a, 24b, and 24c show three common types of contact forces. The **normal force** (see fig. 24a) is exerted on an object by any surface with which it is in contact. The adjective *normal* means that the force always acts perpendicular to the surface of contact, no matter what the angle of that surface. By contrast, the **friction force** (see fig. 24b) exerted on an object by a surface acts *parallel* to the surface, in the direction that opposes sliding. The pulling force exerted by a stretched rope or cord on an object to which it's attached is called a **tension force** (see fig. 24c). When you tug on your dog's leash, the force that pulls on her collar is a tension force.

In addition to contact forces, there are **long-range forces** that act even when the bodies are separated by empty space. The force between two magnets is an example of a long-range force, as is the force of gravity (see fig. 24d); the earth pulls a dropped object toward it even though there is no direct contact between the object and the earth. The gravitational force that the earth exerts on your body is called your **weight**. (a) Normal force \vec{n} : When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.



(c) Tension force \vec{T} : A pulling force exerted on an object by a rope, cord, etc.



(b) Friction force \vec{f} : In addition to the normal force, a surface may exert a frictional force on an object, directed parallel to the surface.





(d) Weight \vec{w} : The pull of gravity on an object

is a long-range force (a force that acts over

Figure 24 - Four types of forces

a distance).

To describe a force vector, we need to describe the *direction* in which it acts as well as its *magnitude*, the quantity that describes "how much" or "how hard" the force pushes or pulls. The SI unit of the magnitude of force is the *newton*, abbreviated N. Table 2 lists some typical force magnitudes.

Table 2 – Typical force magnitudes

Force	Magnitude
Sun's gravitational force on the earth	$3.5 \cdot 10^{22} N$
Thrust of a space shuttle during launch	$3.1 \cdot 10^7 N$
Weight of a large blue whale	$1.9 \cdot 10^{6} N$
Maximum pulling force of a locomotive	8.9 · 10 ⁵ N
Weight of a 250-lb linebacker	$1.1 \cdot 10^3 N$
Weight of a medium apple 1 N	1 <i>N</i>
Weight of smallest insect eggs	$2 \cdot 10^{-6} N$
Electric attraction between the proton and the electron in a	$8.2 \cdot 10^{-8} N$
hydrogen atom	
Weight of a very small bacterium	$1 \cdot 10^{-18} N$
Weight of a hydrogen atom	$1.6 \cdot 10^{-26} N$
Weight of an electron	$8.9 \cdot 10^{-30} N$
Gravitational attraction between the proton and the electron in	$3.6 \cdot 10^{-47} N$
a hydrogen atom	

A common instrument for measuring force magnitudes is the *spring balance*. It consists of a coil spring enclosed in a case with a pointer attached to one end.

When forces are applied to the ends of the spring, it stretches by an amount that depends on the force. We can make a scale for the pointer by using a number of identical bodies with weights of exactly 1 N each. When one, two, or more of these are suspended simultaneously from the balance, the total force stretching the spring is 1 N, 2 N, and so on, and we can label the corresponding positions of the pointer 1 N, 2 N, and so on. Then we can use this instrument to measure the magnitude of an unknown force. We can also make a similar instrument that measures pushes instead of pulls.

Figure 25 shows a spring balance being used to measure a pull or push that we apply to a box. In each case we draw a vector to represent the applied force. The length of the vector shows the magnitude; the longer the vector, the greater the force magnitude.



Figure 25 – Using a vector arrow to denote the force we exert when (a) pulling a block with a string or (b) pushing a block with a stick

When you throw a ball, there are at least two forces acting on it: the push of your hand and the downward pull of gravity. Experiment shows that when two forces $\vec{F_1}$ and $\vec{F_2}$ act at the same time at the same point on a body (see fig. 26), the effect on the body's motion is the same as if a single force \vec{F} were acting equal to

the vector sum of the original forces: $\vec{F} = \vec{F}_1 + \vec{F}_2$. More generally, any number of forces applied at a point on a body have the same effect as a single force equal to the vector sum of the forces. This important principle is called **superposition of forces**.



Figure 26 – Superposition of forces

The principle of superposition of forces is of the utmost importance, and we will use it throughout our study of physics. For example, in Fig. 27a, force \vec{F} acts on a body at point *O*. The component vectors of \vec{F} in the directions *Ox* and *Oy* are F_x and F_y . When F_x and F_y are applied simultaneously, as in Fig. 27b, the effect is exactly the same as the effect of the original force \vec{F} . Hence any force can be replaced by its component vectors, acting at the same point.

(a) Component vectors: \vec{F}_x and \vec{F}_y Components: $F_x = F \cos \theta$ and $\vec{F}_y = F \sin \theta$



(b) Component vectors \vec{F}_x and \vec{F}_y together have the same effect as original force \vec{F} .



Figure 27 – The force \vec{F} , which acts at an angle θ from *x*-axis, may be replaced by its rectangular component vectors \vec{F}_x and \vec{F}_y

It's frequently more convenient to describe a force \vec{F} in terms of its x- and ycomponents F_x and F_y rather than by its component vectors (recall that *component vectors* are vectors, but *components* are just numbers). For the case shown in Fig. 27, both F_x and F_y are positive; for other orientations of the force \vec{F} , either F_x or F_y may be negative or zero.

Our coordinate axes don't have to be vertical and horizontal. Figure 28 shows a crate being pulled up a ramp by a force \vec{F} , represented by its components F_x and F_y parallel and perpendicular to the sloping surface of the ramp.

In Fig. 28 we draw a wiggly line through the force vector \vec{F} to show that we have replaced it by its x- and y-components. Otherwise, the diagram would include the same force twice. We will draw such a wiggly line in any force diagram where a force is replaced by its components. Look for this wiggly line in other figures in this and subsequent chapters.



Figure 28 - F_x and F_y are the components of \vec{F} parallel and perpendicular to the sloping surface of the inclined plane

We will often need to find the vector sum (resultant) of *all* the forces acting on body. We call this the **net force** acting on the body. We will use the Greek letter (capital sigma, equivalent to the Roman *S*) as a shorthand notation for a sum. If the forces are labeled \vec{F}_1 , \vec{F}_2 , \vec{F}_3 , and so on, we abbreviate the sum as

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots = \sum \vec{F}$$
(41)

We read $\sum \vec{F}$ as "the vector sum of the forces" or "the net force." The component version of Eq. (41) is the pair of component equations

$$F_x = \sum F_{xi} , F_y = \sum F_{yi}$$
(42)

Here $\sum F_{xi}$ is the sum of the *x*-components and $\sum F_{yi}$ is the sum of the *y*-components (see fig. 29). Each component may be positive or negative, so be careful with signs when you evaluate these sums.



Figure 29 – Finding the components of the vector sum (resultant) \vec{F} of two forces $\vec{F_1}$ and $\vec{F_2}$

Once we have F_x and F_y we can find the magnitude and direction of the net force $\vec{F} = \sum \vec{F_i}$ acting on the body. The magnitude is

$$F = \sqrt{F_x^2 + F_y^2} \tag{43}$$

and the angle θ between \vec{F} and the +*x*-axis can be found from the relationship $\tan \theta = \frac{F_y}{F_x}$. The components F_x and F_y may be positive, negative, or zero, and the angle θ may be in any of the four quadrants.

In three-dimensional problems, forces may also have z-components; then we add the equation $F_z = \sum F_{zi}$ to Eq. (42). The magnitude of the net force is then

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
(44)

1.2.2 Newton's first law

How do the forces that act on a body affect its motion? To begin to answer this question, let's first consider what happens when the net force on a body is *zero*. You would almost certainly agree that if a body is at rest, and if no net force acts on it (that is, no net push or pull), that body will remain at rest. But what if there is zero net force acting on a body in *motion*?

To see what happens in this case, suppose you slide a hockey puck along a horizontal tabletop, applying a horizontal force to it with your hand (see fig. 30a). After you stop pushing, the puck *does not* continue to move indefinitely; it slows down and stops. To keep it moving, you have to keep pushing (that is, applying a force). You might come to the "common sense" conclusion that bodies in motion naturally come to rest and that a force is required to sustain motion.

But now imagine pushing the puck across a smooth surface of ice (see fig. 30b). After you quit pushing, the puck will slide a lot farther before it stops. Put it on an air-hockey table, where it floats on a thin cushion of air, and it moves still farther (see fig. 30c). In each case, what slows the puck down is *friction*, an interaction between the lower surface of the puck and the surface on which it slides. Each surface exerts a frictional force on the puck that resists the puck's motion; the difference in the three cases is the magnitude of the frictional force. The ice exerts less friction than the tabletop, so the puck travels farther. The gas molecules of the air-hockey table exert the least friction of all. If we could eliminate friction completely, the puck would never slow down, and we would need no force at all to keep the puck moving once it had been started. Thus the "common sense" idea that a force is required to sustain motion is *incorrect*.

Experiments like the ones we've just described show that when *no* net force acts on a body, the body either remains at rest *or* moves with constant velocity in a straight line. Once a body has been set in motion, no net force is needed to keep it moving. We call this observation *Newton's first law of motion*:

Newton's first law of motion: A body acted on by no net force moves with constant velocity (which may be zero) and zero acceleration.


Figure 30 – The slicker the surface, the father a puck slides after being given an initial velocity. On an air-hockey table (c) the friction force is practically zero, so the puck continues with almost constant velocity

The tendency of a body to keep moving once it is set in motion results from a property called **inertia.** You use inertia when you try to get ketchup out of a bottle by shaking it. First you start the bottle (and the ketchup inside) moving forward; when you jerk the bottle back, the ketchup tends to keep moving forward and, you hope, ends up on your burger. The tendency of a body at rest to remain at rest is also due to inertia. You may have seen a tablecloth yanked out from under the china without breaking anything. The force on the china isn't great enough to make it move appreciably during the short time it takes to pull the tablecloth away. It's important to note that the *net* force is what matters in Newton's first law. For example, a physics book at rest on a horizontal tabletop has two forces acting on it: an upward supporting force, or normal force, exerted by the tabletop (see Fig. 24a) and the downward force of the earth's gravitational attraction (a long-range force that acts even if the tabletop is elevated above the ground; see Fig. 24d). The upward push of the surface is just as great as the downward pull of gravity, so the *net* force acting on the book (that is, the vector sum of the two forces) is zero. In agreement with Newton's first law, if the book is at rest on the tabletop, it remains at rest. The same principle applies to a hockey puck sliding on a horizontal, frictionless surface: The vector sum of the upward push of the surface and the downward pull of gravity is zero. Once the puck is in motion, it continues to move with constant velocity because the *net* force acting on it is zero.

Here's another example. Suppose a hockey puck rests on a horizontal surface with negligible friction, such as an air-hockey table or a slab of wet ice. If the puck is initially at rest and a single horizontal force $\vec{F_1}$ acts on it (see fig. 31a), the puck starts to move. If the puck is in motion to begin with, the force changes its speed, its direction, or both, depending on the direction of the force. In this case the net force is equal to $\vec{F_1}$, which is *not* zero. (There are also two vertical forces: the earth's gravitational attraction and the upward normal force exerted by the surface. But as we mentioned earlier, these two forces cancel.)

Now suppose we apply a second force \vec{F}_2 (see fig. 31b), equal in magnitude to \vec{F}_1 but opposite in direction. The two forces are negatives of each other, $\vec{F}_2 = \vec{F}_1$, and their vector sum is zero:

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 = \vec{F}_1 + \left(-\vec{F}_1\right) = 0 \tag{45}$$

Again, we find that if the body is at rest at the start, it remains at rest; if it is initially moving, it continues to move in the same direction with constant speed. These results show that in Newton's first law, *zero net force is equivalent to no force at all.* This is just the principle of superposition of forces.

When a body is either at rest or moving with constant velocity (in a straight line with constant speed), we say that the body is in **equilibrium.** For a body to be in equilibrium, it must be acted on by no forces, or by several forces such that their vector sum—that is, the net force—is zero:

$$\sum \vec{F} = 0 \tag{46}$$

For this to be true, each component of the net force must be zero, so

$$\sum F_x = 0, \sum F_y = 0 \tag{47}$$

We are assuming that the body can be represented adequately as a point particle. When the body has finite size, we also have to consider *where* on the body the forces are applied.



Figure 31 – (a) A hockey puck accelerates in the direction of a net applied force \vec{F}_1 . (b) When the net force is zero, the acceleration is zero, the acceleration is zero, and puck is in equilibrium

Early we introduced the concept of *frame of reference*. This concept is central to Newton's laws of motion. Suppose you are in a bus that is traveling on a straight road and speeding up. If you could stand in the aisle on roller skates, you would start moving *backward* relative to the bus as the bus gains speed. If instead the bus was slowing to a stop, you would start moving forward down the aisle. In either case, it looks as though Newton's first law is not obeyed; there is no net force acting on you, yet your velocity changes. What's wrong?

The point is that the bus is accelerating with respect to the earth and is *not* a suitable frame of reference for Newton's first law. This law is valid in some frames of reference and not valid in others. A frame of reference in which Newton's first law *is* valid is called an **inertial frame of reference**. The earth is at least approximately an inertial frame of reference, but the bus is not. (The earth is not a completely inertial frame, owing to the acceleration associated with its rotation and its motion around the sun. These effects are quite small. Because Newton's first law is used to define what we mean by an inertial frame of reference, it is sometimes called the *law of inertia*.

Figure 32 helps us understand what you experience when riding in a vehicle that's accelerating. In Fig. 32a, a vehicle is initially at rest and then begins to accelerate to the right. A passenger on roller skates (which nearly eliminate the

effects of friction) has virtually no net force acting on her, so she tends to remain at rest relative to the inertial frame of the earth. As the vehicle accelerates around her, she moves backward relative to the vehicle. In the same way, a passenger in a vehicle that is slowing down tends to continue moving with constant velocity relative to the earth, and so moves forward relative to the vehicle (see fig. 32b). A vehicle is also accelerating if it moves at a constant speed but is turning (see fig. 32c). In this case a passenger tends to continue moving relative to the earth at constant speed in a straight line; relative to the vehicle, the passenger moves to the side of the vehicle on the outside of the turn.



Figure 32 – Riding in an accelerating vehicle

In each case shown in Fig. 32, an observer in the vehicle's frame of reference might be tempted to conclude that there *is* a net force acting on the passenger, since the passenger's velocity *relative to the vehicle* changes in each case. This conclusion is simply wrong; the net force on the passenger is indeed zero. The vehicle observer's mistake is in trying to apply Newton's first law in the vehicle's frame of reference, which is *not* an inertial frame and in which Newton's first law isn't valid (see fig. 33). In this book we will use *only* inertial frames of reference.



Figure 33 – From the frame of reference of the car, it seems as though a force is pushing the crash test dummies forward as the car comes to a sudden stop. But there is really no such force: As the car stops, the dummies keep moving forward as a consequence of Newton's first law

We've mentioned only one (approximately) inertial frame of reference: the earth's surface. But there are many inertial frames. If we have an inertial frame of reference *A*, in which Newton's first law is obeyed, then *any* second frame of reference *B* will also be inertial if it moves relative to *A* with constant velocity $\vec{v}_{B/A}$. We can prove this using the relative-velocity relationship

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} \tag{48}$$

Suppose that *P* is a body that moves with constant velocity $\vec{v}_{P/A}$ with respect to an inertial frame *A*. By Newton's first law the net force on this body is zero. The velocity of *P* relative to another frame *B* has a different value, $\vec{v}_{P/B} = \vec{v}_{P/A} - \vec{v}_{B/A}$. But if the relative velocity of the two frames is constant, then $\vec{v}_{P/B}$ is constant as well. Thus *B* is also an inertial frame; the velocity of *P* in this frame is constant, and the net force on *P* is zero, so Newton's first law is obeyed in *B*. Observers in frames *A* and *B* will disagree about the velocity of *P*, but they will agree that *P* has a constant velocity (zero acceleration) and has zero net force acting on it.

There is no single inertial frame of reference that is preferred over all others for formulating Newton's laws. If one frame is inertial, then every other frame moving relative to it with constant velocity is also inertial. Viewed in this light, the state of rest and the state of motion with constant velocity are not very different; both occur when the vector sum of forces acting on the body is zero.

1.2.3 Mass and weight. Newton's second law

Newton's first law tells us that when a body is acted on by zero net force, it moves with constant velocity and zero acceleration. In Fig. 34a, a hockey puck is sliding to the right on wet ice. There is negligible friction, so there are no horizontal forces acting on the puck; the downward force of gravity and the upward normal force exerted by the ice surface sum to zero. So the net force $\sum \vec{F}$ acting on the puck is zero, the puck has zero acceleration, and its velocity is constant.

But what happens when the net force is *not* zero? In Fig. 34b we apply a constant horizontal force to a sliding puck in the same direction that the puck is moving. Then is constant and in the same horizontal direction as .We find that during the time the force is acting, the velocity of the puck changes at a constant rate; that is, the puck moves with constant acceleration. The speed of the puck increases, so the acceleration \vec{a} is in the same direction as \vec{v} and $\sum \vec{F}$.



Figure 34 – Exploring the relationship between the acceleration of a body and the net force acting on the body (in this case, a hockey puck on a frictionless surface)

In Fig. 34c we reverse the direction of the force on the puck so that $\sum \vec{F}$ acts opposite to \vec{v} . In this case as well, the puck has an acceleration; the puck moves more and more slowly to the right. The acceleration \vec{a} in this case is to the left, in the same direction as $\sum \vec{F}$. As in the previous case, experiment shows that the acceleration is constant if $\sum \vec{F}$ is constant.

We conclude that *a net force acting on a body causes the body to accelerate in the same direction as the net force.* If the magnitude of the net force is constant, as in Figs. 34b and 34c, then so is the magnitude of the acceleration.

These conclusions about net force and acceleration also apply to a body moving along a curved path. For example, Fig. 35 shows a hockey puck moving in a horizontal circle on an ice surface of negligible friction. A rope is attached to the puck and to a stick in the ice, and this rope exerts an inward tension force of constant magnitude on the puck. The net force and acceleration are both constant in magnitude and directed toward the center of the circle. The speed of the puck is constant, so this is uniform circular motion.

Figure 36a shows another experiment to explore the relationship between acceleration and net force. We apply a constant horizontal force to a puck on a frictionless horizontal surface with the spring stretched a constant amount. As in Figs. 34b and 34c, this horizontal force equals the net force on the puck. If we change the magnitude of the net force, the acceleration changes in the same proportion. Doubling the net force doubles the acceleration (see fig. 34b), halving the net force halves the acceleration (see fig. 34c), and so on. Many such experiments show that *for any given body, the magnitude of the acceleration is directly proportional to the magnitude of the net force acting on the body.*



At all points, the acceleration \vec{a} and the net force $\Sigma \vec{F}$ point in the same direction—always toward the center of the circle.

Figure 35 – A top view of a hockey puck in uniform circular motion on a frictionless horizontal surface



Figure 36 - For a body of a given mass *m*, the magnitude of the body's acceleration is directly proportional to the magnitude of the acting on the body

Our results mean that for a given body, the ratio of the magnitude $|\sum \vec{F}|$ of the net force to the magnitude $a = |\vec{a}|$ of the acceleration is constant, regardless of the magnitude of the net force. We call this ratio the inertial mass, or simply the mass, of the body and denote it by m. That is,

$$m = \frac{\left|\sum \vec{F}\right|}{a} \tag{49}$$

or

$$\sum \vec{F} = ma \tag{50}$$

or

$$a = \frac{\left|\sum \vec{F}\right|}{m} \tag{51}$$

Mass is a quantitative measure of inertia. The last of the equations in Eqs. (51) says that the greater its mass, the more a body "resists" being accelerated. When you hold a piece of fruit in your hand at the supermarket and move it slightly up and down to estimate its heft, you're applying a force and seeing how much the fruit accelerates up and down in response. If a force causes a large acceleration, the fruit has a small mass; if the same force causes only a small acceleration, the fruit has a large mass. In the same way, if you hit a table-tennis ball and then a basketball with the same force, the basketball has much smaller acceleration because it has much greater mass.

The SI unit of mass is the kilogram. The kilogram is officially defined to be the mass of a cylinder of platinum–iridium alloy kept in a vault near Paris. We can use this standard kilogram, along with Eqs. (51), to define the newton:

One newton is the amount of net force that gives an acceleration of 1 meter per second squared to a body with a mass of 1 kilogram.

This definition allows us to calibrate the spring balances and other instruments used to measure forces. Because of the way we have defined the newton, it is related to the units of mass, length, and time. For Eqs. (51) to be dimensionally consistent, it must be true that

1 newton = (1 kilogram)(1 meter per second squared)
1
$$N = 1 kg \cdot m/s^2$$

or

We will use this relationship many times in the next few chapters, so keep it in mind.

We can also use Eqs. (51) to compare a mass with the standard mass and thus to measure masses. Suppose we apply a constant net force $\sum \vec{F}$ to a body having a known mass m_1 and we find an acceleration of magnitude a_1 (see fig. 37a). We then apply the same force to another body having an unknown mass m_2 and we find an acceleration of magnitude a_2 (see fig. 37b). Then, according to Eqs. (51),

$$m_1 a_1 = m_2 a_2 \tag{52}$$

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \tag{53}$$

For the same net force, the ratio of the masses of two bodies is the inverse of the ratio of their accelerations. In principle we could use Eq. (52) to measure an unknown mass m_2 but it is usually easier to determine mass indirectly by measuring the body's weight.

When two bodies with masses m_1 and m_2 are fastened together, we find that the mass of the composite body is always $m_1 + m_2$ (see fig. 37c). This additive property of mass may seem obvious, but it has to be verified experimentally. Ultimately, the mass of a body is related to the number of protons, electrons, and neutrons it contains. This wouldn't be a good way to define mass because there is no practical way to count these particles. But the concept of mass is the most fundamental way to characterize the quantity of matter in a body.



(b) Applying the same force $\Sigma \vec{F}$ to a second object and noting the acceleration allow us to measure the mass.



(c) When the two objects are fastened together, the same method shows that their composite mass is the sum of their individual masses.



Figure 37 – For a given net force $\sum \vec{F}$ acting on a body, the acceleration is inversely proportional to the mass of the body

We've been careful to state that the net force on a body is what causes that body to accelerate. Experiment shows that if a combination of forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ and so on is applied to a body, the body will have the same acceleration (magnitude and direction) as when only a single force is applied, if that single force is equal to the vector sum $\vec{F_1}$, $+\vec{F_2}$ + $\vec{F_3}$ + In other words, the principle of superposition of forces (see Fig. 26) also holds true when the net force is not zero and the body is accelerating.

Equations (51) relate the magnitude of the net force on a body to the magnitude of the acceleration that it produces. We have also seen that the direction of the net force is the same as the direction of the acceleration, whether the body's path is straight or curved. Newton wrapped up all these relationships and experimental results in a single concise statement that we now call Newton's second law of motion:

Newton's second law of motion: If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The mass of the body times the acceleration of the body equals the net force vector.

In symbols,

$$\sum \vec{F} = m\vec{a} \tag{54}$$

An alternative statement is that the acceleration of a body is in the same direction as the net force acting on the body, and is equal to the net force divided by the body's mass:

$$\vec{a} = \frac{\sum \vec{F}}{m} \tag{55}$$

Newton's second law is a fundamental law of nature, the basic relationship between force and motion. Most of the remainder of this chapter and all of the next are devoted to learning how to apply this principle in various situations.

Equation (54) has many practical applications (see fig. 38). You've actually been using it all your life to measure your body's acceleration. In your inner ear, microscopic hair cells sense the magnitude and direction of the force that they must exert to cause small membranes to accelerate along with the rest of your body. By Newton's second law, the acceleration of the membranes—and hence that of your body as a whole—is proportional to this force and has the same direction. In this way, you can sense the magnitude and direction of your acceleration even with your eyes closed!

There are at least four aspects of Newton's second law that deserve special attention. First, Eq. (54) is a vector equation. Usually we will use it in component form, with a separate equation for each component of force and the corresponding component of acceleration:

$$\sum F_x = ma_x , \sum F_y = ma_y , \sum F_z = ma_z$$
(56)

This set of component equations is equivalent to the single vector equation (54). Each component of the net force equals the mass times the corresponding component of acceleration.



Figure 38 – The design of high-perfomance motorcycles depends fundamentally on Newton's second law. To maximize the forward acceleration, the designer makes the motorcycle as light as possible (that is, minimizes the mass) and uses the most powerful engine possible (thus maximizing the forward force)

Second, the statement of Newton's second law refers to external forces. By this we mean forces exerted on the body by other bodies in its environment. It's impossible for a body to affect its own motion by exerting a force on itself; if it were possible, you could lift yourself to the ceiling by pulling up on your belt! That's why only external forces are included in the sum $\sum \vec{F}$ in Eqs. (54) and (56).

Third, Eqs. (54) and (56) are valid only when the mass m is constant. It's easy to think of systems whose masses change, such as a leaking tank truck, a rocket ship, or a moving railroad car being loaded with coal. But such systems are better handled by using the concept of momentum.

Finally, Newton's second law is valid only in inertial frames of reference, just like the first law. Thus it is not valid in the reference frame of any of the accelerating vehicles in Fig. 32; relative to any of these frames, the passenger accelerates even though the net force on the passenger is zero. We will usually assume that the earth is an adequate approximation to an inertial frame, although because of its rotation and orbital motion it is not precisely inertial.

One of the most familiar forces is the weight of a body, which is the gravitational force that the earth exerts on the body. (If you are on another planet, your weight is the gravitational force that planet exerts on you.) Unfortunately, the terms mass and weight are often misused and interchanged in everyday

conversation. It is absolutely essential for you to understand clearly the distinctions between these two physical quantities.

Mass characterizes the inertial properties of a body. Mass is what keeps the china on the table when you yank the tablecloth out from under it. The greater the mass, the greater the force needed to cause a given acceleration; this is reflected in Newton's second law, $\sum \vec{F} = m\vec{a}$.

Weight, on the other hand, is a force exerted on a body by the pull of the earth. Mass and weight are related: Bodies having large mass also have large weight. A large stone is hard to throw because of its large mass, and hard to lift off the ground because of its large weight.



Figure 39 – The relationship of mass and weight

To understand the relationship between mass and weight, note that a freely falling body has an acceleration of magnitude g. Newton's second law tells us that a force must act to produce this acceleration. If a 1-kg body falls with an acceleration of 9.8 m/s^2 the required force has magnitude

$$F = ma = (1 \ kg) \left(9.8 \frac{m}{s^2}\right) = 9.8 \ kg \cdot \frac{m}{s^2} = 9.8 \ N$$

The force that makes the body accelerate downward is its weight. Any body near the surface of the earth that has a mass of 1 kg must have a weight of 9.8 N to give it the acceleration we observe when it is in free fall. More generally, a body with mass m must have weight with magnitude w given by

$$w = mg \tag{57}$$

Hence the magnitude w of a body's weight is directly proportional to its mass m. The weight of a body is a force, a vector quantity, and we can write Eq. (57) as a vector equation (see fig. 39):

$$\vec{w} = m\vec{g} \tag{58}$$

Remember that g is the magnitude of \vec{g} , the acceleration due to gravity, so g is always a positive number, by definition. Thus w, given by Eq. (57), is the magnitude of the weight and is also always positive.

1.2.4. Newton's third law

A force acting on a body is always the result of its interaction with another body, so forces always come in pairs. You can't pull on a doorknob without the doorknob pulling back on you. When you kick a football, the forward force that your foot exerts on the ball launches it into its trajectory, but you also feel the force the ball exerts back on your foot. If you kick a boulder, the pain you feel is due to the force that the boulder exerts on your foot.

In each of these cases, the force that you exert on the other body is in the opposite direction to the force that body exerts on you. Experiments show that whenever two bodies interact, the two forces that they exert on each other are always *equal in magnitude* and *opposite in direction*. This fact is called *Newton's third law of motion*:

Newton's third law of motion: If body A exerts a force on body B (an "action"), then body B exerts a force on body A (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on *different* bodies.

For example, in Fig. 40 $\vec{F}_{A \text{ on } B}$ is the force applied by body A (first subscript) on body B (second subscript), and $\vec{F}_{B \text{ on } A}$ is the force applied by body B (first subscript) on body A (second subscript). The mathematical statement of Newton's third law is

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \tag{59}$$

It doesn't matter whether one body is inanimate (like the soccer ball in Fig. 40) and the other is not (like the kicker): They necessarily exert forces on each other that obey Eq. (59).

In the statement of Newton's third law, "action" and "reaction" are the two opposite forces (in Fig. 40, $\vec{F}_{A \text{ on } B}$ and $\vec{F}_{B \text{ on } A}$); we sometimes refer to them as an **action-reaction pair.** This is *not* meant to imply any cause-and-effect relationship; we can consider either force as the "action" and the other as the

"reaction." We often say simply that the forces are "equal and opposite," meaning that they have equal magnitudes and opposite directions.



Figure 40 – If body A exerts a force $\vec{F}_{A \text{ on } B}$ on body B, then body B exerts a force $\vec{F}_{B \text{ on } A}$ on body A that equal in magnitude and opposite in direction: $\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$

In Fig. 40 the action and reaction forces are *contact* forces that are present only when the two bodies are touching. But Newton's third law also applies to *longrange* forces that do not require physical contact, such as the force of gravitational attraction. Atable-tennis ball exerts an upward gravitational force on the earth that's equal in magnitude to the downward gravitational force the earth exerts on the ball. When you drop the ball, both the ball and the earth accelerate toward each other. The net force on each body has the same magnitude, but the earth's acceleration is microscopically small because its mass is so great. Nevertheless, it does move!

Abody that has pulling forces applied at its ends, such as the rope in Fig. 41, is said to be in *tension*. The **tension** at any point is the magnitude of force acting at that point (see Fig. 24c). In Fig. 41b the tension at the right end of the rope is the magnitude of $\vec{F}_{M \ on \ R}$ (or of $\vec{F}_{R \ on \ B}$), and the tension at the left end equals the magnitude $\vec{F}_{B \ on \ R}$ of (or of $\vec{F}_{R \ on \ B}$). If the rope is in equilibrium and if no forces act except at its ends, the tension is the *same* at both ends and throughout the rope. Thus, if the magnitudes of $\vec{F}_{B \ on \ R}$ and $\vec{F}_{M \ on \ R}$ are 50 N each, the tension in the rope is 50 N (*not* 100 N). The *total* force vector $\vec{F}_{B \ on \ R} + \vec{F}_{M \ on \ R}$ acting on the rope in this case is zero!

We emphasize once more a fundamental truth: The two forces in an action– reaction pair *never* act on the same body. Remembering this simple fact can often help you avoid confusion about action–reaction pairs and Newton's third law.





1.2.5 Dynamics of circular motion

When a particle moves in a circular path with constant speed, the particle's acceleration is always directed toward the center of the circle (perpendicular to the instantaneous velocity). The magnitude a_{rad} of the acceleration is constant and is given in terms of the speed v and the radius R of the circle by

$$a_{rad} = \frac{v^2}{R} \tag{60}$$

The subscript "rad" is a reminder that at each point the acceleration is radially inward toward the center of the circle, perpendicular to the instantaneous velocity. This acceleration is often called *centripetal acceleration*.

We can also express the centripetal acceleration in terms of the *period* T, the time for one revolution:

$$T = \frac{2\pi R}{v} \tag{61}$$

In terms of the period, a_{rad} is

$$a_{rad} = \frac{4\pi^2 R}{T^2} \tag{62}$$

Uniform circular motion, like all other motion of a particle, is governed by Newton's second law. To make the particle accelerate toward the center of the circle, the net force $\sum \vec{F}$ on the particle must always be directed toward the center (see fig. 42). The magnitude of the acceleration is constant, so the magnitude F_{net} of the net force must also be constant. If the inward net force stops acting, the particle flies off in a straight line tangent to the circle (see fig. 43).



Figure 42 – Net force, acceleration and velocity in uniform circular motion

A ball attached to a string whirls in a circle on a frictionless surface.



Figure 43 – What happens if the inward radial force suddenly ceases to act on a body in circular motion?

The magnitude of the radial acceleration is given by $a_{rad} = \frac{v^2}{R}$ so the magnitude F_{net} of the net force on a particle with mass *m* in uniform circular motion must be

$$F_{net} = ma_{rad} = m\frac{v^2}{R}$$
(63)

Uniform circular motion can result from *any* combination of forces, just so the net force $\sum \vec{F}$ is always directed toward the center of the circle and has a constant magnitude. Note that the body need not move around a complete circle: Equation (63) is valid for *any* path that can be regarded as part of a circular arc.

1.2.6 The fundamental forces of nature

We have discussed several kinds of forces—including weight, tension, friction, fluid resistance, and the normal force—and we will encounter others as we continue our study of physics. But just how many kinds of forces are there? Our current understanding is that all forces are expressions of just four distinct classes of *fundamental* forces, or interactions between particles (see fig. 44). Two are familiar in everyday experience. The other two involve interactions between subatomic particles that we cannot observe with the unaided senses.

Gravitational interactions include the familiar force of your *weight*, which results from the earth's gravitational attraction acting on you. The mutual gravitational attraction of various parts of the earth for each other holds our planet together (see fig. 44a). Newton recognized that the sun's gravitational attraction for the earth keeps the earth in its nearly circular orbit around the sun. Next we will study gravitational interactions in greater detail, and we will analyze their vital role in the motions of planets and satellites.

The second familiar class of forces, **electromagnetic interactions,** includes electric and magnetic forces. If you run a comb through your hair, the comb ends up with an electric charge; you can use the electric force exerted by this charge to pick up bits of paper. All atoms contain positive and negative electric charge, so atoms and molecules can exert electric forces on one another (see fig. 44b). Contact forces, including the normal force, friction, and fluid resistance, are the combination of all such forces exerted on the atoms of a body by atoms in its surroundings. *Magnetic* forces, such as those between magnets or between a magnet and a piece of iron, are actually the result of electric charges in motion. For example, an electromagnet causes magnetic interactions because electric charges move through its wires. We will study electromagnetic interactions in detail in the second half of this book.

On the atomic or molecular scale, gravitational forces play no role because electric forces are enormously stronger: The electrical repulsion between two protons is stronger than their gravitational attraction by a factor of about But in bodies of astronomical size, positive and negative charges are usually present in nearly equal amounts, and the resulting electrical interactions nearly cancel out. Gravitational interactions are thus the dominant influence in the motion of planets and in the internal structure of stars.

The other two classes of interactions are less familiar. One, the **strong interaction,** is responsible for holding the nucleus of an atom together. Nuclei contain electrically neutral neutrons and positively charged protons. The electric force between charged protons tries to push them apart; the strong attractive force between nuclear particles counteracts this repulsion and makes the nucleus stable. In this context the strong interaction is also called the *strong nuclear force*. It has much shorter range than electrical interactions, but within its range it is much stronger. The strong interaction plays a key role in thermonuclear reactions that take place at the sun's core and generate the sun's heat and light (see fig. 44c).

(a) Gravitational forces hold planets together.



(b) Electromagnetic forces hold molecules together.



(C) Strong forces release energy to power the sun.





Figure 44 – Examples of fundamental interactions in nature: (a) The moon and the earth are held together and held in orbit by gravitational force; (b) This molecule of bacterial plasmid DNA is held together be electromagnetic forces between its atoms; (c) The sun shine because in its core, strong forces between nuclear particles cause the release of energy; (d) When a massive star explodes into a supernova, a flood of energy is released by weak interactions between the star's nuclear particles

Finally, there is the **weak interaction.** Its range is so short that it plays a role only on the scale of the nucleus or smaller. The weak interaction is responsible for a common form of radioactivity called beta decay, in which a neutron in a radioactive nucleus is transformed into a proton while ejecting an electron and a nearly massless particle called an antineutrino. The weak interaction between the antineutrino and ordinary matter is so feeble that an antineutrino could easily penetrate a wall of lead a million kilometers thick! Yet when a giant star undergoes a cataclysmic explosion called a supernova, most of the energy is released by way of the weak interaction (see fig. 44d).

In the 1960s physicists developed a theory that described the electromagnetic and weak interactions as aspects of a single *electroweak* interaction. This theory has passed every experimental test to which it has been put. Encouraged by this success, physicists have made similar attempts to describe the strong, electromagnetic, and weak interactions in terms of a single *grand unified theory* (GUT), and have taken steps toward a possible unification of all interactions into a *theory of everything* (TOE). Such theories are still speculative,

and there are many unanswered questions in this very active field of current research.

1.3 Work, kinetic and potential energy

1.3.1 Work

You'd probably agree that it's hard work to pull a heavy sofa across the room, to lift a stack of encyclopedias from the floor to a high shelf, or to push a stalled car off the road. Indeed, all of these examples agree with the everyday meaning of *work*—any activity that requires muscular or mental effort.

In physics, work has a much more precise definition. By making use of this definition we'll find that in any motion, no matter how complicated, the total work done on a particle by all forces that act on it equals the change in its *kinetic energy*—a quantity that's related to the particle's speed. This relationship holds even when the forces acting on the particle aren't constant, a situation that can be difficult or impossible to handle with the techniques you learned in Chapters 4 and 5. The ideas of work and kinetic energy enable us to solve problems in mechanics that we could not have attempted before.

In this section we'll see how work is defined and how to calculate work in avariety of situations involving *constant* forces. Even though we already know how to solve problems in which the forces are constant, the idea of work is still useful in such problems. Later in this chapter we'll relate work and kinetic energy, and then apply these ideas to problems in which the forces are *not* constant.

The three examples of work described above—pulling a sofa, lifting encyclopedias, and pushing a car—have something in common. In each case you do work by exerting a *force* on a body while that body *moves* from one place to another—that is, undergoes a *displacement* (see fig. 45). You do more work if the force is greater (you push harder on the car) or if the displacement is greater (you push the car farther down the road).



Figure 45 – These people are doing work as they push on the stalled car because they exert a force on the car as it moves

The physicist's definition of work is based on these observations. Consider a body that undergoes a displacement of magnitude *s* along a straight line. (For now, we'll assume that any body we discuss can be treated as a particle so that we can ignore any rotation or changes in shape of the body.) While the body moves, a constant force \vec{F} acts on it in the same direction as the displacement (see fig. 46). We define the **work** *W* done by this constant force under these circumstances as the product of the force magnitude *F* and the displacement magnitude *s*:

$$W = Fs \tag{64}$$

The work done on the body is greater if either the force F or the displacement s is greater, in agreement with our observations above.



Figure 46 – The work done be a constant force acting in the same direction as the displacement

The SI unit of work is the **joule** (abbreviated J, pronounced "jool," and named in honor of the 19th-century English physicist James Prescott Joule). From Eq. (64) we see that in any system of units, the unit of work is the unit of force multiplied by the unit of distance. In SI units the unit of force is the newton and the unit of distance is the meter, so 1 joule is equivalent to 1 newton-meter $(N \cdot m)$:

$$1 \text{ joule} = (1 \text{ newton})(1 \text{ meter})\text{ or } 1 \text{ J} = 1 \text{ N} \cdot m$$

As an illustration of Eq. (64), think of a person pushing a stalled car. If he pushes the car through a displacement with a constant force \vec{F} in the direction of

motion, the amount of work he does on the car is given by Eq. (64): W = Fs. But what if the person pushes at an angle α to the car's displacement (see fig. 47)?



Figure 47 – The work done by a constant force acting at an angle to the displacement

Then \vec{F} has a component $F_{\parallel} = F \cos \alpha$ in the direction of the displacement and a component $F_{\perp} = F \sin \alpha$ that acts perpendicular to the displacement. (Other forces must act on the car so that it moves along \vec{s} , not in the direction of \vec{F} . We're interested only in the work that the person does, however, so we'll consider only the force he exerts.) In this case only the parallel component F_{\parallel} is effective in moving the car, so we define the work as the product of this force component and the magnitude of the displacement. Hence $W = F_{\parallel}s = (F \cos \alpha)s$, or

$$W = Fs \cos \alpha \tag{65}$$

We are assuming that *F* and α are constant during the displacement. If $\alpha = 0$, so that \vec{F} and \vec{s} are in the same direction, then $\cos \alpha = 1$ and we are back to Eq. (64).

Equation (65) has the form of the *scalar product* of two vectors: You may want to review that definition. Hence we can write Eq. (65) more compactly as

$$W = \vec{F}\vec{s} \tag{66}$$

In Example 64 the work done in pushing the cars was positive. But it's important to understand that work can also be negative or zero. This is the essential way in which work as defined in physics differs from the "everyday" definition of work. When the force has a component in the same direction as the displacement (α between zero and 90°), $\cos \alpha$ in Eq. (65) is positive and the work W is positive (see fig. 48a). When the force has a component opposite to the displacement (α between 90° and 180°), $\cos \alpha$ is negative and the work is negative (see fig. 48b). When the force is perpendicular to the displacement, $\alpha = 90°$ and the work done by the force is zero (see fig. 48c). The cases of zero work and negative work bear closer examination, so let's look at some examples.



Figure 48 – A constant force \vec{F} can do positive, negative, or zero work depending on the angle between \vec{F} and displacement \vec{s}

There are many situations in which forces act but do zero work. You might think it's "hard work" to hold a barbell motionless in the air for 5 minutes (see fig. 49). But in fact, you aren't doing any work at all on the barbell because there is no displacement.



Figure 49 – A weightlifter does no work on a barbell as long as he holds in stationary

You get tired because the components of muscle fibers in your arm do work as they continually contract and relax. This is work done by one part of the arm exerting force on another part, however, not on the barbell. Even when you walk with constant velocity on a level floor while carrying a book, you still do no work on it. The book has a displacement, but the (vertical) supporting force that you exert on the book has no component in the direction of the (horizontal) motion. Then $\alpha = 90^{\circ}$ in Eq. (65), and $\cos \alpha = 0$. When a body slides along a surface, the work done on the body by the normal force is zero; and when a ball on a string moves in uniform circular motion, the work done on the ball by the tension in the string is also zero. In both cases the work is zero because the force has no component in the direction of motion.

What does it really mean to do negative work? The answer comes from Newton's third law of motion. When a weightlifter lowers a barbell as in Fig. 50a, his hands and the barbell move together with the same displacement The barbell exerts a force on his hands in the same direction as the hands' displacement, so the work done by the barbell on his hands is positive (see fig. 50b). But by Newton's third law the weightlifter's hands exert an equal and opposite force $\vec{F}_{hands on barbel} = -\vec{F}_{barbell on hands}$ on the barbell (see fig. 50c). This force, which keeps the barbell from crashing to the floor, acts opposite to the barbell's displacement. Thus the work done by his hands on the barbell is negative. Because the weightlifter's hands and the barbell have the same displacement, the work that his hands do on the barbell is just the negative of the work that the barbell does on his hands. In general, when one body does negative work on a second body, the second body does an equal amount of *positive* work on the first body.



Figure 50 – This weightlifter's hands do negative work on a barbell as the barbell does work on his hands

How do we calculate work when several forces act on a body? One way is to use Eq. (65) or (66) to compute the work done by each separate force. Then, because work is a scalar quantity, the total work W_{tot} done on the body by all the forces is the algebraic sum of the quantities of work done by the individual forces. An alternative way to find the total work W_{tot} is to compute the vector sum of the forces (that is, the net force) and then use this vector sum as \vec{F} in Eq. (65) or (66). The following example illustrates both of these techniques.

1.3.2 Kinetic energy and the work–energy theorem

The total work done on a body by external forces is related to the body's displacement—that is, to changes in its position. But the total work is also related to changes in the *speed* of the body. To see this, consider Fig. 51, which shows three examples of a block sliding on a frictionless table. The forces acting on the

block are its weight \vec{w} , the normal force \vec{n} , and the force \vec{F} exerted on it by the hand.

In Fig. 51a the net force on the block is in the direction of its motion. From Newton's second law, this means that the block speeds up; from Eq. (64), this also means that the total work done on the block is positive. The total work is *negative* in Fig. 51b because the net force opposes the displacement; in this case the block slows down. The net force is zero in Fig. 51c, so the speed of the block stays the same and the total work done on the block is zero. We can conclude that when a particle undergoes a displacement, it speeds up if $W_{tot} > 0$, slows down if $W_{tot} < 0$ and maintains the same speed if $W_{tot} = 0$.



Figure 51 – The relationship between the total work done on a body how the body's speed changes

Let's make these observations more quantitative. Consider a particle with mass *m* moving along the *x*-axis under the action of a constant net force with magnitude *F* directed along the positive *x*-axis (see fig. 52). The particle's acceleration is constant and given by Newton's second law, $F_x = ma_x$. Suppose the speed changes from v_1 to v_2 while the particle undergoes a displacement $s = x_2 - x_1$ from point x_1 to x_2 . Using a constant-acceleration equation, Eq. (18), and replacing v_{0x} by v_1 , v_x by v_2 , and $(x - x_0)$ by s, we have

$$v_2^2 = v_1^2 + 2a_x s \tag{67}$$

$$v_2^2 - v_2^2 \tag{68}$$

$$a_x = \frac{v_2^2 - v_1^2}{2s} \tag{68}$$



Figure 52 – A constant net force \vec{F} does work on a moving body

When we multiply this equation by m and equate ma_x to the net force F, we find

$$F = ma_x = m \frac{v_2^2 - v_1^2}{2s}$$
(69)

and

$$Fs = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \tag{70}$$

The product Fs is the work done by the net force F and thus is equal to the total work W_{tot} done by all the forces acting on the particle. The quantity is $\frac{1}{2}mv^2$ is called the kinetic energy K of the particle:

$$K = \frac{1}{2}mv^2\tag{71}$$

Like work, the kinetic energy of a particle is a scalar quantity; it depends on only the particle's mass and speed, not its direction of motion (see fig. 53). A car (viewed as a particle) has the same kinetic energy when going north at 10 m/s as

when going east at 10 m/s. Kinetic energy can never be negative, and it is zero only when the particle is at rest.



Figure 53 – Comparing the kinetic energy $K = \frac{1}{2}mv^2$ of different bodies

We can now interpret Eq. (70) in terms of work and kinetic energy. The first term on the right side of Eq. (70) is $K_2 = \frac{1}{2}mv_2^2$, the final kinetic energy of the particle (that is, after the displacement). The second term is the initial kinetic energy, $K_1 = \frac{1}{2}mv_1^2$, and the difference between these terms is the change in kinetic energy. So Eq. (70) says:

The work done by the net force on a particle equals the change in the particle's kinetic energy:

$$W_{tot} = K_2 - K_1 = \Delta K \tag{72}$$

This result is the work–energy theorem.

The work-energy theorem agrees with our observations about the block in Fig. 51. When W_{tot} is *positive*, the kinetic energy *increases* (the final kinetic

energy K_2 is greater than the initial kinetic energy K_1) and the particle is going faster at the end of the displacement than at the beginning. When W_{tot} is *negative*, the kinetic energy *decreases* (K_2 is less than K_1) and the speed is less after the displacement. When $W_{tot} = 0$ the kinetic energy stays the same ($K_2 = K_1$) and the speed is unchanged. Note that the work-energy theorem by itself tells us only about changes in *speed*, not velocity, since the kinetic energy doesn't depend on the direction of motion.

From Eq. (70) or Eq. (72), kinetic energy and work must have the same units. Hence the joule is the SI unit of both work and kinetic energy (and, as we will see later, of all kinds of energy). To verify this, note that in SI units the quantity $K = \frac{1}{2}mv^2$ has units $kg \cdot (m/s)^2$ or $kg \cdot m^2/s^2$ we recall that $1 N = 1 kg \cdot m/s^2$ so

$$1J = 1N \cdot m = (kg \cdot m/s^2) \cdot m = 1kg \cdot m^2/s^2$$

Because we used Newton's laws in deriving the work-energy theorem, we can $\cos \alpha$ use this theorem only in an inertial frame of reference. Note also that the work-energy theorem is valid in *any* inertial frame, but the values of W_{tot} and $K_2 - K_1$ may differ from one inertial frame to another (because the displacement and speed of a body may be different in different frames).

We've derived the work–energy theorem for the special case of straight-line motion with constant forces, and in the following examples we'll apply it to this special case only. We'll find in the next section that the theorem is valid in general, even when the forces are not constant and the particle's trajectory is curved.

In this section we've been careful to apply the work–energy theorem only to bodies that we can represent as *particles*—that is, as moving point masses. New subtleties appear for more complex systems that have to be represented as many particles with different motions. We can't go into these subtleties in detail in this chapter, but here's an example.

Suppose a boy stands on frictionless roller skates on a level surface, facing a rigid wall (see fig. 54). He pushes against the wall, which makes him move to the right. The forces acting on him are his weight \vec{w} , the upward normal forces \vec{n}_1 and \vec{n}_2 exerted by the ground on his skates, and the horizontal force \vec{F} exerted on him by the wall. There is no vertical displacement, so \vec{w}, \vec{n}_1 and \vec{n}_2 do no work. Force \vec{F} accelerates him to the right, but the parts of his body where that force is applied (the boy's hands) do not move while the force acts. Thus the force \vec{F} also does no work. Where, then, does the boy's kinetic energy come from?



Figure 54 – The external forces acting on a skater pushing off a wall. The work done by these forces is zero, but the skater's kinetic energy changes nonetheless

The explanation is that it's not adequate to represent the boy as a single point mass. Different parts of the boy's body have different motions; his hands remain stationary against the wall while his torso is moving away from the wall. The various parts of his body interact with each other, and one part can exert forces and do work on another part. Therefore the *total* kinetic energy of this *composite* system of body parts can change, even though no work is done by forces applied by bodies (such as the wall) that are outside the system. We'll discover that just as for the boy in this example, the total kinetic energy of such a system can change even when no work is done on any part of the system by anything outside it.

1.3.3 Power

The definition of work makes no reference to the passage of time. If you lift a barbell weighing 100 N through a vertical distance of 1.0 m at constant velocity, you do (100 N)(1.0 m)=100 J of work whether it takes you 1 second, 1 hour, or 1 year to do it. But often we need to know how quickly work is done. We describe this in terms of *power*. In ordinary conversation the word "power" is often synonymous with "energy" or "force." In physics we use a much more precise definition: **Power** is the time *rate* at which work is done. Like work and energy, power is a scalar quantity.

When a quantity of work ΔW is done during a time interval Δt , the average work done per unit time or **average power** P_{av} is defined to be

$$P_{av} = \frac{\Delta W}{\Delta t} \tag{73}$$

The rate at which work is done might not be constant. We can define **instantaneous power** *P* as the quotient in Eq. (73) as Δt approaches zero:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$
(74)

The SI unit of power is the **watt** (W), named for the English inventor James Watt. One watt equals 1 joule per second: 1 W = 1 J/s (see fig. 55). The kilowatt ($1kW=10^3 \text{ W}$) and the megawatt ($1MW=10^6 \text{ W}$) are also commonly used.



Figure 55 – The same amount of work is done in both of these situations, but the power (the rate at which work is done) is different

The watt is a familiar unit of electrical power; a 100-W light bulb converts 100 J of electrical energy into light and heat each second. But there's nothing inherently electrical about a watt. A light bulb could be rated in horsepower, and an engine can be rated in kilowatts.

The *kilowatt-hour* (kW·h) is the usual commercial unit of electrical energy. One kilowatt-hour is the total work done in 1 hour (3600 s) when the power is 1 kilowatt (10^3 J/s), so

$$1 kW \cdot h = (10^3 J/s)(3600 s) = 3.6 \times 10^6 J = 3.6 MJ$$

The kilowatt-hour is a unit of work or energy, not power.

In mechanics we can also express power in terms of force and velocity. Suppose that a force \vec{F} acts on a body while it undergoes a vector displacement $\Delta \vec{s}$. If F_{\parallel} is the component of \vec{F} tangent to the path (parallel to $\Delta \vec{s}$), then the work done by the force is $\Delta W = F_{\parallel} \Delta s$. The average power is

$$P_{av} = \frac{F_{\parallel}\Delta s}{\Delta t} = F_{\parallel}\frac{\Delta s}{\Delta t} = F_{\parallel}v_{av}$$
(75)

Instantaneous power P is the limit of this expression as $\Delta t \rightarrow 0$:

$$P = F_{\parallel} v \tag{76}$$

where v is the magnitude of the instantaneous velocity. We can also express Eq. (76) in terms of the scalar product:

$$P = \vec{F} \cdot \vec{v} \tag{77}$$

1.3.4 Potential energy

Gravitational Potential Energy. We learned early that a particle gains or loses kinetic energy because it interacts with other objects that exert forces on it. During any interaction, the change in a particle's kinetic energy is equal to the total work done on the particle by the forces that act on it.

In many situations it seems as though energy has been stored in a system, to be recovered later. For example, you must do work to lift a heavy stone over your head. It seems reasonable that in hoisting the stone into the air you are storing energy in the system, energy that is later converted into kinetic energy when you let the stone fall.

This example points to the idea of an energy associated with the *position* of bodies in a system. This kind of energy is a measure of the *potential* or *possibility* for work to be done; when a stone is raised into the air, there is a potential for work to be done on it by the gravitational force, but only if the stone is allowed to fall to

the ground. For this reason, energy associated with position is called **potential energy.** Our discussion suggests that there is potential energy associated with a body's weight and its height above the ground. We call this *gravitational potential energy* (see fig. 56).



Figure 56 – As a basketball descends, gravitational potential energy is conserved to kinetic energy and basketball's speed increases

We now have *two* ways to describe what happens when a body falls without air resistance. One way is to say that gravitational potential energy decreases and the falling body's kinetic energy increases. The other way, is that a falling body's kinetic energy increases because the force of the earth's gravity (the body's weight) does work on the body. Later in this section we'll use the work–energy theorem to show that these two descriptions are equivalent.

To begin with, however, let's derive the expression for gravitational potential energy. Suppose a body with mass *m* moves along the (vertical) *y*-axis, as in Fig. 57. The forces acting on it are its weight, with magnitude and possibly some other forces; we call the vector sum (resultant) of all the other forces We'll assume that the body stays close enough to the earth's surface that the weight is constant. We want to find the work done by the weight when the body moves downward from a height above the origin to a lower height (see fig. 57a). The weight and displacement are in the same direction, so the work W_{grav} done on the body by its weight is positive;

$$W_{grav} = Fs = w(y_1 - y_2) = mgy_1 - mgy_2$$
(78)

This expression also gives the correct work when the body moves upward and y_2 is greater than y_1 (see fig. 57b). In that case the quantity $(y_1 - y_2)$ is negative, and W_{grav} is negative because the weight and displacement are opposite in direction.

(a) A body moves downward



Figure 57 –When a body moves vertically from an initial height y_1 to a final height y_2 , the gravitational force \vec{w} does work and the gravitational potential energy changes

Equation (78) shows that we can express W_{grav} in terms of the values of the quantity mgy at the beginning and end of the displacement. This quantity, the product of the weight mg and the height y above the origin of coordinates, is called the **gravitational potential energy**, U_{grav} :

$$U_{grav} = mgy \tag{79}$$

Its initial value is $U_{grav1} = mgy_1$ and its final value is $U_{grav2} = mgy_2$. The change in U_{grav} is the final value minus the initial value, $\Delta U_{grav} = U_{grav2} - U_{grav1}$ or We can express the work W_{grav} done by the gravitational force during the displacement from y_1 to y_2 as

$$W_{grav} = U_{grav1} - U_{grav2} = -(U_{grav2} - U_{grav1}) = -\Delta U_{grav}$$
(80)

The negative sign in front of ΔU_{grav} is *essential*. When the body moves up, y increases, the work done by the gravitational force is negative, and the gravitational potential energy increases ($\Delta U_{grav} > 0$). When the body moves down, y decreases, the gravitational force does positive work, and the gravitational potential energy decreases ($\Delta U_{grav} < 0$). It's like drawing money out of the bank (decreasing) and spending it (doing positive work). The unit of potential energy is the joule (J), the same unit as is used for work.

Elastic Potential Energy. There are many situations in which we encounter potential energy that is not gravitational in nature. One example is a rubber-band slingshot. Work is done on the rubber band by the force that stretches it, and that work is stored in the rubber band until you let it go. Then the rubber band gives kinetic energy to the projectile.

This is the same pattern we saw with the pile driver: Do work on the system to store energy, which can later be converted to kinetic energy. We'll describe the process of storing energy in a deformable body such as a spring or rubber band in terms of *elastic potential energy* (see fig. 58). A body is called *elastic* if it returns to its original shape and size after being deformed. To be specific, we'll consider storing energy in an ideal spring. To keep such an ideal spring stretched by a distance x, we must exert a force F = kx, where k is the force constant of the spring. The ideal spring is a useful idealization because many elastic bodies show this same direct proportionality between force \vec{F} and displacement x, provided that x is sufficiently small.



Figure 58 – The Achilles tendon, with runs along the back the back of the ankle to the heel bone, acts like a natural spring. When it stretches and then relaxes, this tendon stores and then release elastic potential energy. This spring action reduce the amount of work your leg muscles must do as you run

Let's proceed just as we did for gravitational potential energy. We begin with the work done by the elastic (spring) force and then combine this with the work–energy theorem. The difference is that gravitational potential energy is a shared property of a body and the earth, but elastic potential energy is stored just in the spring (or other deformable body).

Figure 59 shows the ideal spring from, with its left end held stationary and its right end attached to a block with mass *m* that can move along the *x*-axis. In Fig. 59a the body is at x = 0 when the spring is neither stretched nor compressed. We move the block to one side, thereby stretching or compressing the spring, and then let it go. As the block moves from one position x_1 to another position x_2 how much work does the elastic (spring) force do on the block?


Figure 59 – Calculating the work done by a spring attached to a block on a horizontal surface. The quantity x is the extension to compression of the spring

We found early that the work we must do *on* the spring to move one end from an elongation x_1 to a different elongation x_2 is

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 \tag{81}$$

where k is the force constant of the spring. If we stretch the spring farther, we do positive work on the spring; if we let the spring relax while holding one end, we do negative work on it. We also saw that this expression for work is still correct if the spring is compressed, not stretched, so that x_1 or x_2 or both are negative. Now we need to find the work done by the spring. From Newton's third law the two quantities of work are just negatives of each other. Changing the signs in this equation, we find that in a displacement from x_1 to x_2 the spring does an amount of work W_{el} given by

$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 \tag{82}$$

The subscript "el" stands for *elastic*. When x_1 and x_2 are both positive and $x_2 > x_1$ (see fig. 59b), the spring does negative work on the block, which moves in the +xdirection while the spring pulls on it in the -x-direction. The spring stretches farther, and the block slows down. When x_1 and x_2 are both positive and $x_2 < x_1$ (see fig. 59c), the spring does positive work as it relaxes and the block speeds up. If the spring can be compressed as well as stretched, x_1 or x_2 or both may be negative, but the expression for W_{el} is still valid. In Fig. 59d, both x_1 and x_2 are negative, but x_2 is less negative than x_1 ; the compressed spring does positive work as it relaxes, speeding the block up.

Just as for gravitational work, we can express the work done by the spring in terms of a given quantity at the beginning and end of the displacement. This quantity is $\frac{1}{2}kx^2$ and we define it to be the elastic potential energy:

$$U_{el} = \frac{1}{2}kx^2\tag{83}$$

Figure 60 is a graph of Eq. (83). The unit of U_{el} is the joule (J), the unit used for all energy and work quantities; to see this from Eq. (83), recall that the units of k are N/m and that $1 N \cdot m = 1 J$.



Figure 60 – The graph of elastic potential energy for an ideal spring is a parabola: $U_{el} = \frac{1}{2}kx^2$, where x is the extension or compression of the spring. Elastic potential energy U_{el} is never negative

We can use Eq. (83) to express the W_{el} work done on the block by the elastic force in terms of the change in elastic potential energy:

$$W_{el} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = U_{el1} - U_{el2} = -\Delta U_{el}$$
(84)

When a stretched spring is stretched farther, as in Fig. 59b, W_{el} is negative and U_{el} increases; a greater amount of elastic potential energy is stored in the spring. When a stretched spring relaxes, as in Fig. 59c, x decreases, W_{el} is positive, and U_{el} decreases; the spring loses elastic potential energy. Negative values of x refer to a compressed spring. But, as Fig. 60 shows, U_{el} is positive for both positive and negative x, and Eqs. (83) and (84) are valid for both cases. The more a spring is compressed or stretched, the greater its elastic potential energy.

The work–energy theorem says that $W_{tot} = K_2 - K_1$ no matter what kind of forces are acting on a body. If the elastic force is the only force that does work on the body, then

$$W_{tot} = W_{el} = U_{el1} - U_{el2} \tag{85}$$

The work–energy theorem, $W_{tot} = K_2 - K_1$, then gives us

$$K_1 + U_{el1} = K_2 - U_{el2} \tag{86}$$

Here U_{el} is given by Eq. (83), so

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$
(87)

In this case the total mechanical energy $E = K + U_{el}$ — the sum of kinetic and elastic potential energy—is conserved. An example of this is the motion of the block in Fig. 59, provided the horizontal surface is frictionless so that no force does work other than that exerted by the spring.

For Eq. (87) to be strictly correct, the ideal spring that we've been discussing must also be *massless*. If the spring has a mass, it also has kinetic energy as the coils of the spring move back and forth. We can neglect the kinetic energy of the spring if its mass is much less than the mass m of the body attached to the spring. For instance, a typical automobile has a mass of 1200 kg or more. The springs in its suspension have masses of only a few kilograms, so their mass can be neglected if we want to study how a car bounces on its suspension.

1.4 Rotational of rigid body

1.4.1 Angular velocity and acceleration

In analyzing rotational motion, let's think first about a rigid body that rotates about a *fixed axis*—an axis that is at rest in some inertial frame of reference and does not change direction relative to that frame. The rotating rigid body might be a motor shaft, a chunk of beef on a barbecue skewer, or a merry-go-round.

Figure 61 shows a rigid body (in this case, the indicator needle of a speedometer) rotating about a fixed axis. The axis passes through point O and isperpendicular to the plane of the diagram, which we choose to call the *xy*-plane. One way to describe the rotation of this body would be to choose a particular point P on the body and to keep track of the *x*- and *y*-coordinates of this point. This isn't a terribly convenient method, since it takes two numbers (the two coordinates x and y) to specify the rotational position of the body. Instead, we notice that the line OP is fixed in the body and rotates with it. The angle that this line makes with the describes the rotational position of the body; we will use this single quantity as a *coordinate* for rotation.

The angular coordinate θ of a rigid body rotating around a fixed axis can be positive or negative. If we choose positive angles to be measured counterclockwise from the positive x-axis, then the angle θ in Fig. 61 is positive. If we instead choose the positive rotation direction to be clockwise, then θ in Fig. 61 is negative. When we considered the motion of a particle along a straight line, it was essential to specify the direction of positive displacement along that line; when we discuss rotation around a fixed axis, it's just as essential to specify the direction of positive rotation.



Figure 61 – A speedometer needle (an example of a rigid body) rotating counterclockwise about a fixed axis

To describe rotational motion, the most natural way to measure the angle θ is not in degrees, but in **radians.** As shown in Fig. 62a, one radian (1 rad) is the angle subtended at the center of a circle by an arc with a length equal to the radius of the circle. In Fig. 62b an angle θ is subtended by an arc of length *s* on a circle of radius *r*. The value of (in radians) is equal to *s* divided by *r*:

$$\theta = \frac{s}{r} \tag{88}$$

or

$$s = r\theta \tag{89}$$

An angle in radians is the ratio of two lengths, so it is a pure number, without dimensions. If s = 3.0 m and r = 2.0 m then $\theta = 1.5$ but we will often write this as 1.5 rad to distinguish it from an angle measured in degrees or revolutions.

The circumference of a circle (that is, the arc length all the way around the circle) is 2π times the radius, so there are 2π (about 6.283) radians in one complete revolution (360 °). Therefore

$$1 rad = \frac{360^{\circ}}{2\pi} 57.3^{\circ}$$

Similarly, $360^\circ = \pi rad$, $90^\circ = \frac{\pi}{2} rad$, and so on. If we had insisted on measuring the angle in degrees, we would have needed to include an extra factor of $2\pi/360$ on the right-hand side of $s = r\theta$ in Eq. (89). By measuring angles in radians, we

keep the relationship between angle and distance along an arc as simple as possible.



Figure 62 – Measuring angels in radians

The coordinate θ shown in Fig. 61 specifies the rotational position of a rigid body at a given instant. We can describe the rotational *motion* of such a rigid body in terms of the rate of change of θ . We'll do this in an analogous way to our description of straight-line motion. In Fig. 63a, a reference line *OP* in a rotating body makes an angle θ_1 with the +x-axis at time t_1 . At a later time t_2 the angle has changed to θ_2 . We define the **average angular velocity** ω_{av-z} (the Greek letter omega) of the body in the time interval $\Delta = t_2 - t_1$ as the ratio of the **angular displacement** $\Delta = \theta_2 - \theta_1$ to $\Delta \theta$:

$$\omega_{a\nu-z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \tag{90}$$

The subscript z indicates that the body in Fig. 63a is rotating about the z-axis, which is perpendicular to the plane of the diagram. The **instantaneous angular**

velocity ω_z is the limit of ω_{av-z} as Δt approaches zero—that is, the derivative of θ with respect to *t*:

$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$
(91)

When we refer simply to "angular velocity," we mean the instantaneous angular velocity, not the average angular velocity.

The angular velocity ω_z can be positive or negative, depending on the direction in which the rigid body is rotating (see fig. 64). The angular speed ω is the magnitude of angular velocity. Like ordinary (linear) speed v the angular speed is never negative.



Figure 63 – (a) Angular displacement $\Delta\theta$ of a rotating body. (b) Every part of a rotating rigid body has the same average angular velocity $\Delta\theta/\Delta t$

Different points on a rotating rigid body move different distances in a given time interval, depending on how far each point lies from the rotation axis. But because the body is rigid, *all* points rotate through the same angle in the same time (see fig. 63b). Hence *at any instant, every part of a rotating rigid body has the same angular velocity*. The angular velocity is positive if the body is rotating in the direction of increasing θ and negative if it is rotating in the direction of decreasing θ .

If the angle is in radians, the unit of angular velocity is the radian per second (rad/s). Other units, such as the revolution per minute (rev/min or rpm), are often used. Since $1 rev = 2\pi rad$, two useful conversions are

$$1\frac{rev}{s} = 2\pi \frac{rad}{s}$$

and

$$1\frac{rev}{min} = 1 rpm = \frac{2\pi}{60} rad/s$$

That is, 1 rad/s is about 10 rpm.



Figure 64 – A rigid body's average angular velocity (shown here) and instantaneous angular velocity can be positive or negative

As we have seen, our notation for the angular velocity ω_z about the z-axis is reminiscent of the notation v_x for the ordinary velocity along the x-axis. Just as v_x is the x-component of the velocity vector \vec{v}, ω_z is the z-component of an angular velocity vector $\vec{\omega}$ directed along the axis of rotation. As Fig. 65a shows, the direction $\vec{\omega}$ of is given by the right-hand rule that we used to define the vector product. If the rotation is about the z-axis, then $\vec{\omega}$ has only z-component; this component is positive if $\vec{\omega}$ is along the positive z-axis and negative if $\vec{\omega}$ is along the negative z-axis (see fig. 65b).

The vector formulation is especially useful in situations in which the direction of the rotation axis *changes*. In this chapter, however, we'll consider only situations in which the rotation axis is fixed. Hence throughout this chapter we'll use "angular velocity" to refer ω_z to the component of the angular velocity vector $\vec{\omega}$ along the axis.



Figure 65 – (a) The right-hand rule for the direction of the angular velocity vector $\vec{\omega}$. Reserving the direction of rotation reverses the direction of $\vec{\omega}$. (b) The sign of ω_z for rotation along the z-axis

When the angular velocity of a rigid body changes, it has an *angular acceleration*. When you pedal your bicycle harder to make the wheels turn faster or apply the brakes to bring the wheels to a stop, you're giving the wheels an angular acceleration. You also impart an angular acceleration whenever you change the rotation speed of a piece of spinning machinery such as an automobile engine's crankshaft.

If ω_{z1} and ω_{z2} are the instantaneous angular velocities at times t_1 and t_2 , we define the **average angular acceleration** ε_{av-z} over the interval $\Delta t = t_2 - t_1$ as the change in angular velocity divided by Δt (see fig. 66):

$$\varepsilon_{av-z} = \frac{\omega_{z2} - \omega_{z1}}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \tag{92}$$

The instantaneous angular acceleration ε_z is the limit of ε_{av-z} as $\Delta t \to 0$:

$$\varepsilon_z = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$
(93)

The usual unit of angular acceleration is the radian per second per second, or rad/s^2 . From now on we will use the term "angular acceleration" to mean the instantaneous angular acceleration rather than the average angular acceleration.

The average angular acceleration is the change in angular velocity divided by the time interval:



Figure 66 – Calculating the average angular acceleration of a rotating body

Because $\omega_z = d\theta/dt$, we can also express angular acceleration as the second derivative of the angular coordinate:

$$\varepsilon_z = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} \tag{94}$$

You have probably noticed that we are using Greek letters for angular kinematic quantities: θ for angular position, ω_z for angular velocity, and ε_z for angular acceleration. These are analogous to x for position, v_x for velocity, and a_x for acceleration, respectively, in straight-line motion. In each case, velocity is the rate of change of position with respect to time and acceleration is the rate of change of velocity with respect to time. We will sometimes use the terms "*linear* velocity" and "*linear* acceleration" for the familiar quantities we defined early to distinguish clearly between these and the *angular* quantities introduced in this chapter.

In rotational motion, if the angular acceleration ε_z is positive, then the angular velocity ω_z is increasing; if ε_z is negative, then ω_z is decreasing. The rotation is speeding up if ε_z and ω_z have the same sign and slowing down if ε_z and ω_z have opposite signs.

Just as we did for angular velocity, it's useful to define an angular acceleration vector $\vec{\varepsilon}$. Mathematically, $\vec{\varepsilon}$ is the time derivative of the angular velocity vector $\vec{\omega}$. If the object rotates around the fixed z-axis, then $\vec{\varepsilon}$ has only a z-component; the quantity ε_z is just that component. In this case, $\vec{\varepsilon}$ is in the same

direction as $\vec{\omega}$ if the rotation is speeding up and opposite $\vec{\omega}$ to if the rotation is slowing down (see fig. 67).



Figure 67 – When the rotation axis is fixed, the angular acceleration and angular velocity vectors both lie along that axis

1.4.2 Rotation with constant angular acceleration

Early we found that straight-line motion is particularly simple when the acceleration is constant. This is also true of rotational motion about a fixed axis. When the angular acceleration is constant, we can derive equations for angular velocity and angular position using exactly the same procedure that we used for straight-line motion. In fact, the equations we are about to derive are identical to Eqs. (8), (13), (18), and (19) if we replace x with θ , v_x with ω_z , and a_x with ε_z .

Let ω_{0z} be the angular velocity of a rigid body at time t = 0, and let ω_z be its angular velocity at any later time t. The angular acceleration ε_z is constant and equal to the average value for any interval. Using Eq. (92) with the interval from 0 to t, we find

$$\varepsilon_z = \frac{\omega_z - \omega_{0z}}{t - 0} \tag{95}$$

or

$$\omega_z = \omega_{0z} + \varepsilon_z t \tag{96}$$

The product $\varepsilon_z t$ is the total change in ω_z between t = 0 and the later time t; the angular velocity ω_z at time t is the sum of the initial value ω_{0z} and this total change.

With constant angular acceleration, the angular velocity changes at a uniform rate, so its average value between 0 and t is the average of the initial and final values:

$$\omega_{av-z} = \frac{\omega_{0z} + \omega_z}{2} \tag{97}$$

We also know that ω_{av-z} is the total angular displacement $(\theta - \theta_0)$ divided by the time interval (t - 0)

$$\omega_{av-z} = \frac{\theta - \theta_0}{t - 0} \tag{98}$$

When we equate Eqs. (97) and (98) and multiply the result by t, we get

$$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t \tag{99}$$

To obtain a relationship between θ and t that doesn't contain ω_z , we substitute Eq. (96) into Eq. (99):

$$\theta - \theta_0 = \frac{1}{2} [\omega_{0z} + (\omega_{0z} + \varepsilon_z t)]t$$
⁽¹⁰⁰⁾

or

$$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\varepsilon_z t^2 \tag{101}$$

That is, if at the initial time t = 0 the body is at angular position θ_0 and has angular velocity ω_{0z} , then its angular position θ at any later time t is the sum of three terms: its initial angular position θ_0 , plus the rotation $\omega_{0z}t$ it would have if the angular velocity were constant, plus an additional rotation $\frac{1}{2}\varepsilon_z t^2$ caused by the changing angular velocity.Following the same procedure as for straight-line motion, we can combine Eqs. (96) and (101) to obtain a relationship between θ and ω_z that does not contain t. We invite you to work out the details, following the same procedure we used to get Eq. (18). In fact, because of the perfect analogy between straight-line and rotational quantities, we can simply take Eq. (18) and replace each straight-line quantity by its rotational analog. We get

$$\omega_z^2 = \omega_{0z}^2 + 2\varepsilon_z(\theta - \theta_0) \tag{102}$$

L	0		
Straight-line motion with		Fixed-axis rotation with	
constant linear acceleration		constant angular acceleration	
$a_x = constant$		$\varepsilon_z = constant$	
$v_x = v_{0x} + a_x t$	(8)	$\omega_z = \omega_{0z} + \varepsilon_z t$	(96)
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	(13)	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\varepsilon_z t^2$	(101)
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	(18)	$\omega_z^2 = \omega_{0z}^2 + 2\varepsilon_z(\theta - \theta_0)$	(102)
$x - x_0 = \left(\frac{v_x - v_{0x}}{2}\right)t$	(19)	$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$	(99)

Table 3 Comparison of linear and angular motion with constant acceleration

1.4.3 Relating linear and angular kinematics

How do we find the linear speed and acceleration of a particular point in a rotating rigid body? We need to answer this question to proceed with our study of rotation. For example, to find the kinetic energy of a rotating body, we have to start from $K = \frac{1}{2}mv^2$ for a particle, and this requires knowing the speed for each particle in the body. So it's worthwhile to develop general relationships between the *angular* speed and acceleration of a rigid body rotating about a fixed axis and the *linear* speed and acceleration of a specific point or particle in the body.

When a rigid body rotates about a fixed axis, every particle in the body moves in a circular path. The circle lies in a plane perpendicular to the axis and is centered on the axis. The speed of a particle is directly proportional to the body's angular velocity; the faster the body rotates, the greater the speed of each particle. In Fig. 68, point *P* is a constant distance *r* from the axis of rotation, so it moves in a circle of radius *r*. At any time, the angle θ (in radians) and the arc length *s* are related by

$$s = r\theta \tag{103}$$

We take the time derivative of this, noting that r is constant for any specific particle, and take the absolute value of both sides:

$$\left|\frac{ds}{dt}\right| = r \left|\frac{d\theta}{dt}\right| \tag{104}$$

Now $\left|\frac{ds}{dt}\right|$ is the absolute value of the rate of change of arc length, which is equal to the instantaneous *linear* speed v of the particle. Analogously, $\left|\frac{d\theta}{dt}\right|$, the absolute value of the rate of change of the angle, is the instantaneous **angular speed** ω — that is, the magnitude of the instantaneous angular velocity in rad/s. Thus

$$v = \omega r \tag{105}$$

The farther a point is from the axis, the greater its linear speed. The *direction* of the linear velocity *vector* is tangent to its circular path at each point (see fig. 68).



Figure 68 – A rigid body rotating about a fixed axis through point O

We can represent the acceleration of a particle moving in a circle in terms of its centripetal and tangential components, a_{rad} and a_{tan} (see fig. 69). It would be a good idea to review that section now. We found that the **tangential component of acceleration** the component parallel to the instantaneous velocity, acts to change the *magnitude* of the particle's velocity (i.e., the speed) and is equal to the rate of change of speed. Taking the derivative of Eq. (105), we find

$$a_{tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\varepsilon \tag{106}$$

This component of a particle's acceleration is always tangent to the circular path of the particle.

The quantity $\varepsilon = \frac{d\omega}{dt}$ in Eq. (106) is the rate of change of the angular *speed*. It is not quite the same as $\varepsilon_z = d\omega_z/dt$, which is the rate of change of the angular *velocity*. For example, consider a body rotating so that its angular velocity vector points in the -z-direction (see Fig. 65b). If the body is gaining angular speed at a rate of 10 rad/s per second, then $\varepsilon = 10 \ rad/s^2$. But ω_z is negative and becoming more negative as the rotation gains speed, so $\varepsilon_z = -10 \ rad/s^2$ The rule for rotation about a fixed axis is that ε is equal to ε_z if ω_z is positive but equal to $-\varepsilon_z$ if ω_z is negative.

The component of the particle's acceleration directed toward the rotation axis, the **centripetal component of acceleration** a_{rad} , is associated with the change of *direction* of the particle's velocity. Early we worked out the relationship $a_{rad} = \frac{v^2}{r}$. We can express this in terms of ω by using Eq. (105):

$$a_{rad} = \frac{v^2}{r} = \omega^2 r \tag{107}$$

Radial and tangential acceleration components:

• $a_{\rm rad} = \omega^2 r$ is point *P*'s centripetal acceleration.

• $a_{tan} = r\alpha$ means that *P*'s rotation is speeding up (the body has angular acceleration).



Figure 69 – A rigid body whose rotation is speeding up. The acceleration of point P has a component a_{rad} towards the rotation axis (perpendicular to \vec{v}) and a component a_{tan} along the circle that point P follows (parallel to \vec{v})

This is true at each instant, *even when and are not constant*. The centripetal component always points toward the axis of rotation.

The vector sum of the centripetal and tangential components of acceleration of a particle in a rotating body is the linear acceleration \vec{a} (see fig. 69).

Equations (89), (105), and (106) also apply to any particle that has the same tangential velocity as a point in a rotating rigid body. For example, when a rope

wound around a circular cylinder unwraps without stretching or slipping, its speed and acceleration at any instant are equal to the speed and tangential acceleration of the point at which it is tangent to the cylinder. The same principle holds for situations such as bicycle chains and sprockets, belts and pulleys that turn without slipping, and so on. We will have several opportunities to use these relationships later in this chapter. Note that Eq. (107) for the centripetal component is applicable to the rope or chain *only* at points that are in contact with the cylinder or sprocket. Other points do not have the same acceleration toward the center of the circle that points on the cylinder or sprocket have.

It's important to remember that Eq. (89), is valid only when is angle measured in radians. The same is true of any equation derived from this, including Eqs. (105), (106), and (107). When you use these equations, you must express the angular quantities in radians, not revolutions or degrees (see fig. 70).





1.4.4 Energy in rotational motion

A rotating rigid body consists of mass in motion, so it has kinetic energy. As we will see, we can express this kinetic energy in terms of the body's angular speed and a new quantity, called *moment of inertia*, that depends on the body's mass and how the mass is distributed.

To begin, we think of a body as being made up of a large number of particles, with masses $m_1, m_2, ...$ at distances $r_1, r_2, ...$ from the axis of rotation. We label the particles with the index *i*: The mass of the *i*th particle is m_i and its

distance from the axis of rotation is r_i . The particles don't necessarily all lie in the same plane, so we specify that r_i is the *perpendicular* distance from the axis to the *i*th particle.

When a rigid body rotates about a fixed axis, the speed v_i of the *i*th particle is given by Eq. (105), $v_i = r_i \omega$ where ω is the body's angular speed. Different particles have different values of *r*, but ω is the same for all (otherwise, the body wouldn't be rigid). The kinetic energy of the *i*th particle can be expressed as

$$\frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2 \omega^2 \tag{108}$$

The *total* kinetic energy of the body is the sum of the kinetic energies of all its particles:

$$K = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \dots = \sum_i \frac{1}{2}m_ir_i^2\omega^2$$
(109)

Taking the common factor $\omega^2/2$ out of this expression, we get

$$K = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + \dots)\omega^2 = \frac{1}{2} \left(\sum_i m_i r_i^2\right) \omega^2$$
(110)

The quantity in parentheses, obtained by multiplying the mass of each particle by the square of its distance from the axis of rotation and adding these products, is denoted by *I* and is called the **moment of inertia** of the body for this rotation axis:

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots = \sum_i m_i r_i^2$$
(111)

The word "moment" means that *I* depends on how the body's mass is distributed in space; it has nothing to do with a "moment" of time. For a body with a given rotation axis and a given total mass, the greater the distance from the axis to theparticles that make up the body, the greater the moment of inertia. In a rigid body, the distances r_i are all constant and *I* is independent of how the body rotates around the given axis. The SI unit of moment of inertia is the kilogram-meter² (kg·m²).

In terms of moment of inertia I, the **rotational kinetic energy** K of a rigid body is

$$K = \frac{1}{2}I\omega^2 \tag{112}$$

The kinetic energy given by Eq. (112) is *not* a new form of energy; it's simply the sum of the kinetic energies of the individual particles that make up the rotating rigid body. To use Eq. (112), ω must be measured in radians per second, not revolutions or degrees per second, to give K in joules. That's because we used $v_i = r_i \omega$ in our derivation.

Equation (112) gives a simple physical interpretation of moment of inertia: *The greater the moment of inertia, the greater the kinetic energy of a rigid body rotating with a given angular speed* ω . We learned that the kinetic energy of a body equals the amount of work done to accelerate that body from rest. So the greater a body's moment of inertia, the harder it is to start the bodyrotating if it's at rest and the harder it is to stop its rotation if it's already rotating (see fig. 71). For this reason, *I* is also called the *rotational inertia*.



Figure 71 – An apparatus free to rotate around a vertical axis. To vary the moment of inertia, the two equal-mass cylinders can be locked into different positions on the horizontal shaft

1.5 Conservation's laws

1.5.1 Conservation's law of energy

In our discussions of potential energy we have talked about "storing" kinetic energy by converting it to potential energy. We always have in mind that later we may retrieve it again as kinetic energy. For example, when you throw a ball up in the air, it slows down as kinetic energy is converted to gravitational potential energy. But on the way down, the conversion is reversed, and the ball speeds up as potential energy is converted back to kinetic energy. If there is no air resistance, the ball is moving just as fast when you catch it as when you threw it.

Another example is a glider moving on a frictionless horizontal air track that runs into a spring bumper at the end of the track. The glider stops as it compresses the spring and then bounces back. If there is no friction, the glider ends up with the same speed and kinetic energy it had before the collision. Again, there is a twoway conversion from kinetic to potential energy and back. In both cases we can define a potential-energy function so that the total mechanical energy, kinetic plus potential, is constant or conserved during the motion.

A force that offers this opportunity of two-way conversion between kinetic and potential energies is called a **conservative force**. We have seen two examples of conservative forces: the gravitational force and the spring force. An essential feature of conservative forces is that their work is always *reversible*. Anything that we deposit in the energy "bank" can later be withdrawn without loss. Another important aspect of conservative forces is that a body may move from point 1 to point 2 by various paths, but the work done by a conservative force is the same for all of these paths (see fig. 72). Thus, if a body stays close to the surface of the earth, the gravitational force $m\vec{g}$ is independent of height, and the work done by this force depends only on the change in height. If the body moves around a closed path, ending at the same point where it started, the total work done by the gravitational force is always zero.



Figure 72 - The work done by a conservative force such as gravity depends only on the end points of a path, not on the specific path taken between those points

The work done by a conservative force always has four properties:

1. It can be expressed as the difference between the initial and final values of a potential-energy function.

2. It is reversible.

3. It is independent of the path of the body and depends only on the starting and ending points.

4. When the starting and ending points are the same, the total work is zero.

When the only forces that do work are conservative forces, the total mechanical energy E = K + U is constant.

Not all forces are conservative. Consider the friction force acting on the crate sliding on a ramp. When the body slides up and then back down to the starting point, the total work done on it by the friction force is not zero. When the direction of motion reverses, so does the friction force, and friction does negative work in both directions. When a car with its brakes locked skids across the pavement with decreasing speed (and decreasing kinetic energy), the lost kinetic energy cannot be recovered by reversing the motion or in any other way, and mechanical energy is not conserved. There is no potential-energy function for the friction force.

In the same way, the force of fluid resistance is not conservative. If you throw a ball up in the air, air resistance does negative work on the ball while it's rising and while it's descending. The ball returns to your hand with less speed and less kinetic energy than when it left, and there is no way to get back the lost mechanical energy.

A force that is not conservative is called a **nonconservative force**. The work done by a nonconservative force cannot be represented by a potential-energy function. Some nonconservative forces, like kinetic friction or fluid resistance, cause mechanical energy to be lost or dissipated; a force of this kind is called a **dissipative force**. There are also nonconservative forces that increase mechanical energy. The fragments of an exploding firecracker fly off with very large kinetic energy, thanks to a chemical reaction of gunpowder with oxygen. The forces unleashed by this reaction are nonconservative because the process is not reversible. (The fragments never spontaneously reassemble themselves into a complete firecracker!)

Nonconservative forces cannot be represented in terms of potential energy. But we can describe the effects of these forces in terms of kinds of energy other than kinetic and potential energy. When a car with locked brakes skids to a stop, the tires and the road surface both become hotter. The energy associated with this change in the state of the materials is called **internal energy**. Raising the temperature of a body increases its internal energy; lowering the body's temperature decreases its internal energy.

To see the significance of internal energy, let's consider a block sliding on a rough surface. Friction does *negative* work on the block as it slides, and the change in internal energy of the block and surface (both of which get hotter) is *positive*. Careful experiments show that the increase in the internal energy is exactly equal to the absolute value of the work done by friction. In other words,

$$\Delta U_{int} = -W_{other} \tag{113}$$

where ΔU_{int} is the change in internal energy. If we substitute this into Eq. (7.7) or (7.14), we find

$$K_1 + U_1 - \Delta U_{int} = K_2 + U_2 \tag{114}$$

Writing
$$\Delta K = K_2 - K_1$$
 and $\Delta U = U_2 - U_1$ we can finally express this as
 $\Delta K + \Delta U - \Delta U_{int} = 0$ (115)

This remarkable statement is the general form of the **law of conservation of energy**. In a given process, the kinetic energy, potential energy, and internal energy of a system may all change. But the sum of those changes is always zero. If there is a decrease in one form of energy, it is made up for by an increase in the other forms (see fig. 73). When we expand our definition of energy to include internal energy, Eq. (115) says: *Energy is never created or destroyed; it only changes form*. No exception to this rule has ever been found.



Figure 73 – When 1 liter of gasoline is burned in an automotive engine, it releases $3.3 \times 10^7 J$ of internal energy. Hence $\Delta U_{int} = -3.3 \times 10^7 J$, where the minus sign means that the amount of energy stored in the gasoline has decreased. This energy can be converted to kinetic energy (making the car go faster) or to potential energy (enabling the car to climb uphill)

The concept of work has been banished from Eq. (115); instead, it suggests that we think purely in terms of the conversion of energy from one form to another. For example, when you throw a baseball straight up, you convert a portion of the internal energy of your molecules to kinetic energy of the baseball. This is converted to gravitational potential energy as the ball climbs and back to kinetic energy as the ball falls. If there is air resistance, part of the energy is used to heat up the air and the ball and increase their internal energy. Energy is converted back to the kinetic form as the ball falls. If you catch the ball in your hand, whatever energy was not lost to the air once again becomes internal energy; the ball and your hand are now warmer than they were at the beginning.

1.5.2 Momentum and impulse

Early we re-expressed Newton's second law for a particle, in terms of the work–energy theorem. This theorem helped us tackle a great number of physics problems and led us to the law of conservation of energy. Let's now return to and see yet another useful way to restate this fundamental law.

Consider a particle of constant mass m. (Later we'll see how to deal with situations in which the mass of a body changes.) Because $\vec{a} = d\vec{v}/dt$, we can write Newton's second law for this particle as

$$\sum \vec{F} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \tag{116}$$

We can move the mass m inside the derivative because it is constant. Thus Newton's second law says that the net force $\sum \vec{F}$ acting on a particle equals the time rate of change of the combination $m\vec{v}$, the product of the particle's mass and velocity. We'll call this combination the **momentum**, or **linear momentum**, of the particle. Using the symbol \vec{p} for momentum, we have

$$\vec{p} = m\vec{v} \tag{117}$$

The greater the mass mand speed \vec{v} of a particle, the greater is its magnitude of momentum mv. Keep in mind, however, that momentum is a vector quantity with the same direction as the particle's velocity (see fig. 8.1). Hence a car driving north at 20 m/s and an identical car driving east at 20 m/s have the same magnitude of momentum (mv) but different momentum vectors $(m\vec{v})$ because their directions are different.



Momentum \vec{p} is a vector quantity; a particle's momentum has the same direction as its velocity \vec{v} .

Figure 74 – The velocity and momentum vectors of a particle

We often express the momentum of a particle in terms of its components. If the particle has velocity components v_x , v_y and v_z then its momentum components p_x , p_y and p_z (which we also call the x-momentum, y-momentum, and zmomentum) are given by

$$p_x = mv_x, p_y = mv_y, p_z = mv_z \tag{118}$$

These three component equations are equivalent to Eq. (117).

The units of the magnitude of momentum are units of mass times speed; the SI units of momentum are $kg \cdot m/s$. The plural of momentum is "momenta."

If we now substitute the definition of momentum, Eq. (117), into Eq. (116), we get

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \tag{119}$$

. This, not $\sum \vec{F} = m\vec{a}$ is the form in which Newton originally stated his second law (although he called momentum the "quantity of motion"). This law is valid only in inertial frames of reference.

According to Eq. (119), a rapid change in momentum requires a large net force, while a gradual change in momentum requires less net force. This principle is used in the design of automobile safety devices such as air bags (see fig. 75).

A particle's momentum $\vec{p} = m\vec{v}$ and its kinetic energy $K = \frac{1}{2}mv^2$ both depend on the mass and velocity of the particle. What is the fundamental difference between these two quantities? A purely mathematical answer is that momentum is a vector whose magnitude is proportional to speed, while kinetic

energy is a scalar proportional to the speed squared. But to see the physical difference between momentum and kinetic energy, we must first define a quantity closely related to momentum called *impulse*.

Let's first consider a particle acted on by a *constant* net force $\sum \vec{F}$ during a time interval Δt from t_1 to t_2 (We'll look at the case of varying forces shortly.) The **impulse** of the net force, denoted by \vec{J} is defined to be the product of the net force and the time interval:

$$\vec{J} = \sum \vec{F} (t_2 - t_1) = \sum \vec{F} \Delta t$$
 (120)

Impulse is a vector quantity; its direction is the same as the net force $\sum \vec{F}$. Its magnitude is the product of the magnitude of the net force and the length of time that the net force acts. The SI unit of impulse is the newton-second $(N \cdot s)$ Because $1 N = 1 kg \cdot m/s^2$, an alternative set of units for impulse is $kg \cdot m/s^2$, the same as the units of momentum.



Figure 75 – If a fast moving automatic stops suddenly in a collision, the driver's momentum (mass times velocity) changes from a large value to zero in a short time. An air bag cause the drivers to lose momentum more gradually than would an abrupt collision with the steering wheel, reducing the force exerted on the drivers as well as the possibility of injury

To see what impulse is good for, let's go back to Newton's second law as restated in terms of momentum, Eq. (119). If the net force $\sum \vec{F}$ is constant, then

 $d\vec{p}/dt$ is also constant. In that case, $d\vec{p}/dt$ is equal to the *total* change in momentum $\vec{p}_2 - \vec{p}_1$ during the time interval $t_2 - t_1$, divided by the interval:

$$\sum \vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1} \tag{121}$$

Multiplying this equation by $(t_2 - t_1)$ we have

(122)
$$\sum \vec{F} (t_2 - t_1) = \vec{p}_2 - \vec{p}_1$$

Comparing with Eq. (120), we end up with a result called the **impulse-momentum theorem:**

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \tag{123}$$

The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval.

The impulse-momentum theorem also holds when forces are not constant. To see this, we integrate both sides of Newton's second law $\sum \vec{F} = d\vec{p}/dt$ over time between the limits t_1 and t_2 :

$$\int_{t_1}^{t_2} \sum_{\vec{r}} \vec{F} dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt} dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$
(124)

The integral on the left is defined to be the impulse \vec{J} of the net force $\sum \vec{F}$ during this interval:

$$\vec{J} = \int_{t_1}^{t_2} \sum_{\vec{F}} \vec{F} \, dt$$
(125)

With this definition, the impulse–momentum theorem $\vec{J} = \vec{p}_2 - \vec{p}_1$ Eq. (123), is valid even when the net force $\sum \vec{F}$ varies with time.

We can define an *average* net \vec{J} force \vec{F}_{av} such that even when $\sum \vec{F}$ is not constant, the impulse is given by

$$\vec{J} = \vec{F}_{av}(t_2 - t_1)$$
(126)

When $\sum \vec{F}$ is constant, $\sum \vec{F} = \vec{F}_{av}$ and Eq. (126) reduces to Eq. (120).

Figure 76a shows the x-component of net force $\sum \vec{F}$ as a function of time during a collision. This might represent the force on a soccer ball that is in contact with a player's foot from time t_1 to t_2 . The x-component of impulse during this interval is represented by the red area under the curve between t_1 and t_2 . This area is equal to the green rectangular area bounded by t_1, t_2 and $(\vec{F}_{av})_x$ so $(\vec{F}_{av})_x(t_2 - t1)$ is equal to the impulse of the actual time-varying force during the same interval. Note that a large force acting for a short time can have the same impulse as a smaller force acting for a longer time if the areas under the force-time curves are the same (see fig. 8.3b). In this language, an automobile airbag (see Fig. 62) provides the same impulse to the driver as would the steering wheel or the dashboard by applying a weaker and less injurious force for a longer time.



Figure 76 – The meaning of the area under a graph of $\sum F_x$ versus t

Impulse and momentum are both vector quantities, and Eqs. (120)–(126) are all vector equations. In specific problems, it is often easiest to use them in component form:

$$J_{x} = \int_{t_{1}}^{t_{2}} \sum F_{x} dt = (\vec{F}_{av})_{x} (t_{2} - t_{1}) = \vec{p}_{2x} - \vec{p}_{1x} = \vec{v}_{2x} - \vec{v}_{1x}$$

$$J_{y} = \int_{t_{1}}^{t_{2}} \sum F_{y} dt = (\vec{F}_{av})_{y} (t_{2} - t_{1}) = \vec{p}_{2y} - \vec{p}_{1y} = \vec{v}_{2y} - \vec{v}_{1y}$$
(127)

and similarly for the *z*-component.

1.5.3 Conservation's law of momentum

The concept of momentum is particularly important in situations in which we have two or more bodies that *interact*. To see why, let's consider first an idealized system of two bodies that interact with each other but not with anything else—for example, two astronauts who touch each other as they float freely in the zero-gravity environment of outer space (see fig. 8.8). Think of the astronauts as particles. Each particle exerts a force on the other; according to Newton's third law, the two forces are always equal in magnitude and opposite in direction. Hence, the *impulses* that act on the two particles are equal and opposite, and the changes in momentum of the two particles are equal and opposite.

Let's go over that again with some new terminology. For any system, the forces that the particles of the system exert on each other are called **internal forces.** Forces exerted on any part of the system by some object outside it are called **external forces.** For the system shown in Fig. 8.8, the internal forces are $\vec{F}_{B \text{ on } A}$, exerted by particle *B* on particle *A*, and $\vec{F}_{A \text{ on } B}$ exerted by particle *A* on particle *B*. There are *no* external forces; when this is the case, we have an **isolated system**.



No external forces act on the two-astronaut system, so its total momentum is conserved.



The forces the astronauts exert on each other form an action-reaction pair.

Figure 77 – Two astronauts push each other as they float freely in the zero-gravity environment of space

The net force on particle *A* is $\vec{F}_{B on A}$ and the net force on particle *B* is $\vec{F}_{A on B}$ so from Eq. (119) the rates of change of the momenta of the two particles are

$$\vec{F}_{B \text{ on } A} = \frac{d\vec{p}_A}{dt}, \vec{F}_{A \text{ on } B} = \frac{d\vec{p}_B}{dt}$$
(128)

The momentum of each particle changes, but these changes are related to each other by Newton's third law: The two forces $\vec{F}_{B on A}$ and $\vec{F}_{A on B}$ are always equal in magnitude and opposite in direction. That is, $\vec{F}_{B on A} = -\vec{F}_{A on B}$ so $\vec{F}_{B on A} + \vec{F}_{A on B} = 0$. Adding together the two equations in Eq. (128), we have

$$\vec{F}_{B \ on \ A} + \vec{F}_{A \ on \ B} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = 0 \tag{129}$$

The rates of change of the two momenta are equal and opposite, so the rate of change of the vector sum $\vec{p}_A + \vec{p}_B$ is zero. We now define the **total momentum** \vec{P} of the system of two particles as the vector sum of the momenta of the individual particles; that is,

$$\vec{P} = \vec{p}_A + \vec{p}_B \tag{130}$$

Then Eq. (129) becomes, finally,

$$\vec{F}_{B \ on \ A} + \vec{F}_{A \ on \ B} = \frac{d\vec{P}}{dt} = 0 \tag{131}$$

The time rate of change of the *total* momentum \vec{P} is zero. Hence the total momentum of the system is constant, even though the individual momenta of the particles that make up the system can change.

If external forces are also present, they must be included on the left side of Eq. (131) along with the internal forces. Then the total momentum is, in general, not constant. But if the vector sum of the external forces is zero, as in Fig. 8.9, these forces have no effect on the left side of Eq. (131), and $\frac{d\vec{P}}{dt}$ is again zero. Thus we have the following general result:

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

This is the simplest form of the **principle of conservation of momentum.** This principle is a direct consequence of Newton's third law. What makes this principle useful is that it doesn't depend on the detailed nature of the internal forces that act between members of the system. This means that we can apply conservation of momentum even if (as is often the case) we know very little about the internal forces. We have used Newton's second law to derive this principle, so we have to be careful to use it only in inertial frames of reference.

We can generalize this principle for a system that contains any number of particles A, B, C, . . . interacting only with one another. The total momentum of such a system is

$$\vec{P} = \vec{p}_A + \vec{p}_B + \dots = m_A \vec{v}_A + m_B \vec{v}_B + \dots$$
 (132)

We make the same argument as before: The total rate of change of momentum of the system due to each action–reaction pair of internal forces is zero. Thus the total rate of change of momentum of the entire system is zero whenever the vector sum of the external forces acting on it is zero. The internal forces can change the momenta of individual particles in the system but not the total momentum of the system.

In some ways the principle of conservation of momentum is more general than the principle of conservation of mechanical energy. For example, mechanical energy is conserved only when the internal forces are *conservative*—that is, when the forces allow two-way conversion between kinetic and potential energy—but conservation of momentum is valid even when the internal forces are *not* conservative. In this chapter we will analyze situations in which both momentum

and mechanical energy are conserved, and others in which only momentum is conserved. These two principles play a fundamental role in all areas of physics, and we will encounter them throughout our study of physics.

1.5.4 Torque and angular momentum

We know that forces acting on a body can affect its **translational motion** that is, the motion of the body as a whole through space. Now we want to learn which aspects of a force determine how effective it is in causing or changing *rotational* motion. The magnitude and direction of the force are important, but so is the point on the body where the force is applied. In Fig. 78 a wrench is being used to loosen a tight bolt. Force \vec{F}_b applied near the end of the handle, is more effective than an equal force \vec{F}_a applied near the bolt. Force doesn't do any good at all; it's applied at the same point and has the same magnitude as but it's directed along the length of the handle. The quantitative measure of the tendency of a force to cause or change a body's rotational motion is called torque; we say that \vec{F}_a applies a torque about point O to the wrench in Fig. 78, \vec{F}_b applies a greater torque about O, and \vec{F}_c applies zero torque about O.



Figure 78 – Which of these three equal-magnitude forces is most likely loosen the tight bolt?

Figure 79 shows three examples of how to calculate torque. The body in the figure can rotate about an axis that is perpendicular to the plane of the figure and passes through point O. Three forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ act on the body in the plane of

the figure. The tendency of the first of these forces \vec{F}_1 , to cause a rotation about O depends on its magnitude F_1 . It also depends on the *perpendicular* distance l_1 between point O and the **line of action** of the force (that is, the line along which the force vector lies). We call the distance l_1 the **lever arm** (or **moment arm**) of force \vec{F}_1 about O. The twisting effort is directly proportional to both F_1 and l_1 so we define the **torque** (or moment) of the force \vec{F}_1 with respect to O as the product $F_1 l_1$. We use the Greek letter τ (tau) for torque. In general, for a force of magnitude F whose line of action is a perpendicular distance l from O, the torque is

$$\tau = Fl \tag{133}$$

Physicists usually use the term "torque," while engineers usually use "moment" (unless they are talking about a rotating shaft). Both groups use the term "lever arm" or "moment arm" for the distance l.



Figure 79 – The torque of a force about a point is the product of the magnitude and the lever arm of the force

The lever arm of \vec{F}_1 in Fig. 79 is the perpendicular distance l_1 and the lever arm of \vec{F}_2 is the perpendicular distance l_2 . The line of action of passes through point O, so the lever arm for \vec{F}_3 is zero and its torque with respect to O is zero. In the same way, force $\vec{F_c}$ in Fig. 78 has zero torque with respect to point O; $\vec{F_b}$ has a greater torque than \vec{F}_a because its lever arm is greater.

Force \vec{F}_1 in Fig. 79 tends to cause *counterclockwise* rotation about O, while \vec{F}_2 tends to cause clockwise rotation. To distinguish between these two possibilities, we need to choose a positive sense of rotation. With the choice that counterclockwise torques are positive and clockwise torques are negative, the torques of \vec{F}_1 and \vec{F}_2 about O are

$$\tau_1 = +F_1 l_1, \tau_2 = -F_2 l_2 \tag{134}$$

Figure 79 shows this choice for the sign of torque. We will often use the symbol

+ to indicate our choice of the positive sense of rotation.

The SI unit of torque is the newton-meter. In our discussion of work and energy we called this combination the joule. But torque is not work or energy, and torque should be expressed in newton-meters, not joules.

Figure 80 shows a force \vec{F} applied at a point P described by a position vector \vec{r} with respect to the chosen point O. There are three ways to calculate the torque of this force:

1. Find the lever arm $\tau = Fl$ and use

2. Determine the angle α between the vectors \vec{r} and \vec{F} ; the lever arm is $r \sin \alpha$, so $\tau = rF \sin \alpha$

3. Represent \vec{F} in terms of a radial component F_{rad} along the direction of \vec{r} and a tangential component F_{tan} at right angles, perpendicular to \vec{r} . (We call this a tangential component because if the body rotates, the point where the force acts moves in a circle, and this component is tangent to that circle.) Then $F_{tan} =$ $F \sin \alpha$ and $\tau = r(F \sin \alpha) = F_{tan}r$. The component F_{tan} produces no torque with respect to O because its lever arm with respect to that point is zero (compare to forces \vec{F}_c in Fig. 78 and \vec{F}_3 in Fig. 79).



Figure 80 – Three ways to calculate the torque of the force about the point O. In this figure \vec{r} and \vec{F} are in the plane of the page and the torque vector $\vec{\tau}$ points out of the page toward you

Summarizing these three expressions for torque, we have

$$\tau = Fl = rF\sin\alpha = F_{tan}r\tag{135}$$

We saw that angular velocity and angular acceleration can be represented as vectors; the same is true for torque. To see how to do this, note that the quantity in Eq. (10.2) is the magnitude of the *vector product* $\vec{r} \times \vec{F}$. We now generalize the definition of torque as follows: When a force \vec{F} acts at a point having a position vector \vec{r} with respect to an origin O, as in Fig. 10.3, the $\vec{\tau}$ torque of the force with respect to O is the *vector* quantity

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{136}$$

The torque as defined in Eq. (10.2) is just the magnitude of the torque vector $\vec{r} \times \vec{F}$. The direction of $\vec{\tau}$ is perpendicular to both \vec{r} and \vec{F} . In particular, if both \vec{r} and \vec{F} lie in a plane perpendicular to the axis of rotation, as in Fig. 80, then the torque vector $\vec{\tau} = \vec{r} \times \vec{F}$ is directed along the axis of rotation, with a sense given by the right-hand rule (see fig. 81).





Figure 81 – (a) The vector product $\vec{A} \times \vec{B}$ determined by the right-hand rule. (b) $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$; the vector product in anticommutative

In diagrams that involve \vec{r}, \vec{F} and $\vec{\tau}$ it's common to have one of the vectors oriented perpendicular to the page. (Indeed, by the very nature of the cross product, $\vec{\tau} = \vec{r} \times \vec{F}$ must be perpendicular to the plane of the vectors \vec{r} and \vec{F}) We use a dot (·) to represent a vector that points out of the page (see Fig. 80) and a cross (×) to represent a vector that points into the page.

In the following sections we will usually be concerned with rotation of a body about an axis oriented in a specified constant direction. In that case, only the component of torque along that axis is of interest, and we often call that component the torque with respect to the specified *axis*.

Every rotational quantity is the analog of some quantity in the translational motion of a particle. The analog of momentum of a particle is **angular momentum**, a vector quantity denoted as \vec{L} . Its relationship to momentum \vec{p} (which we will often call linear momentum for clarity) is exactly the same as the relationship of torque to force, $\vec{\tau} = \vec{r} \times \vec{F}$. For a particle with constant mass *m*, velocity \vec{v} , momentum $\vec{p} = m\vec{v}$ and position vector \vec{r} relative to the origin O of an inertial frame, we define angular momentum \vec{L} as

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} \tag{137}$$

The value of \vec{L} depends on the choice of origin O, since it involves the particle's position vector relative to O. The units of angular momentum are $kg \cdot m^2/s$.

In Fig. 82 a particle moves in the *xy*-plane; its position vector and momentum $\vec{p} = m\vec{v}$ are shown. The angular momentum vector \vec{L} is perpendicular to the *xy*-plane. The right-hand rule for vector products shows that its direction is along the +*z*-axis and its magnitude is

$$L = mvr\sin\varphi = mvl \tag{138}$$

where *l* is the perpendicular distance from the line of \vec{v} to O. This distance plays the role of "lever arm" for the momentum vector.

When a net force \vec{F} acts on a particle, its velocity and momentum change, so its angular momentum may also change. We can show that the *rate of change* of angular momentum is equal to the torque of the net force. We take the time derivative of Eq. (137), using the rule for the derivative of a product:

$$\frac{d\vec{L}}{dt} = \left(\frac{d\vec{r}}{dt} \times m\vec{v}\right) + \left(\vec{r} \times m\frac{d\vec{v}}{dt}\right) = (\vec{v} \times m\vec{v}) + (\vec{r} \times m\vec{a})$$
(139)

The first term is zero because it contains the vector product of the vector $\vec{v} = \frac{d\vec{r}}{dt}$ with itself. In the second term we replace with the net force \vec{F} obtaining

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau} \tag{140}$$

The rate of change of angular momentum of a particle equals the torque of the net force acting on it. Compare this result to Eq. (119), which states that the rate of change $d\vec{p}/dt$ of the linear momentum of a particle equals the net force that acts on it.



Figure 82 – Calculating the angular momentum $\vec{L} = \vec{r} \times m\vec{v} = \vec{r} \times \vec{p}$ of a particle with mass *m* moving in the *xy*-plane

We can use Eq. (138) to find the total angular momentum of a *rigid body* rotating about the *z*-axis with angular speed ω . First consider a thin slice of the body lying in the *xy*-plane (see fig. 83). Each particle in the slice moves in a circle centered at the origin, and at each instant its velocity \vec{v} is perpendicular to its position vector \vec{r}_i as shown. Hence in Eq. (138), $\varphi = 90^\circ$ for every particle. A particle with mass m_i at a distance \vec{r}_i from O has a speed v_i equal to From Eq. (138) the magnitude L_i of its angular momentum is

$$L_i = m_i (\vec{r}_i \omega) \vec{r}_i = m_i r_i^2 \omega \tag{141}$$

The direction of each particle's angular momentum, as given by the right-hand rule for the vector product, is along the +z-axis.


Figure 83 – Calculating the angular momentum of a particle of mass m_i in a rigid body rotating at angular speed ω

The *total* angular momentum of the slice of the body lying in the *xy*-plane is the sum $\sum L_i$ of the angular momenta L_i of the particles. Summing Eq. (141), we have

$$L = \sum L_i = \left(\sum m_i r_i^2\right) \omega = I\omega \tag{142}$$

where *I* is the moment of inertia of the slice about the *z*-axis.

We can do this same calculation for the other slices of the body, all parallel to the xy-plane. For points that do not lie in the xy-plane, a complication arises because the \vec{r} vectors have components in the z-direction as well as the x- and ydirections; this gives the angular momentum of each particle a component perpendicular to the z-axis. But *if the z-axis is an axis of symmetry*, the perpendicular components for particles on opposite sides of this axis add up to zero (see fig. 84). So when a body rotates about an axis of symmetry, its angular momentum vector \vec{L} lies along the symmetry axis, and its magnitude is $L = I\omega$.



Figure 84 – Two particles of the same mass located symmetrically on either side of the rotation axis of a rigid body. The angular momentum vectors \vec{L}_1 and \vec{L}_2 of the two particles do not lie along the rotation axis, but their vector sum $\vec{L}_1 + \vec{L}_2$

The angular velocity vector $\vec{\omega}$ also lies along the rotation axis. Hence for a rigid body rotating around an axis of symmetry, \vec{L} and $\vec{\omega}$ are in the same direction (see fig. 85). So we have the *vector* relationship

$$\vec{L} = I\vec{\omega}$$
(143)
If you curl the fingers of your right hand in the direction of rotation ...
$$\vec{v}$$
... your right thumb points in the direction of $\vec{\omega}$. If the rotation axis is an axis of symmetry, this is also the direction of \vec{L} .

Figure 85 – For rotation about an axis of symmetry, $\vec{\omega}$ and \vec{L} are parallel and along the axis. The directions of both vectors are given by the right-hand rule

From Eq. (140) the rate of change of angular momentum of a particle equals the torque of the net force acting on the particle. For any system of particles (including both rigid and nonrigid bodies), the rate of change of the *total* angular momentum equals the sum of the torques of all forces acting on all the particles. The torques of the *internal* forces add to zero if these forces act along the line from one particle to another, as in Fig. 86, and so the sum of the torques includes only the torques of the *external* forces. If the total angular momentum of the system of particles is \vec{L} and the sum of the external torques is $\sum \vec{\tau}$ then

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt} \tag{144}$$



Figure 86 – Two particles in a rigid body exert equal and opposite forces on each other. If the forces act along the line joining the particles, the lever arms of the forces with respect to an axis through *O* are the same and the torques due to the two forces are equal and opposite. Only external torques affect the body's rotation

Finally, if the system of particles is a rigid body rotating about a symmetry axis (the z-axis), then $L_z = I\omega_z$ and I is constant. If this axis has a fixed direction in space, then the vectors \vec{L} and $\vec{\omega}$ change only in magnitude, not in direction. In that case, $dL_z/dt = Id\omega_z/dt = I\alpha_z$ or

$$\sum \vec{\tau} = I\alpha_z \tag{145}$$

which is again our basic relationship for the dynamics of rigid-body rotation. If the body is *not* rigid, I may change, and in that case, L changes even when is constant. For a nonrigid body, Eq. (144) is still valid, even though Eq. (10.7) is not.

When the axis of rotation is *not* a symmetry axis, the angular momentum is in general *not* parallel to the axis (see fig. 87). As the body turns, the angular momentum vector \vec{L} traces out a cone around the rotation axis. Because \vec{L} changes, there must be a net external torque acting on the body even though the angular velocity magnitude may be constant. If the body is an unbalanced wheel on a car, this torque is provided by friction in the bearings, which causes the bearings to wear out. "Balancing" a wheel means distributing the mass so that the rotation axis is an axis of symmetry; then \vec{L} points along the rotation axis, and no net torque is required to keep the wheel turning.



Figure 87 -If the rotation axis of a rigid body is not a symmetry axis, \vec{L} does not in general lie along the rotation axis. Even if $\vec{\omega}$ is constant, the direction of \vec{L} changes and a net torque is required to maintain rotation

In fixed-axis rotation we often use the term "angular momentum of the body" to refer to only the *component* of \vec{L} along the rotation axis of the body (the *z*-axis in Fig. 87), with a positive or negative sign to indicate the sense of rotation just as with angular velocity.

1.5.5 Conservation's law of angular momentum

We have just seen that angular momentum can be used for an alternative statement of the basic dynamic principle for rotational motion. It also forms the basis for the **principle of conservation of angular momentum**. Like conservation

of energy and of linear momentum, this principle is a universal conservation law, valid at all scales from atomic and nuclear systems to the motions of galaxies. This principle follows directly from Eq. (144): $\sum \vec{\tau} = \frac{d\vec{L}}{dt}$. If $\sum \vec{\tau} = 0$, then $\frac{d\vec{L}}{dt} = 0$, and \vec{L} is constant.

When the net external torque acting on a system is zero, the total angular momentum of the system is constant (conserved).

A circus acrobat, a diver, and an ice skater pirouetting on the toe of one skate all take advantage of this principle. Suppose an acrobat has just left a swing with arms and legs extended and rotating counterclockwise about her center of mass. When she pulls her arms and legs in, her moment of inertia I_{cm} with respect to her center of mass changes from a large value I_1 to a much smaller value I_2 . The only external force acting on her is her weight, which has no torque with respect to an axis through her center of mass. So her angular momentum $L_z = I_{cm}\omega_z$ remains constant, and her angular velocity ω_z increases as I_{cm} decreases. That is,

$$I_1 \omega_{1z} = I_2 \omega_2 \tag{146}$$

When a skater or ballerina spins with arms outstretched and then pulls her arms in, her angular velocity increases as her moment of inertia decreases. In each case there is conservation of angular momentum in a system in which the net external torque is zero.

When a system has several parts, the internal forces that the parts exert on one another cause changes in the angular momenta of the parts, but the total angular momentum doesn't change. Here's an example. Consider two bodies A and B that interact with each other but not with anything else. Suppose body A exerts a force $\vec{F}_{A \text{ on } B}$ on body B; the corresponding torque (with respect to whatever point we choose) is $\vec{\tau}_{A \text{ on } B}$. According to Eq. (144), this torque is equal to the rate of change of angular momentum of B:

$$\vec{\tau}_{A \text{ on } B} = \frac{d\vec{L}_B}{dt} \tag{147}$$

At the same time, body B exerts a force $\vec{F}_{B on A}$ on body A, with a corresponding torque $\vec{\tau}_{B on A}$ and

$$\vec{\tau}_{B \text{ on } A} = \frac{d\vec{L}_A}{dt} \tag{148}$$

From Newton's third law $\vec{F}_{B on A} = -\vec{F}_{A on B}$, Furthermore, if the forces act along the same line, as in Fig. 86, their lever arms with respect to the chosen axis are equal. Thus the *torques* of these two forces are equal and opposite, and $\vec{\tau}_{B on A} = -\vec{\tau}_{A on B}$. So if we add the two preceding equations, we find

$$\frac{d\vec{L}_A}{dt} + \frac{d\vec{L}_B}{dt} = 0 \tag{149}$$

or, because $\vec{L}_A + \vec{L}_B$ is the *total* angular momentum \vec{L} of the system,

$$\frac{d\vec{L}}{dt} = 0 \tag{150}$$

That is, the total angular momentum of the system is constant. The torques of the internal forces can transfer angular momentum from one body to the other, but they can't change the *total* angular momentum of the system (see fig. 88).



Figure 88 – A falling cat twists different parts of its body in different directions so that it lands feet first. At all times during this process the angular momentum of the cat as a whole remains zero

1.6 Periodic motion

1.6.1 Describing Oscillation

Many kinds of motion repeat themselves over and over: the vibration of a quartz crystal in a watch, the swinging pendulum of a grandfather clock, the sound vibrations produced by a clarinet or an organ pipe, and the back-and-forth motion of the pistons in a car engine. This kind of motion, called **periodic motion** or **oscillation**, is the subject of this chapter. Understanding periodic motion will be essential for our later study of waves, sound, alternating electric currents, and light.

A body that undergoes periodic motion always has a stable equilibrium position. When it is moved away from this position and released, a force or torque comes into play to pull it back toward equilibrium. But by the time it gets there, it has picked up some kinetic energy, so it overshoots, stopping somewhere on the other side, and is again pulled back toward equilibrium. Picture a ball rolling back and forth in a round bowl or a pendulum that swings back and forth past its straight-down position.

In this chapter we will concentrate on two simple examples of systems that can undergo periodic motions: spring-mass systems and pendulums. We will also study why oscillations often tend to die out with time and why some oscillations can build up to greater and greater displacements from equilibrium when periodically varying forces act.

Figure 89 shows one of the simplest systems that can have periodic motion. A body with mass *m* rests on a frictionless horizontal guide system, such as a linear air track, so it can move only along the *x*-axis. The body is attached to a spring of negligible mass that can be either stretched or compressed. The left end of the spring is held fixed and the right end is attached to the body. The spring force is the only horizontal force acting on the body; the vertical normal and gravitational forces always add to zero.



Figure 89 – A system that can have periodic motion

It's simplest to define our coordinate system so that the origin O is at the equilibrium position, where the spring is neither stretched nor compressed. Then x

is the *x*-component of the **displacement** of the body from equilibrium and is also the change in the length of the spring. The *x*-component of the force that the spring exerts on the body is F_x and the *x*-component of acceleration a_x is given by $a_x = F_x/m$.

Figure 90 shows the body for three different displacements of the spring. Whenever the body is displaced from its equilibrium position, the spring force tends to restore it to the equilibrium position. We call a force with this character a **restoring force.** Oscillation can occur only when there is a restoring force tending to return the system to equilibrium.

Let's analyze how oscillation occurs in this system. If we displace the body to the right to x = A and then let go, the net force and the acceleration are to the left (see fig. 90a). The speed increases as the body approaches the equilibrium position O. When the body is at O, the net force acting on it is zero (see fig. 90b), but because of its motion it *overshoots* the equilibrium position. On the other side of the equilibrium position the body is still moving to the left, but the net force and the acceleration are to the right (see fig. 90c); hence the speed decreases until the body comes to a stop. We will show later that with an ideal spring, the stopping point is at x = -A. The body then accelerates to the right, overshoots equilibrium again, and stops at the starting point ready to repeat the whole process. The body is oscillating! If there is no friction or other force to remove mechanical energy from the system, this motion repeats forever; the restoring force perpetually draws the body back toward the equilibrium position, only to have the body overshoot time after time.

In different situations the force may depend on the displacement x from equilibrium in different ways. But oscillation *always* occurs if the force is a *restoring* force that tends to return the system to equilibrium.

Amplitude, Period, Frequency, and Angular Frequency are some terms that we'll use in discussing periodic motions of all kinds.

The **amplitude** of the motion, denoted by A, is the maximum magnitude of displacement from equilibrium—that is, the maximum value of |x|. It is always positive. If the spring in Fig. 90 is an ideal one, the total overall range of the motion is 2A. The SI unit of A is the meter. A complete vibration, or **cycle**, is one complete round trip—say, from A to -A and back to A, or from O to A, back through O to -A and back to O. Note that motion from one side to the other (say, to A) is a half-cycle, not a whole cycle.

The **period**, T, is the time for one cycle. It is always positive. The SI unit is the second, but it is sometimes expressed as "seconds per cycle."

The **frequency**, is the number of cycles in a unit of time. It is always positive. The SI unit of frequency is the hertz:

$$1 Hertz = 1 Hz = 1 \frac{cycle}{s} = 1 s^{-1}$$

This unit is named in honor of the German physicist Heinrich Hertz (1857–1894), a pioneer in investigating electromagnetic waves.

The **angular frequency**, ω , is 2π times the frequency:

$$\omega = 2\pi\nu \tag{151}$$



Figure 90 – Model for periodic motion. When the body is displaced from its equilibrium position at x = 0, the spring exerts a restoring force back toward the equilibrium position

We'll learn shortly why ω is a useful quantity. It represents the rate of change of an angular quantity (not necessarily related to a rotational motion) that is always measured in radians, so its units are *rad/s*. Since ν is in *cycle/s*, we may regard the number 2π as having units *cycle/s*.

From the definitions of period T and frequency ν we see that each is the reciprocal of the other:

$$\nu = \frac{1}{T} , T = \frac{1}{\nu}$$
⁽¹⁵²⁾

Also, from the definition of ω

$$\omega = 2\pi\nu = \frac{2\pi}{T} \tag{153}$$

1.6.2 Simple harmonic motion

The simplest kind of oscillation occurs when the restoring force F_x is *directly proportional* to the displacement from equilibrium x. This happens if the spring in Figs. 89 and 90 is an ideal one that obeys Hooke's law. The constant of proportionality between and x is the force constant k. On either side of the equilibrium position, F_x and x always have opposite signs. Early we represented the force acting *on* a stretched ideal spring as $F_x = kx$. The *x*-component of force the spring exerts *on the body* is the negative of this, so the *x*-component of force F_x on the body is

$$F_x = -kx \tag{154}$$

This equation gives the correct magnitude and sign of the force, whether x is positive, negative, or zero (see fig. 91). The force constant k is always positive and has units of N/m. We are assuming that there is no friction, so Eq. (154) gives the *net* force on the body.

When the restoring force is directly proportional to the displacement from equilibrium, as given by Eq. (154), the oscillation is called **simple harmonic motion**, abbreviated **SHM**. The acceleration $a_x = \frac{d^2x}{dt^2} = F_x/m$ of a body in SHM is given by

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x\tag{155}$$

The minus sign means the acceleration and displacement always have opposite signs. This acceleration is *not* constant, so don't even think of using the constant-acceleration equations. We'll see shortly how to solve this equation to find the

displacement x as a function of time. A body that undergoes simple harmonic motion is called a **harmonic oscillator**.



Figure 91 – An idealized spring exerts a restoring force obeys Hooke's law, $F_x = -kx$. Oscillation which such a restoring force is called simple harmonic motion

Why is simple harmonic motion important? Keep in mind that not all periodic motions are simple harmonic; in periodic motion in general, the restoring force depends on displacement in a more complicated way than in Eq. (154). But in many systems the restoring force is *approximately* proportional to displacement if the displacement is sufficiently small (see fig. 92). That is, if the amplitude is small enough, the oscillations of such systems are approximately simple harmonic and therefore approximately described by Eq. (155). Thus we can use SHM as an approximate model for many different periodic motions, such as the vibration of the quartz crystal in a watch, the motion of a tuning fork, the electric current in an alternating-current circuit, and the oscillations of atoms in molecules and solids.

To explore the properties of simple harmonic motion, we must express the displacement x of the oscillating body as a function of time, x(t). The second derivative of this function, $\frac{d^2x}{dt^2}$, must be equal to $\left(-\frac{k}{m}\right)$ times the function itself, as required by Eq. (155). As we mentioned, the formulas for constant acceleration

are no help because the acceleration changes constantly as the displacement x changes. Instead, we'll find by noticing a striking similarity between SHM and another form of motion that we've already studied.



Figure 92 – In most real oscillations Hooke's law applies provided the body doesn't move too far from equilibrium. In such a case small-amplitude oscillations are approximately simple harmonic

Figure 93a shows a top view of a horizontal disk of radius A with a ball attached to its rim at point Q. The disk rotates with constant angular speed (measured in so the ball moves in uniform circular motion. A horizontal light beam shines on the rotating disk and casts a shadow of the ball on a screen. The shadow at point P oscillates back and forth as the ball moves in a circle. We then arrange a body attached to an ideal spring, like the combination shown in Figs. 89 and 90, so that the body oscillates parallel to the shadow. We will prove that the motion of the body and the motion of the ball's shadow are *identical* if the amplitude of the body's oscillation is equal to the disk radius A, and if the angular frequency $2\pi\nu$ of the oscillating body is equal to the angular speed of the rotating disk. That is, simple harmonic motion is the projection of uniform circular motion onto a diameter.

(a) Apparatus for creating the reference circle





Figure 93 – (a) Relating uniform circular motion and simple harmonic motion. (b) The ball's shadow moves exactly like a body oscillating on an ideal spring

We can verify this remarkable statement by finding the acceleration of the shadow at P and comparing it to the acceleration of a body undergoing SHM, given by Eq. (155). The circle in which the ball moves so that its projection matches the motion of the oscillating body is called the **reference circle**; we will call the point Q the *reference point*. We take the reference circle to lie in the xy-plane, with the origin O at the center of the circle (see fig. 93b). At time t the vector OQ from the origin to the reference point Q makes an angle with the positive x-axis. As the point Q moves around the reference circle with constant angular speed the vector OQ rotates with the same angular speed. Such a rotating vector is called a **phasor**. The phasor method for analyzing oscillations is useful in many areas of physics.

The *x*-component of the phasor at time *t* is just the *x*-coordinate of the point *Q*:

$$x = A\cos\theta \tag{156}$$

This is also the x-coordinate of the shadow P, which is the projection of Q onto the x-axis. Hence the x-velocity of the shadow P along the x-axis is equal to the x-component of the velocity vector of point Q (see fig. 94a), and the x-acceleration of P is equal to the x-component of the acceleration vector of Q (see fig. 94b). Since point Q is in uniform circular motion, its acceleration vector \vec{a}_Q is always directed toward O. Furthermore, the magnitude of \vec{a}_Q is constant and given by the angular speed squared times the radius of the circle:

$$a_0 = \omega^2 A \tag{157}$$

Figure 94b shows that the *x*-component of \vec{a}_Q is $a_x = -a_Q \cos \theta$. Combining this with Eqs. (156) and (157), we get that the acceleration of point *P* is

$$a_x = -a_0 \cos \theta = -\omega^2 \cos \theta \tag{158}$$

or

$$a_x = -\omega^2 x \tag{159}$$

The acceleration of point P is directly proportional to the displacement x and always has the opposite sign. These are precisely the hallmarks of simple harmonic motion.

Equation (159) is *exactly* the same as Eq. (155) for the acceleration of a harmonic oscillator, provided that the angular speed of the reference point Q is related to the force constant k and mass m of the oscillating body by

$$\omega^2 = \frac{k}{m} \tag{160}$$

or

$$\omega = \sqrt{\frac{k}{m}}$$
(161)

We have been using the same symbol for the angular *speed* of the reference point Q and the angular *frequency* of the oscillating point P. The reason is that these quantities are equal! If point Q makes one complete revolution in time T, then point P goes through one complete cycle of oscillation in the same time; hence T is the period of the oscillation. During time T the point Q moves through 2π radians, so its angular speed is $\omega = 2\pi/T$. But this is just the same as Eq. (153) for the angular frequency of the point P, which verifies our statement about the two interpretations of ω . This is why we introduced angular frequency early; this quantity makes the connection between oscillation and circular motion. So we reinterpret Eq. (161) as an expression for the angular frequency of simple harmonic motion for a body of mass m, acted on by a restoring force with force constant k:

$$\omega = \sqrt{\frac{k}{m}}$$
(162)

When you start a body oscillating in SHM, the value of ω is not yours to choose; it is predetermined by the values of k and m. The units of k are N/m or kg/s^2 so k/m is in $(kg/s^2)/kg = s^{-2}$. When we take the square root in Eq. (162), we get s^{-1} or more properly because this is an *angular* frequency (recall that a radian is not a true unit).



(b) Using the reference circle to determine the *x*-acceleration of point *P*



Figure 94 – The (a) *x*-velocity and (b) *x*-acceleration of the ball's shadow P are the *x*-components of the velocity and acceleration vectors, respectively, of the ball Q

According to Eqs. (152) and (153), the frequency and period T are

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$
(163)

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$
⁽¹⁶⁴⁾
¹²³

We see from Eq. (164) that a larger mass m, with its greater inertia, will have less acceleration, move more slowly, and take a longer time for a complete cycle (see fig. 95). In contrast, a stiffer spring (one with a larger force constant k) exerts a greater force at a given deformation x, causing greater acceleration, higher speeds, and a shorter time T per cycle.

Tines with large mass m: low frequency f = 128 Hz



Tines with small mass m:⁷ high frequency f = 4096 Hz

Figure 95 – The greater the mass m in a tuning fork's tines, the lower the frequency of oscillation $v = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ and the lower the pitch of the sound that the tuning fork produces

Equations (163) and (164) show that the period and frequency of simple harmonic motion are completely determined by the mass m and the force constant k. In simple harmonic motion the period and frequency do not depend on the amplitude A. For given values of m and k, the time of one complete oscillation is the same whether the amplitude is large or small. Equation (154) shows why we should expect this. Larger A means that the body reaches larger values of and is subjected to larger restoring forces. This increases the average speed of the body

over a complete cycle; this exactly compensates for having to travel a larger distance, so the same total time is involved.

The oscillations of a tuning fork are essentially simple harmonic motion, which means that it always vibrates with the same frequency, independent of amplitude. This is why a tuning fork can be used as a standard for musical pitch. If it were not for this characteristic of simple harmonic motion, it would be impossible to make familiar types of mechanical and electronic clocks run accurately or to play most musical instruments in tune. If you encounter an oscillating body with a period that *does* depend on the amplitude, the oscillation is *not* simple harmonic motion.

We still need to find the displacement x as a function of time for a harmonic oscillator. Equation (155) for a body in simple harmonic motion along the *x*-axis is identical to Eq. (159) for the *x*-coordinate of the reference point in uniform

circular motion with constant angular speed $\omega = \sqrt{\frac{k}{m}}$. Hence Eq. (156), $x = A \cos \theta$, describes the *x*-coordinate for both of these situations. If at t = 0 the phasor *OQ* makes an angle (the Greek letter phi) with the positive *x*-axis,

then at any φ later time t this angle is $\theta = \omega t + \varphi$. We substitute this into Eq. (156) to obtain

$$x = A\cos(\omega t + \varphi) \tag{165}$$

Where $\omega = \sqrt{\frac{k}{m}}$. Figure 96 shows a graph of Eq. (165) for the particular case $\varphi = 0$. The displacement x is a periodic function of time, as expected for SHM. We could also have written Eq. (165) in terms of a sine function rather than a cosine by using the identity $\cos \alpha = \sin(\alpha + \pi/2)$. In simple harmonic motion the position is a periodic, sinusoidal function of time. There are many other periodic functions, but none so simple as a sine or cosine function.



Figure 96 – Graph of *x* versus *t* for simple harmonic motion. The case shown has $\varphi = 0$

The value of the cosine function is always between -1 and 1, so in Eq. (165), x is always between -A and A. This confirms that A is the amplitude of the motion.

The period T is the time for one complete cycle of oscillation, as Fig. 96 shows. The cosine function repeats itself whenever the quantity in parentheses in Eq. (165) increases by 2π radians. Thus, if we start at time t = 0 the time T to complete one cycle is given by

$$\omega T = \sqrt{\frac{k}{m}}T = 2\pi \tag{166}$$

or

$$T = 2\pi \sqrt{\frac{m}{k}} \tag{167}$$

which is just Eq. (164). Changing either m or k changes the period of oscillation, as shown in Figs. 97a and 97b. The period does not depend on the amplitude A (see fig. 97c).



Figure 97 – Variations of simple harmonic motion. All cases shown have $\varphi = 0$

The constant φ in Eq. (165) is called the **phase angle.** It tells us at what point in the cycle the motion was at t = 0 (equivalent to where around the circle the point Q was at t = 0). We denote the position at t = 0 by x_0 . Putting t = 0 and $x = x_0$ in Eq. (165), we get

$$x_0 = A\cos\varphi \tag{168}$$

If $\varphi = 0$ then $x_0 = A \cos 0 = A$ and the body starts at its maximum positive displacement. If $\varphi = \pi$ then $x_0 = A \cos \pi = -A$ and the particle starts at its maximum *negative* displacement. If $\varphi = \pi/2$ then $x_0 = A \cos \frac{\pi}{2} = 0$ and the particle is initially at the origin. Figure 98 shows the displacement *x* versus time for three different phase angles.

These three curves show SHM with the same period *T* and amplitude *A* but with different phase angles ϕ .



Figure 98 – Variations of SHM: displacement versus time for the same harmonic oscillator with different phase angles φ

We find the velocity and acceleration as functions of time for a harmonic oscillator by taking derivatives of Eq. (165) with respect to time:

$$v_x = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi)$$
(169)

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A \sin(\omega t + \varphi)$$
(170)

The velocity v_x oscillates between $v_{max} = +\omega A$ and $v_{min} = -\omega A$, and the acceleration a_x oscillates between $a_{max} = +\omega^2 A$ and $v_{min} = -\omega^2 A$ (see fig. 99). Comparing Eq. (170) with Eq. (165) and recalling that $\omega^2 = \frac{k}{m}$ from Eq. (161), we see that

$$a_x = -\omega^2 x = -\frac{k}{m} x \tag{171}$$

which is just Eq. (155) for simple harmonic motion. This confirms that Eq. (165) for x as a function of time is correct.

We actually derived Eq. (170) earlier in a geometrical way by taking the xcomponent of the acceleration vector of the reference point Q. This was done in Fig. 94b and Eq. (158) (recall that $\theta = \omega t + \varphi$). In the same way, we could have derived Eq. (169) by taking the x-component of the velocity vector of Q, as shown in Fig. 94b. We'll leave the details for you to work out.

Note that the sinusoidal graph of displacement versus time (see fig. 99a) is shifted by one-quarter period from the graph of velocity versus time (see fig. 99b) and by one-half period from the graph of acceleration versus time (see fig. 99c). Figure 91 shows why this is so. When the body is passing through the equilibrium position so that the displacement is zero, the velocity equals either v_{max} or $-v_{max}$ (depending on which way the body is moving) and the acceleration is zero. When the body is at either its maximum positive displacement, x = +A, or its maximum negative displacement, x = +A, the velocity is zero and the body is instantaneously at rest. At these points, the restoring force $F_x = -kx$ and the acceleration of the body have their maximum magnitudes. At x = +A the acceleration is negative and equal to $-a_{max}$. At the acceleration is positive: $a_x = +a_{max}$.

If we are given the initial position and initial velocity v_{0x} for the oscillating body, we can determine the amplitude A and the phase angle φ . Here's how to do it. The initial velocity v_{0x} is the velocity at time t = 0; putting $v_x = v_{0x}$ and t = 0in Eq. (169), we find

$$v_{0x} = -\omega A \sin \varphi \tag{172}$$

To find φ we divide Eq. (172) by Eq. (168). This eliminates A and gives an equation that we can solve for φ :

$$\frac{v_{0x}}{x_0} = \frac{-\omega A \sin \varphi}{A \cos \varphi} = -\omega \tan \varphi$$
(173)

$$\varphi = \arctan\left(-\frac{v_{0x}}{\omega x_0}\right) \tag{174}$$

It is also easy to find the amplitude A if we are given x_0 and v_{0x} .We'll sketch the derivation, and you can fill in the details. Square Eq. (168); then divide Eq. (172) by ω , square it, and add to the square of Eq. (168). The right side will be $A^2(\sin^2 \varphi + \cos^2 \varphi)$ which is equal to A^2 . The final result is

$$A = \sqrt{x_0^2 + \frac{v_{0x}^2}{\omega^2}}$$
(175)

Note that when the body has both an initial displacement and a nonzero initial velocity the amplitude A is *not* equal to the initial displacement. That's reasonable; if you start the body at a positive x_0 but give it a positive velocity v_{0x} , it will go *farther* than x_0 before it turns and comes back.

(a) Displacement x as a function of time t



(b) Velocity v_x as a function of time t



(c) Acceleration a_x as a function of time t



The a_x -t graph is shifted by $\frac{1}{4}$ cycle from the v_x -t graph and by $\frac{1}{2}$ cycle from the x-t graph.

Figure 99 – Graphs (a) x versus t, (b) v_x versus t, and (c) a_x versus t for a body in SHM. For the motion depicted in these graphs, $\varphi = \pi/3$

1.6.3 Damped and forced oscillations. Resonance

The idealized oscillating systems we have discussed so far are frictionless. There are no nonconservative forces, the total mechanical energy is constant, and a system set into motion continues oscillating forever with no decrease in amplitude. Real-world systems always have some dissipative forces, however, and oscillations die out with time unless we replace the dissipated mechanical energy (see fig. 100). A mechanical pendulum clock continues to run because potential energy stored in the spring or a hanging weight system replaces the mechanical energy lost due to friction in the pivot and the gears. But eventually the spring runs down or the weights reach the bottom of their travel. Then no more energy is available, and the pendulum swings decrease in amplitude and stop.



Figure 100 – A swinging bell left to itself will eventually stop oscillating due to damping forces (air resistance and friction at the point of suspension)

The decrease in amplitude caused by dissipative forces is called **damping**, and the corresponding motion is called **damped oscillation**. The simplest case to analyze in detail is a simple harmonic oscillator with a frictional damping force that is directly proportional to the *velocity* of the oscillating body. This behavior occurs in friction involving viscous fluid flow, such as in shock absorbers or sliding between oil-lubricated surfaces. We then have an additional force on the body due to friction, $F_x = -bv_x$, where $v_x = dx/dt$ is the velocity and b is a constant that describes the strength of the damping force. The negative sign shows

that the force is always opposite in direction to the velocity. The *net* force on the body is then

$$\sum F_x = -kx - bv_x \tag{176}$$

and Newton's second law for the system is

$$-kx - bv_x = ma_x \tag{177}$$

or

$$-kx - bv_x = m\frac{d^2x}{dt^2} \tag{178}$$

Equation (178) is a differential equation for x; it would be the same as Eq. (155), the equation for the acceleration in SHM, except for the added term -bdx/dt. Solving this equation is a straightforward problem in differential equations, but we won't go into the details here. If the damping force is relatively small, the motion is described by

$$x = Ae^{-(b/2m)t}\cos(\omega't + \varphi)$$
(179)

The angular frequency of oscillation ω' is given by

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \tag{180}$$

You can verify that Eq. (179) is a solution of Eq. (178) by calculating the first and second derivatives of x, substituting them into Eq. (178), and checking whether the left and right sides are equal. This is a straightforward but slightly tedious procedure.

The motion described by Eq. (179) differs from the undamped case in two ways. First, the amplitude $Ae^{-(b/2m)t}$ is not constant but decreases with time because of the decreasing exponential factor $e^{-(b/2m)t}$. Figure 101 is a graph of Eq. (179) for the case $\varphi = 0$ it shows that the larger the value of *b*, the more quickly the amplitude decreases.



Figure 101 – Graph of displacement versus time for an oscillator with little damping and with phase angle $\varphi = 0$. The curves are for two values of the damping constant *b*

Second, the angular frequency given by Eq. (180), is no longer equal to $\omega = \sqrt{\frac{k}{m}}$ but is somewhat smaller. It becomes zero when *b* becomes so large that

$$\frac{k}{m} - \frac{b^2}{4m^2} \tag{181}$$

or

$$b = 2\sqrt{km} \tag{182}$$

When Eq. (182) is satisfied, the condition is called **critical damping.** The system no longer oscillates but returns to its equilibrium position without oscillation when it is displaced and released.

If *b* is greater than $2\sqrt{km}$, the condition is called **overdamping.** Again there is no oscillation, but the system returns to equilibrium more slowly than with critical damping. For the overdamped case the solutions of Eq. (178) have the form

$$x = C_1 e^{-a_1 t} + C_2 e^{-a_2 t} (183)$$

where C_1 and C_2 are constants that depend on the initial conditions and a_1 and a_2 are constants determined by m, k, and b.

When b is less than the critical value, as in Eq. (179), the condition is called **underdamping.** The system oscillates with steadily decreasing amplitude. In a vibrating tuning fork or guitar string, it is usually desirable to have as little damping as possible. By contrast, damping plays a beneficial role in the oscillations of an automobile's suspension system. The shock absorbers provide a velocitydependent damping force so that when the car goes over a bump, it doesn't continue bouncing forever. For optimal passenger comfort, the system should be critically damped or slightly underdamped. Too much damping would be counterproductive; if the suspension is overdamped and the car hits a second bump just after the first one, the springs in the suspension will still be compressed somewhat from the first bump and will not be able to fully absorb the impact.

A damped oscillator left to itself will eventually stop moving altogether. But we can maintain a constant-amplitude oscillation by applying a force that varies with time in a periodic or cyclic way, with a definite period and frequency. As an example, consider your cousin Throckmorton on a playground swing. You can keep him swinging with constant amplitude by giving him a little push once each cycle. We call this additional force a **driving force**.

If we apply a periodically varying driving force with angular frequency ω_d to a damped harmonic oscillator, the motion that results is called a **forced oscillation** or a *driven oscillation*. It is different from the motion that occurs when the system is simply displaced from equilibrium and then left alone, in which case the system oscillates with a **natural angular frequency** ω' determined by *m*, *k*, and *b*, as in Eq. (180). In a forced oscillation, however, the angular frequency with which the mass oscillates is equal to the driving angular frequency ω_d . This does *not* have to be equal to the angular frequency ω' with which the system would oscillate without a driving force. If you grab the ropes of Throckmorton's swing, you can force the swing to oscillate with any frequency you like.

Suppose we force the oscillator to vibrate with an angular frequency ω_d that is nearly *equal* to the angular frequency ω' it would have with no driving force. What happens? The oscillator is naturally disposed to oscillate at $\omega = \omega'$, so we expect the amplitude of the resulting oscillation to be larger than when the two frequencies are very different. Detailed analysis and experiment show that this is just what happens. The easiest case to analyze is a *sinusoidally* varying force say, $F(t) = F_{max} \cos \omega_d t$. If we vary the frequency ω_d of the driving force, the amplitude of the resulting forced oscillation varies in an interesting way (see fig. 102). When there is very little damping (small *b*), the amplitude goes through a sharp peak as the driving angular frequency ω_d nears the natural oscillation angular frequency ω . When the damping is increased (larger *b*), the peak becomes broader and smaller in height and shifts toward lower frequencies. Each curve shows the amplitude A for an oscillator subjected to a driving force at various angular frequencies ω_d . Successive curves from blue to gold represent successively greater damping.



Driving frequency ω_d equals natural angular frequency ω of an undamped oscillator.

Figure 102 – Graph of the amplitude A of forced oscillation as a function of the angular frequency ω_d of the driving force. The horizontal axis shows the ratio of ω_d to the angular frequency $\omega = \sqrt{k/m}$ of an undamped oscillator. Each curve has a different value of the damping constant b

We could work out an expression that shows how the amplitude A of the forced oscillation depends on the frequency of a sinusoidal driving force, with maximum value F_{max} . That would involve more differential equations than we're ready for, but here is the result:

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2 \omega_d^2}}$$
(184)

When $k - m\omega_d^2$ the first term under the radical is zero, so *A* has a maximum near $\omega_d = \sqrt{\frac{k}{m}}$. The height of the curve at this point is proportional to 1/b; the less damping, the higher the peak. At the low-frequency extreme, when $\omega_d = 0$, we get $A = \frac{F_{max}}{k}$. This corresponds to a *constant* force F_{max} and a constant displacement $A = \frac{F_{max}}{k}$ from equilibrium, as we might expect.

The fact that there is an amplitude peak at driving frequencies close to the natural frequency of the system is called **resonance**. Physics is full of examples of resonance; building up the oscillations of a child on a swing by pushing with a frequency equal to the swing's natural frequency is one. A vibrating rattle in a car

that occurs only at a certain engine speed or wheel-rotation speed is an alltoofamiliar example. Inexpensive loudspeakers often have an annoying boom or buzz when a musical note happens to coincide with the resonant frequency of the speaker cone or the speaker housing.

Resonance in mechanical systems can be destructive. A company of soldiers once destroyed a bridge by marching across it in step; the frequency of their steps was close to a natural vibration frequency of the bridge, and the resulting oscillation had large enough amplitude to tear the bridge apart. Ever since, marching soldiers have been ordered to break step before crossing a bridge. Some years ago, vibrations of the engines of a particular airplane had just the right frequency to resonate with the natural frequencies of its wings. Large oscillations built up, and occasionally the wings fell off.

1.7 Gravitation

1.7.1 Newton's Law of Gravitation

The example of gravitational attraction that's probably most familiar to you is your *weight*, the force that attracts you toward the earth. During his study of the motions of the planets and of the moon, Newton discovered the fundamental character of the gravitational attraction between *any* two bodies. Along with his three laws of motion, Newton published the **law of gravitation** in 1687. It may be stated as follows:

Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.

Translating this into an equation, we have

$$F_g = G \frac{m_1 m_2}{r^2}$$
(185)

where F_g is the magnitude of the gravitational force on either particle, m_1 and m_2 are their masses, r is the distance between them (see fig. 103), and G is a fundamental physical constant called the **gravitational constant.** The numerical value of G depends on the system of units used.

Equation (185) tells us that the gravitational force between two particles decreases with increasing distance r: If the distance is doubled, the force is only one-fourth as great, and so on. Although many of the stars in the night sky are far more massive than the sun, they are so far away that their gravitational force on the earth is negligibly small.

Gravitational forces always act along the line joining the two particles, and they form an action–reaction pair. Even when the masses of the particles are different, the two interaction forces have equal magnitude (see fig. 103). The attractive force that your body exerts on the earth has the same magnitude as the force that the earth exerts on you. When you fall from a diving board into a swimming pool, the entire earth rises up to meet you! (You don't notice this because the earth's mass is greater than yours by a factor of about 10^{-23} . Hence the earth's acceleration is only 10^{-23} as great as yours.)



Figure 103 – The gravitational forces between two particles of masses m_1 and m_2

Gravitational forces are negligible between ordinary household-sized objects, but very substantial between objects that are the size of stars. Indeed, gravitation is *the* most important force on the scale of planets, stars, and galaxies (see fig. 104). It is responsible for holding our earth together and for keeping the planets in orbit about the sun. The mutual gravitational attraction between different parts of the sun compresses material at the sun's core to very high densities and temperatures, making it possible for nuclear reactions to take place there. These reactions generate the sun's energy output, which makes it possible for life to exist on earth and for you to read these words.

The gravitational force is so important on the cosmic scale because it acts *at a distance*, without any direct contact between bodies. Electric and magnetic forces have this same remarkable property, but they are less important on astronomical scales because large accumulations of matter are electrically neutral; that is, they contain equal amounts of positive and negative charge. As a result, the electric and magnetic forces between stars or planets are very small or zero. The strong and weak interactions also act at a distance, but their influence is negligible at distances much greater than the diameter of an atomic nucleus (about 10^{-14} m).



Figure 104 - Our solar system is part a spiral galaxy like this one, which contains roughly 10^{11} stars as well gas, dust, and other matter. The entire assemblage is held together by the mutual gravitational attraction of all the matter in the galaxy

A useful way to describe forces that act at a distance is in terms of a *field*. One body sets up a disturbance or field at all points in space, and the force that acts on a second body at a particular point is its response to the first body's field at that point. There is a field associated with each force that acts at a distance, and so we refer to gravitational fields, electric fields, magnetic fields, and so on. We won't need the field concept for our study of gravitation in this chapter, so we won't discuss it further here. But in later chapters we'll find that the field concept is an extraordinarily powerful tool for describing electric and magnetic interactions.

1.7.2 Gravitational potential energy

When we first introduced gravitational potential energy, we assumed that the gravitational force on a body is constant in magnitude and direction. This led to the expression U = mgy. But the earth's gravitational force on a body of mass m at any point outside the earth is given more generally by Eq. $F_g = G \frac{m_E m_2}{r^2}$ where m_E is the mass of the earth and r is the distance of the body from the earth's center. For problems in which r changes enough that the gravitational force can't be

considered constant, we need a more general expression for gravitational potential energy.

To find this expression, we consider a body of mass *m* outside the earth, and first compute the work W_{geav} done by the gravitational force when the body moves directly away from or toward the center of the earth from $r = r_1$ to $r = r_2$ as in Fig. 105. This work is given by

$$W_{geav} = \int_{r_1}^{r_2} F_r dr \tag{186}$$

where F_r is the radial component of the gravitational force \vec{F} —that is, the component in the direction *outward* from the center of the earth. Because \vec{F} points directly *inward* toward the center of the earth, is negative. It differs from Eq. (185), the magnitude of the gravitational force, by a minus sign:

$$F_r = -G \,\frac{m_E m_2}{r^2} \tag{187}$$

Substituting Eq. (187) into Eq. (186), we see that is given by

$$W_{geav} = -Gm_E m \int_{r_1}^{r_2} \frac{dr}{r^2} = \frac{Gm_E m}{r_2} - \frac{Gm_E m}{r_1}$$
(188)

The path doesn't have to be a straight line; it could also be a curve like the one in Fig. 105. This work depends only on the initial and final values of r, not on the path taken. This also proves that the gravitational force is always *conservative*.

We now define the corresponding potential energy U so that $W_{geav} = U_1 - U_2$, as in Eq. (80). Comparing this with Eq. (188), we see that the appropriate definition for **gravitational potential energy** is

$$U = -\frac{Gm_Em}{r} \tag{189}$$

Figure 106 shows how the gravitational potential energy depends on the distance r between the body of mass m and the center of the earth. When the body moves away from the earth, r increases, the gravitational force does negative work, and U increases (i.e., becomes less negative). When the body "falls" toward earth, r decreases, the gravitational work is positive, and the potential energy decreases (i.e., becomes more negative).



Figure 105 – Calculating the work done on a body by the gravitational force as the body moves from radial coordinate r_1 to r_2



Figure 106 – A graph of the gravitational potential energy U for the system of the earth (mass m_E) and an astronaut (mass m) versus the astronaut's distance r from the center of the earth

You may be troubled by Eq. (189) because it states that gravitational potential energy is always negative. But in fact you've seen negative values of U before. We found that U was negative whenever the body of mass m was at a value of y below the arbitrary height we chose to be y = 0 —that is, whenever the body and the earth were closer together than some certain arbitrary distance. In defining U by Eq. (189), we have chosen U to be zero when the body of mass m is infinitely far from the earth $(r = \infty)$. As the body moves toward the earth, gravitational potential energy decreases and so becomes negative.

If we wanted, we could make at the surface of the earth, where $r = R_E$ by simply adding the quantity $\frac{Gm_Em}{R_E}$ to Eq. (189). This would make U positive when $r > R_E$. We won't do this for two reasons: One, it would make the expression for U more complicated; and two, the added term would not affect the *difference* in potential energy between any two points, which is the only physically significant quantity. Armed with Eq. (189), we can now use general energy relationships for problems in which the behavior of the $\frac{1}{r^2}$ earth's gravitational force has to be included. If the gravitational force on the body is the only force that does work, the total mechanical energy of the system is constant, or *conserved*. In the following example we'll use this principle to calculate **escape speed**, the speed required for a body to escape completely from a planet.

1.8 Fluid mechanics

1.8.1 Pressure in a Fluid

When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid. This is the force that you feel pressing on your legs when you dangle them in a swimming pool. While the fluid as a whole is at rest, the molecules that make up the fluid are in motion; the force exerted by the fluid is due to molecules colliding with their surroundings.

If we think of an imaginary surface *within* the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface. (Otherwise, the surface would accelerate and the fluid would not remain at rest.) Consider a small surface of area dA centered on a point in the fluid; the normal force exerted by the fluid on each side is dF_{\perp} (see fig. 107). We define the **pressure** *p* at that point as the normal force per unit area—that is, the ratio of dF_{\perp} to dA (see fig. 108):

$$p = \frac{dF_{\perp}}{A} \tag{190}$$



The surface does not accelerate, so the surrounding fluid exerts equal normal forces on both sides of it. (The fluid cannot exert any force parallel to the surface, since that would cause the surface to accelerate.)

Figure 107 – Forces acting on a small surface within a fluid at rest



Figure 108 – The pressure on either side of a surface is force divided by area. Pressure is a scalar with units of newtons per square meter. By contract, force is a vector with units of newtons If the pressure is the same at all points of a finite plane surface with area A, then

$$p = \frac{F_{\perp}}{A} \tag{191}$$

where is the net normal force on one side of the surface. The SI unit of pressure is the **pascal**, where

$$1 \ pascal = 1 \ Pa = 1 \ N/m^2$$

Two related units, used principally in meteorology, are the *bar*, equal to 10^5 and the *millibar*, equal to 100 Pa.

Atmospheric pressure is the pressure of the earth's atmosphere, the pressure at the bottom of this sea of air in which we live. This pressure varies with weather changes and with elevation. Normal atmospheric pressure at sea level (an average value) is 1 *atmosphere* (atm), defined to be exactly 101,325 Pa. To four significant figures,

$$(p_a)_{av} = 1 atm 1.013 \times 10^5 Pa = 1.013 bar 1013 milliar$$

If the weight of the fluid can be neglected, the pressure in a fluid is the same throughout its volume. We used that approximation in our discussion of bulk stress and strain. But often the fluid's weight is *not* negligible. Atmospheric pressure is less at high altitude than at sea level, which is why an airplane cabin has to be pressurized when flying at 35,000 feet. When you dive into deep water, your ears tell you that the pressure increases rapidly with increasing depth below the surface.

We can derive a general relationship between the pressure p at any point in a fluid at rest and the elevation y of the point. We'll assume that the density has the same value throughout the fluid (that is, the density is *uniform*), as does the acceleration due to gravity g. If the fluid is in equilibrium, every volume element is in equilibrium. Consider a thin element of fluid with thickness dy (see fig. 109a). The bottom and top surfaces each have area A, and they are at elevations y and y+dy above some reference level where y = 0. The volume of the fluid element is dV = Ady, its mass is $dm = \rho dV = \rho Ady$, and its weight is $dw = dmg = \rho gAdy$.

What are the other forces on this fluid element (Fig 12.4b)? Let's call the pressure at the bottom surface p; then the total y-component of upward force on this surface is pA. The pressure at the top surface is p + dp and the total y-component of (downward) force on the top surface is -(p + dp). The fluid element is in equilibrium, so the total y-component of force, including the weight and the forces at the bottom and top surfaces, must be zero:

$$\sum F_{\mathcal{Y}} = 0 \tag{192}$$

$$pA - (p + dp)A - \rho gAdy = 0 \tag{193}$$

(a)



(b)



Because the fluid is in equilibrium, the vector sum of the vertical forces on the fluid element must be zero: pA - (p + dp)A - dw = 0.

Figure 109 – The forces on an element of fluid in equilibrium

When we divide out the area A and rearrange, we get

$$\frac{dp}{dt} = -\rho g \tag{194}$$

This equation shows that when y increases, p decreases; that is, as we move upward in the fluid, pressure decreases, as we expect. If p_1 and p_2 are the pressures at elevations y_1 and y_2 , respectively, and if ρ and g are constant, then

$$p_2 - p_1 = -\rho g(y_2 - y_1) \tag{195}$$

It's often convenient to express Eq. (195) in terms of the *depth* below the surface of a fluid (see fig. 110). Take point 1 at any level in the fluid and let p represent the pressure at this point. Take point 2 at the *surface* of the fluid, where the pressure is (subscript zero for zero depth). The depth of point 1 below the surface is $h = y_2 - y_1$ and Eq. (195) becomes

$$p_0 - p = -\rho g(y_2 - y_1) = -\rho gh$$
(196)

or

$$p = p_0 + \rho g h \tag{197}$$



Pressure difference between levels 1 and 2: $p_2 - p_1 = -\rho g(y_2 - y_1)$ The pressure is greater at the lower level.


The pressure p at a depth h is greater than the pressure p_0 at the surface by an amount ρgh . Note that the pressure is the same at any two points at the same level in the fluid. The *shape* of the container does not matter (see fig. 111).



The pressure at the bottom of each liquid column has the same value p.

The difference between p and p_0 is ρgh , where h is the distance from the top to the bottom of the liquid column. Hence all columns have the same height.

Figure 111 – Each fluid column has the same height, no matter what is shape

Equation (197) shows that if we increase the pressure at the top surface, possibly by using a piston that fits tightly inside the container to push down on the fluid surface, the pressure p at any depth increases by exactly the same amount. This fact was recognized in 1653 by the French scientist Blaise Pascal (1623–1662) and is called *Pascal's law*.

Pascal's law: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

The hydraulic lift shown schematically in Fig. 112 illustrates Pascal's law. A piston with small cross-sectional area S_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $p = \frac{F_1}{S_1}$ is transmitted through the connecting pipe to a larger piston of area S_2 . The applied pressure is the same in both cylinders, so

$$p = \frac{F_1}{S_1} = \frac{F_2}{S_2} \tag{198}$$

The hydraulic lift is a force-multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, many elevators, and hydraulic brakes all use this principle.





For gases the assumption that the density is uniform is realistic only over short vertical distances. In a room with a ceiling height of 3.0 m filled with air of uniform density the difference in pressure between floor and ceiling, given by Eq. (197), is

$$\rho gh = (1.2 \ kg/m^3)(9.8 \ m/s^2)(3.0 \ m) = 35 \ Pa$$

or about 0.00035 atm, a very small difference. But between sea level and the summit of Mount Everest (8882 m) the density of air changes by nearly a factor of 3, and in this case we cannot use Eq. (197). Liquids, by contrast, are nearly incompressible, and it is usually a very good approximation to regard their density as independent of pressure. A pressure of several hundred atmospheres will cause only a few percent increase in the density of most liquids.

1.8.2 Buoyancy

Buoyancy is a familiar phenomenon: A body immersed in water seems to weigh less than when it is in air. When the body is less dense than the fluid, it floats. The human body usually floats in water, and a helium-filled balloon floats in air.

Archimedes's principle: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

To prove this principle, we consider an arbitrary element of fluid at rest. In Fig. 113a the irregular outline is the surface boundary of this element of fluid. The arrows represent the forces exerted on the boundary surface by the surrounding fluid.

The entire fluid is in equilibrium, so the sum of all the *y*-components of force on this element of fluid is zero. Hence the sum of the *y*-components of the *surface* forces must be an upward force equal in magnitude to the weight *mg* of the fluid inside the surface. Also, the sum of the torques on the element of fluid must be zero, so the line of action of the resultant *y*-component of surface force must pass through the center of gravity of this element of fluid.

Now we remove the fluid inside the surface and replace it with a solid body having exactly the same shape (see fig. 113b). The pressure at every point is exactly the same as before. So the total upward force exerted on the body by the fluid is also the same, again equal in magnitude to the weight *mg* of the fluid displaced to make way for the body. We call this upward force the **buoyant force** on the solid body. The line of action of the buoyant force again passes through the center of gravity of the displaced fluid (which doesn't necessarily coincide with the center of gravity of the body).

When a balloon floats in equilibrium in air, its weight (including the gas inside it) must be the same as the weight of the air displaced by the balloon. A fish's flesh is denser than water, yet a fish can float while submerged because it has a gas-filled cavity within its body. This makes the fish's *average* density the same as water's, so its net weight is the same as the weight of the water it displaces. A body whose average density is *less* than that of a liquid can float partially submerged at the free upper surface of the liquid. The greater the density of the liquid, the less of the body is submerged. When you swim in seawater (density 1030 kg/m^3) density your body floats higher than in fresh water (1000 kg/m^3 .



The forces on the fluid element due to pressure must sum to a buoyant force equal in magnitude to the element's weight.

(b) Fluid element replaced with solid body of the same size and shape



The forces due to pressure are the same, so the body must be acted upon by the same buoyant force as the fluid element, regardless of the body's weight.

Figure 113 – Archimede's principle

A practical example of buoyancy is the hydrometer, used to measure the density of liquids (see fig. 114a). The calibrated float sinks into the fluid until the weight of the fluid it displaces is exactly equal to its own weight. The hydrometer floats *higher* in denser liquids than in less dense liquids, and a scale in the top stem permits direct density readings. Figure 114b shows a type of hydrometer that is commonly used to measure the density of battery acid or antifreeze. The bottom of the large tube is immersed in the liquid; the bulb is squeezed to expel air and is then released, like a giant medicine dropper. The liquid rises into the outer tube, and the hydrometer floats in this sample of the liquid.

> (b) Using a hydrometer to measure the density of battery acid or antifreeze



scale float upright.

Figure 114 – Measuring the density of a fluid

An object less dense than water, such as an air-filled beach ball, floats with part of its volume below the surface. Conversely, a paper clip can rest *atop* a water surface even though its density is several times that of water. This is an example of **surface tension:** The surface of the liquid behaves like a membrane under tension (see fig. 115). Surface tension arises because the molecules of the liquid exert attractive forces on each other. There is zero net force on a molecule inside the volume of the liquid, but a surface molecule is drawn into the volume (see fig.

116). Thus the liquid tends to minimize its surface area, just as a stretched membrane does.



Figure 115 – The surface of the water acts like a membrane under tension, allowing this water strider to literally "walk on water"

Molecules in a liquid are attracted by neighboring molecules.



Figure 116 – A molecule at the surface of a liquid is attracted into the bulk liquid, which tends reduce the liquid's surface area

Surface tension explains why freely falling raindrops are spherical (*not* teardropshaped): A sphere has a smaller surface area for its volume than any other shape. It also explains why hot, soapy water is used for washing. To wash clothing thoroughly, water must be forced through the tiny spaces between the fibers (see fig. 117). To do so requires increasing the surface area of the water, which is difficult to achieve because of surface tension. The job is made easier by increasing the temperature of the water and adding soap, both of which decrease the surface tension.



Figure 117 – Surface tension makes it difficult to force water through small crevices. The required water pressure p can be reduced by using hot, soapy water, which has less surface tension

Surface tension is important for a millimeter-sized water drop, which has a relatively large surface area for its volume. (A sphere of radius r has surface area $4\pi r^2$ and volume $(4\pi/3)r^3$. The ratio of surface area to volume is which increases with decreasing radius.) For large quantities of liquid, however, the ratio of surface area to volume is relatively small, and surface tension is negligible compared to pressure forces. For the remainder of this chapter, we will consider only fluids in bulk and hence will ignore the effects of surface tension.

1.8.3 Fluid Flow

We are now ready to consider *motion* of a fluid. Fluid flow can be extremely complex, as shown by the currents in river rapids or the swirling flames of a campfire. But some situations can be represented by relatively simple idealized models. An **ideal fluid** is a fluid that is *incompressible* (that is, its density cannot change) and has no internal friction (called **viscosity**). Liquids are approximately incompressible in most situations, and we may also treat a gas as incompressible if the pressure differences from one region to another are not too great. Internal friction in a fluid causes shear stresses when two adjacent layers of fluid move relative to each other, as when fluid flows inside a tube or around an obstacle. In

some cases we can neglect these shear forces in comparison with forces arising from gravitation and pressure differences.

The path of an individual particle in a moving fluid is called a **flow line**. If the overall flow pattern does not change with time, the flow is called **steady flow**. In steady flow, every element passing through a given point follows the same flow line. In this case the "map" of the fluid velocities at various points in space remains constant, although the velocity of a particular particle may change in both magnitude and direction during its motion. A **streamline** is a curve whose tangent at any point is in the direction of the fluid velocity at that point. When the flow pattern changes with time, the streamlines do not coincide with the flow lines. We will consider only steady-flow situations, for which flow lines and streamlines are identical.

The flow lines passing through the edge of an imaginary element of area, such as the area *A* in Fig. 118, form a tube called a **flow tube.** From the definition of a flow line, in steady flow no fluid can cross the side walls of a flow tube; the fluids in different flow tubes cannot mix.



Figure 118 – A flow tube bounded by flow lines. In steady flow, fluid cannot cross the walls of a flow tube

Figure 119 shows patterns of fluid flow from left to right around three different obstacles. The photographs were made by injecting dye into water flowing between two closely spaced glass plates. These patterns are typical of **laminar flow**, in which adjacent layers of fluid slide smoothly past each other and the flow is steady. (A *lamina* is a thin sheet.) At sufficiently high flow rates, or when boundary surfaces cause abrupt changes in velocity, the flow can become irregular and chaotic. This is called **turbulent flow** (see fig. 120). In turbulent flow there is no steady-state pattern; the flow pattern changes continuously.



Figure 119 - Laminar flow obstacles of different shapes



Figure 120 – The flow of smoke rising from these intense sticks is laminar up to a certain point, and then becomes turbulent

The mass of a moving fluid doesn't change as it flows. This leads to an important quantitative relationship called the **continuity equation.** Consider a portion of a flow tube between two stationary cross sections with areas S_1 and S_2 (see fig. 121). The fluid speeds at these sections are v_1 and v_2 respectively. No fluid flows in or out across the sides of the tube because the fluid velocity is tangent to the wall at every point on the wall. During a small time interval dt, the fluid at S_1 moves a distance $v_1 dt$, so a cylinder of fluid with height $v_1 dt$ and

volume $dV_1 = S_1v_1dt$ flows into the tube across S_1 . During this same interval, a cylinder of volume $dV_2 = S_2v_2dt$ flows out of the tube across S_2 .



Figure 121 - A flow tube with changing cross-sectional area. If the fluid is incompressible, the product Sv has the same value at all points along the tube

Let's first consider the case of an incompressible fluid so that the density ρ has the same value at all points. The mass dm_1 flowing into the tube across S_1 in time dt is $dm_1 = \rho S_1 v_1 dt$. Similarly, the mass dm_2 that flows out across S_2 in the same time is $dm_2 = \rho S_2 v_2 dt$. In steady flow the total mass in the tube is constant, so $dm_1 = dm_2$ and

$$\rho S_1 v_1 dt = \rho S_2 v_2 dt \tag{199}$$

or

$$S_1 v_1 = S_2 v_2 \tag{200}$$

The product is the *volume flow rate* the rate at which volume crosses a section of the tube:

$$\frac{dV}{dt} = Sv \tag{201}$$

The *mass* flow rate is the mass flow per unit time through a cross section. This is equal to the density times the volume flow rate $\frac{dV}{dt}$.

Equation (200) shows that the volume flow rate has the same value at all points along any flow tube. When the cross section of a flow tube decreases, the speed increases, and vice versa. A broad, deep part of a river has larger cross section and slower current than a narrow, shallow part, but the volume flow rates are the same in both. This is the essence of the familiar maxim, "Still waters run deep." The stream of water from a faucet narrows as it gains speed during its fall, but $\frac{dV}{dt}$ is the same everywhere along the stream. If a water pipe with 2-cm diameter is connected to a pipe with 1-cm diameter, the flow speed is four times as great in the 1-cm part as in the 2-cm part.

We can generalize Eq. (200) for the case in which the fluid is *not* incompressible. If ρ_1 and are ρ_2 the densities at sections 1 and 2, then

$$\rho S_1 v_1 = \rho S_2 v_2 \tag{202}$$

If the fluid is denser at point 2 than at point 1 ($\rho_2 > \rho_1$) the volume flow rate at point 2 will be less than at point 1 ($S_1v_1 < \rho S_2v_2$). We leave the details to you. If the fluid is incompressible so that ρ_1 and ρ_2 are always equal, Eq. (202) reduces to Eq. (200).

According to the continuity equation, the speed of fluid flow can vary along the paths of the fluid. The pressure can also vary; it depends on height as in the static situation, and it also depends on the speed of flow. We can derive an important relationship called *Bernoulli's equation* that relates the pressure, flow speed, and height for flow of an ideal, incompressible fluid. Bernoulli's equation is an essential tool in analyzing plumbing systems, hydroelectric generating stations, and the flight of airplanes.

The dependence of pressure on speed follows from the continuity equation, Eq. (200). When an incompressible fluid flows along a flow tube with varying cross section, its speed *must* change, and so an element of fluid must have an acceleration. If the tube is horizontal, the force that causes this acceleration has to be applied by the surrounding fluid. This means that the pressure *must* be different in regions of different cross section; if it were the same everywhere, the net force on every fluid element would be zero. When a horizontal flow tube narrows and a fluid element speeds up, it must be moving toward a region of lower pressure in order to have a net forward force to accelerate it. If the elevation also changes, this causes an additional pressure difference.

To derive Bernoulli's equation, we apply the work–energy theorem to the fluid in a section of a flow tube. In Fig. 122 we consider the element of fluid that at some initial time lies between the two cross sections a and c. The speeds at the

lower and upper ends are and In a small time interval dt, the fluid that is initially at a moves to b, a distance $dl_1 = v_1 dt$ and the fluid that is initially at c moves to d, a distance $dl_2 = v_2 dt$. The cross-sectional areas at the two ends are S_1 and S_2 as shown. The fluid is incompressible; hence by the continuity equation, Eq. (200), the volume of fluid dV passing *any* cross section during time dt is the same. That is, $dV = S_1 dl_1 = S_2 dl_2$.



Figure 122 – Deriving Bernoulli's equation. The net work done on a fluid equals the change in the kinetic energy plus the change in the gravitational potential energy

Let's compute the *work* done on this fluid element during dt. We assume that there is negligible internal friction in the fluid (i.e., no viscosity), so the only nongravitational forces that do work on the fluid element are due to the pressure of the surrounding fluid. The pressures at the two ends are p_1 and p_2 ; the force on the cross section at a is p_1S_1 and the force at c is p_2S_2 . The net work dW done on the element by the surrounding fluid during this displacement is therefore

$$dW = p_1 S_1 dl_1 - p_2 S_2 dl_2 = (p_1 - p_2) dV$$
(203)

The second term has a negative sign because the force at c opposes the displacement of the fluid.

The work dW is due to forces other than the conservative force of gravity, so it equals the change in the total mechanical energy (kinetic energy plus gravitational potential energy) associated with the fluid element. The mechanical energy for the fluid between sections b and c does not change. At the beginning of dt the fluid between a and b has volume S_1dl_1 , mass $\rho_1S_1dl_1$ and kinetic energy $\frac{1}{2}\rho(S_1dl_1)v_1^2$. At the end of dt the fluid between c and d has kinetic energy $\frac{1}{2}\rho(S_2dl_2)v_2^2$. The net change in kinetic energy dK during time dt is

$$dK = \frac{1}{2}\rho dV (v_2^2 - v_1^2)$$
⁽²⁰⁴⁾

What about the change in gravitational potential energy? At the beginning of dt, the potential energy for the mass between a and b is $dmgy_1 = \rho dVgy_1$. At the end of dt, the potential energy for the mass between c and d is $dmgy_2 = \rho dVgy_2$. The net change in potential energy dU during dt is

$$dU = \rho dV g(y_2 - y_1) \tag{205}$$

Combining Eqs. (203), (204), and (205) in the energy equation dW = dK + dU we obtain

$$(p_1 - p_2)dV = \frac{1}{2}\rho dV(v_2^2 - v_1^2) + \rho dVg(y_2 - y_1)$$
(206)

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$
⁽²⁰⁷⁾

This is **Bernoulli's equation.** It states that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow. We may also interpret Eq. (207) in terms of pressures. The first term on the right is the pressure difference associated with the change of speed of the fluid. The second term on the right is the additional pressure difference caused by the weight of the fluid and the difference in elevation of the two ends.

We can also express Eq. (207) in a more convenient form as

$$p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$
(208)

The subscripts 1 and 2 refer to *any* two points along the flow tube, so we can also write

$$p + \rho gy + \frac{1}{2}\rho v^2 = constant$$
⁽²⁰⁹⁾

Note that when the fluid is *not* moving (so $v_1 = v_2 = 0$) Eq. (208) reduces to the pressure relationship we derived for a fluid at rest, Eq. (195).

2.1 Temperature and heat

2.1.1 Temperature and thermal equilibrium

The concept of **temperature** is rooted in qualitative ideas of "hot" and "cold" based on our sense of touch. A body that feels hot usually has a higher temperature than a similar body that feels cold. That's pretty vague, and the senses can be deceived. But many properties of matter that we can *measure* depend on temperature. The length of a metal rod, steam pressure in a boiler, the ability of a wire to conduct an electric current, and the color of a very hot glowing object—all these depend on temperature.

Temperature is also related to the kinetic energies of the molecules of a material. In general this relationship is fairly complex, so it's not a good place to start in *defining* temperature. We will look at the relationship between temperature and the energy of molecular motion for an ideal gas. It is important to understand, however, that temperature and heat can be defined independently of any detailed molecular picture. In this section we'll develop a *macroscopic* definition of temperature.

To use temperature as a measure of hotness or coldness, we need to construct a temperature scale. To do this, we can use any measurable property of a system that varies with its "hotness" or "coldness." Figure 123a shows a familiar system that is used to measure temperature. When the system becomes hotter, the colored liquid (usually mercury or ethanol) expands and rises in the tube, and the value of L increases. Another simple system is a quantity of gas in a constant-volume container (see fig. 123b). The pressure p, measured by the gauge, increases or decreases as the gas becomes hotter or colder. A third example is the electrical resistance R of a conducting wire, which also varies when the wire becomes hotter or colder. Each of these properties gives us a number (L, p, or R) that varies with hotness and coldness, so each property can be used to make a **thermometer**.

To measure the temperature of a body, you place the thermometer in contact with the body. If you want to know the temperature of a cup of hot coffee, you stick the thermometer in the coffee; as the two interact, the thermometer becomes hotter and the coffee cools off a little. After the thermometer settles down to a steady value, you read the temperature. The system has reached an *equilibrium* condition, in which the interaction between the thermometer and the coffee causes no further change in the system. We call this a state of **thermal equilibrium**.

If two systems are separated by an insulating material or **insulator** such as wood, plastic foam, or fiberglass, they influence each other more slowly. Camping coolers are made with insulating materials to delay the ice and cold food inside from warming up and attaining thermal equilibrium with the hot summer air outside. An *ideal insulator* is a material that permits no interaction at all between

the two systems. It prevents the systems from attaining thermal equilibrium if they aren't in thermal equilibrium at the start. An ideal insulator is just that, an idealization; real insulators, like those in camping coolers, aren't ideal, so the contents of the cooler will warm up eventually.





(b) Changes in temperature cause the pressure of the gas to change.



Figure 123 – Two devices for measuring temperature

The Zeroth Law of Thermodynamics. We can discover an important property of thermal equilibrium by considering three systems, *A*, *B*, and *C*, that initially are not in thermal equilibrium (see fig. 124). We surround them with an ideal insulating box so that they cannot interact with anything except each other. We separate systems *A* and *B* with an ideal insulating wall (the green slab in Fig. 124a), but we let system *C* interact with both systems *A* and *B*. This interaction is shown in the figure by a yellow slab representing a thermal equilibrium is attained; then *A* and *B* are each in thermal equilibrium with *C*. But are they in thermal equilibrium *with each other*?



(a) If systems A and B are each in thermal equilibrium with system C ...

(b) ... then systems A and B are in thermal equilibrium with each other.



Figure 124 – The zeroth law of thermodynamics

To find out, we separate system C from systems A and B with an ideal insulating wall (see fig. 124b), and then we replace the insulating wall between A and B with a *conducting* wall that lets A and B interact. What happens? Experiment shows that *nothing* happens; there are no additional changes to A or B. We conclude:

If C is initially in thermal equilibrium with both A and B, then A and B are also in thermal equilibrium with each other. This result is called the zeroth law of thermodynamics.

(The importance of this law was recognized only after the first, second, and third laws of thermodynamics had been named. Since it is fundamental to all of them, the name "zeroth" seemed appropriate.)

Now suppose system C is a thermometer, such as the liquid-in-tube system of Fig. 123a. In Fig. 124a the thermometer C is in contact with both A and B. In thermal equilibrium, when the thermometer reading reaches a stable value, the thermometer measures the temperature of both A and B; hence A and B both have the *same* temperature. Experiment shows that thermal equilibrium isn't affected by adding or removing insulators, so the reading of thermometer C wouldn't change if it were in contact only with A or only with B. We conclude:

Two systems are in thermal equilibrium if and only if they have the same temperature.

This is what makes a thermometer useful; a thermometer actually measures *its own* temperature, but when a thermometer is in thermal equilibrium with another body, the temperatures must be equal. When the temperatures of two systems are different, they *cannot* be in thermal equilibrium.

2.1.2 Quantity of heat

When you put a cold spoon into a cup of hot coffee, the spoon warms up and the coffee cools down as they approach thermal equilibrium. The interaction that causes these temperature changes is fundamentally a transfer of *energy* from one substance to another. Energy transfer that takes place solely because of a temperature difference is called *heat flow* or *heat transfer*, and energy transferred in this way is called **heat**.

An understanding of the relationship between heat and other forms of energy emerged during the 18th and 19th centuries. Sir James Joule (1818–1889) studied how water can be warmed by vigorous stirring with a paddle wheel (see fig. 125a). The paddle wheel adds energy to the water by doing *work* on it, and Joule found that *the temperature rise is directly proportional to the amount of work done*. The same temperature change can also be caused by putting the water in contact with some hotter body (see fig. 125b); hence this interaction must also involve an energy exchange. We will explore the relationship between heat and mechanical energy in next chapter.



(a) Raising the temperature of water by doing work on it

Figure 125 – The same temperature change of the same system may be accomplished by (a) doing work on it or (b) adding heat to it

We can define a *unit* of quantity of heat based on temperature changes of some specific material. The **calorie** (abbreviated cal) is defined as *the amount of heat required to raise the temperature of 1 gram of water from 14.5°C to 15.5°C*. The kilocalorie (kcal), equal to 1000 cal, is also used; a food-value calorie is actually a kilocalorie (see fig. 126).

Because heat is energy in transit, there must be a definite relationship between these units and the familiar mechanical energy units such as the joule. Experiments similar in concept to Joule's have shown that

$$1 cal = 4.186 J$$

 $1 kcal = 1000 cal = 4186 J$

The calorie is not a fundamental SI unit. The International Committee on Weights and Measures recommends using the joule as the basic unit of energy in all forms, including heat. We will follow that recommendation in this book.



Figure 126 – The word "energy" is of Greek origin. This label on a can of Greek coffee shows that 100 milliliters of prepared coffee have an energy content of 9.6 kilojoules or 2.3 kilokalories

We use the symbol Q for quantity of heat. When it is associated with an infinitesimal temperature change dT, we call it dQ. The quantity of heat Q required to increase the temperature of a mass m of a certain material from T_1 to T_2 is found to be approximately proportional to the temperature change $\Delta T = T_1 - T_2$. It is also proportional to the mass m of material. When you're heating water to make tea, you need twice as much heat for two cups as for one if the temperature change is the same. The quantity of heat needed also depends on the nature of the material; raising the temperature of 1 kilogram of water by requires 4190 J of heat, but only 910 J is needed to raise the temperature of 1 kilogram of aluminum by 1 C°.

Putting all these relationships together, we have

$$Q = mc\Delta T \tag{210}$$

where c is a quantity, different for different materials, called the **specific heat** of the material. For an infinitesimal temperature change dT and corresponding quantity of heat dQ,

$$dQ = mcdT \tag{211}$$

$$c = \frac{1}{m} \frac{dQ}{dT} \tag{212}$$

In Eqs. (210), (211), and (212), Q (or dQ) and ΔT (or dT) can be either positive or negative. When they are positive, heat enters the body and its temperature increases; when they are negative, heat leaves the body and its temperature decreases.

The specific heat of water is approximately

The specific heat of a material always depends somewhat on the initial temperature and the temperature interval. Figure 127 shows this dependence for water. In the problems and examples in this chapter we will usually ignore this small variation.



Figure 127 – Specific heat of water as a function of temperature. The value of c varies by less than 1% between $0^{\circ}C$ and $100^{\circ}C$

Sometimes it's more convenient to describe a quantity of substance in terms of the number of *moles n* rather than the *mass m* of material. Recall from your study of chemistry that a mole of any pure substance always contains the same number of molecules. The *molar mass* of any substance, denoted by M, is the mass per mole. (The quantity M is sometimes called *molecular weight*, but *molar mass* is preferable; the quantity depends on the mass of a molecule, not its weight.) For example, the molar mass of water is $18 \frac{g}{mol} = 18 \times 10^{-3} kg/mol$; 1 mole of water has a mass of 18 g = 0.018 kg. The total mass m of material is equal to the mass per mole M times the number of moles n:

$$m = nM \tag{213}$$

Replacing the mass m in Eq. (210) by the product nM, we find

$$Q = nMc\Delta T \tag{214}$$

The product Mc is called the **molar heat capacity** (or *molar specific heat*) and is denoted by C (capitalized). With this notation we rewrite Eq. (214) as

$$Q = nC\Delta T \tag{215}$$

Comparing to Eq. (212), we can express the molar heat capacity C (heat per mole per temperature change) in terms of the specific heat c (heat per mass per temperature change) and the molar mass M (mass per mole):

$$C = \frac{1}{n} \frac{dQ}{dT} = Mc \tag{216}$$

For example, the molar heat capacity of water is

$$C = Mc = (0.018 \, kg/mol)(4190 \, J/kg \cdot K) = 75.4 \, J/mol \cdot K$$

Values of specific heat and molar heat capacity for several substances are given in Table 4. Note the remarkably large specific heat for water (see fig. 128).



Figure 128 – Water has a much higher specific heat than the glass or metals used to make cookware. This helps explain why it takes several minutes to boil water on a stove, even through the pot kettle reaches a high temperature very quickly

Substance	Specific	heat,	Molar	mass,	Molar	heat
	$c, (J/kg \cdot K)$		М, (<i>kg/mo</i>	ol)	capacity,	с,(]/
					$mol \cdot K)$	
Aluminium	910		0.02	7	24.6	
Beryllium	1970		0.009	01	17.7	
Copper	390		0.063	85	24.8	
Ethanol	2428		0.046	51	111.9	
Ethylene glycol	2386		0.062	20	148.0	
Ice (near 0°C)	2100		0.018	30	37.8	
Iron	470		0.055	59	26.3	
Lead	130		0.20	7	26.9	
Marble ($CaCO_3$)	879		0.10	0	87.9	
Mercury	138		0.20	1	27.7	
Salt (NaCl)	879		0.058	35	51.4	
Silver	234		0.10	8	25.3	
Water (liquid)	4190		0.018	30	75.4	

Table 4 Approximate specific heats and molar heat capacities

Precise measurements of specific heats and molar heat capacities require great experimental skill. Usually, a measured quantity of energy is supplied by an electric current in a heater wire wound around the specimen. The temperature change ΔT is measured with a resistance thermometer or thermocouple embedded in the specimen. This sounds simple, but great care is needed to avoid or compensate for unwanted heat transfer between the sample and its surroundings. Measurements for solid materials are usually made at constant atmospheric pressure; the corresponding values are called the specific heat and molar heat capacity at constant pressure, denoted by c_p and C_p . For a gas it is usually easier to keep the substance in a container with constant volume; the corresponding values are called the specific heat and molar heat capacity at constant volume, denoted by c_V and C_V . For a given substance, C_V and C_p are different. If the system can expand while heat is added, there is additional energy exchange through the performance of work by the system on its surroundings. If the volume is constant, the system does no work. For gases the difference between C_p and C_V is substantial.

The last column of Table 4 shows something interesting. The molar heat capacities for most elemental solids are about the same: about This correlation, named the *rule of Dulong and Petit* (for its discoverers), forms the basis for a very important idea. The number of atoms in 1 mole is the same for all elemental substances. This means that on a per atom basis, about the same amount of heat is required to raise the temperature of each of these elements by a given amount, even though the masses of the atoms are very different. The heat required for a given temperature increase depends only on *how many* atoms the sample contains, not on the mass of an individual atom. We will see the reason the rule of Dulong and Petit

works so well when we study the molecular basis of heat capacities in greater detail in next chapters.

2.1.3 Phase changes

We use the term **phase** to describe a specific state of matter, such as a solid, liquid, or gas. The compound exists in the *solid phase* as ice, in the *liquid phase* as ater, and in the *gaseous phase* as steam. (These are also referred to as **states of matter:** the solid state, the liquid state, and the gaseous state.) A transition from one phase to another is called a **phase change** or *phase transition*. For any given pressure a phase change takes place at a definite temperature, usually accompanied by absorption or emission of heat and a change of volume and density.

A familiar example of a phase change is the melting of ice. When we add heat to ice at and normal atmospheric pressure, the temperature of the ice *does not* increase. Instead, some of it melts to form liquid water. If we add the heat slowly, to maintain the system very close to thermal equilibrium, the temperature remains at until all the ice is melted (see fig. 129). The effect of adding heat to this system is not to raise its temperature but to change its *phase* from solid to liquid.



Figure 129 – The surrounding air is at room temperature, but this ice-water mixture remains at 0°C until all of the ice has melted and the phase change is complete

To change 1 kg of ice at 0°C to 1 kg of liquid water at 0°C and normal atmospheric pressure requires $3.34 \times 10^5 J$ of heat. The heat required per unit mass is called the **heat of fusion** (or sometimes *latent heat of fusion*), denoted by L_f . For water at normal atmospheric pressure the heat of fusion is

$$L_f = 3.34 \times 10^5 \frac{J}{kg} = 79.6 \ cal/g$$

More generally, to melt a mass m of material that has a heat of fusion requires a quantity of heat Q given by

$$Q = mL_f \tag{217}$$

This process is *reversible*. To freeze liquid water to ice at $0^{\circ}C$ we have to *remove* heat; the magnitude is the same, but in this case, Q is negative because heat is removed rather than added. To cover both possibilities and to include other kinds of phase changes, we write

$$Q = \pm mL \tag{218}$$

The plus sign (heat entering) is used when the material melts; the minus sign (heat leaving) is used when it freezes. The heat of fusion is different for different materials, and it also varies somewhat with pressure.

For any given material at any given pressure, the freezing temperature is the same as the melting temperature. At this unique temperature the liquid and solid phases (liquid water and ice, for example) can coexist in a condition called **phase equilibrium.**

We can go through this whole story again for *boiling* or *evaporation*, a phase transition between liquid and gaseous phases. The corresponding heat (per unit mass) is called the **heat of vaporization** L_V . At normal atmospheric pressure the heat of vaporization L_V for water is

$$L_V = 2.256 \times 10^6 \frac{J}{kg} = 539 \ cal/g$$

That is, it takes $2.256 \times 10^6 J/kg$ to change 1 kg of liquid water at 100°C to 1 kg of water vapour at 100°C. By comparison, to raise the temperature of 1 kg of water from 0°C to 100°C requires $Q = mc\Delta T = (1 kg)(4190 J/kg \cdot °C)(100°C) =$

= $4.19 \cdot 10^5 J$, less than one-fifth as much heat as is required for vaporization at 100°C. This agrees with everyday kitchen experience; a pot of water may reach boiling temperature in a few minutes, but it takes a much longer time to completely evaporate all the water away.

Like melting, boiling is a reversible transition. When heat is removed from a gas at the boiling temperature, the gas returns to the liquid phase, or *condenses*,

giving up to its surroundings the same quantity of heat (heat of vaporization) that was needed to vaporize it. At a given pressure the boiling and condensation temperatures are always the same; at this temperature the liquid and gaseous phases can coexist in phase equilibrium.

Both L_V and the boiling temperature of a material depend on pressure. Water boils at a lower temperature (about 95°C) in Denver than in Pittsburgh because Denver is at higher elevation and the average atmospheric pressure is lower. The heat of vaporization is somewhat greater at this lower pressure, about 2.27 × $10^6 \frac{J}{kq}$.

Table 5 lists heats of fusion and vaporization for some materials and their melting and boiling temperatures at normal atmospheric pressure. Very few *elements* have melting temperatures in the vicinity of ordinary room temperatures; one of the few is the metal gallium, shown in Fig. 130.



Figure 130 – The metal gallium, shown here melting in a person's hand, is one of the few elements that melt in the vicinity of room temperature. Its melting temperature is 29.8°C, and its heat of fusion is $8.04 \times 10^4 J/kg$

Figure 131 shows how the temperature varies when we add heat continuously to a specimen of ice with an initial temperature below $0^{\circ}C$ (point *a*). The temperature rises until we reach the melting point (point *b*). As more heat is added, the temperature remains constant until all the ice has melted (point *c*). Then the temperature rises again until the boiling temperature is reached (point *d*). At that point the temperature again is constant until all the water is transformed into the vapor phase (point *e*). If the rate of heat input is constant, the line for the solid

phase (ice) has a steeper slope than does the line for the liquid phase (water). Do you see why? (see table 4)

Normal melting		melting	Heat of	Normal	boiling	Heat of
Substance	point		fusion,	ро	int	vaporization,
	Κ	°C	L_f , (J/kg)	K	Ъ°	L,(J/kg)
Helium	-	-	-	4.216	-268.93	$20.9 \cdot 10^3$
Hydrogen	13.84	-259.31	$58.6 \cdot 10^3$	20.26	-252.89	$452 \cdot 10^3$
Nitrogen	63.18	-209.97	$25.5 \cdot 10^3$	77.34	-195.8	$201 \cdot 10^3$
Oxygen	54.36	-218.79	$13.8 \cdot 10^3$	90.18	-183	$213 \cdot 10^{3}$
Ethanol	159	-114	$104.2 \cdot 10^3$	351	78	$854 \cdot 10^{3}$
Mercury	234	-39	$11.8 \cdot 10^3$	630	357	$272 \cdot 10^{3}$
Water	273.15	0	$334 \cdot 10^{3}$	373.15	100	$2256 \cdot 10^3$
Sulfur	392	119	$38.1 \cdot 10^3$	717.75	444.6	$326 \cdot 10^3$
Lead	600.5	327.3	$24.5 \cdot 10^3$	2023	1750	$871 \cdot 10^{3}$
Antinomy	903.65	630.5	$165 \cdot 10^{3}$	1713	1440	$561 \cdot 10^{3}$
Silver	1233.95	960.8	$88.3 \cdot 10^3$	2466	2193	$2336 \cdot 10^3$
Gold	1336.15	1063	$64.5 \cdot 10^3$	2933	2660	$1578 \cdot 10^3$
Copper	1356	1083	$134 \cdot 10^{3}$	1460	1187	$5069 \cdot 10^3$

Table 5 - Heats of fusion and vaporization

A substance can sometimes change directly from the solid to the gaseous phase. This process is called *sublimation*, and the solid is said to *sublime*. The corresponding heat is called the *heat of sublimation*, L_s . Liquid carbon dioxide cannot exist at a pressure lower than about 5×10^5 (about 5 atm), and "dry ice" (solid carbon dioxide) sublimes at atmospheric pressure. Sublimation of water from frozen food causes freezer burn. The reverse process, a phase change from gas to solid, occurs when frost forms on cold bodies such as refrigerator cooling coils.

Very pure water can be cooled several degrees below the freezing temperature without freezing; the resulting unstable state is described as *supercooled*. When a small ice crystal is dropped in or the water is agitated, it crystallizes within a second or less. Supercooled water *vapor* condenses quickly into fog droplets when a disturbance, such as dust particles or ionizing radiation, is introduced. This principle is used in "seeding" clouds, which often contain supercooled water vapor, to cause condensation and rain.

A liquid can sometimes be *superheated* above its normal boiling temperature. Any small disturbance such as agitation causes local boiling with bubble formation.

Steam heating systems for buildings use a boiling–condensing process to transfer heat from the furnace to the radiators. Each kilogram of water that is turned to steam in the boiler absorbs over (the heat of vaporization of water) from the boiler and gives it up when it condenses in the radiators. Boiling–condensing processes are also used in refrigerators, air conditioners, and heat pumps.

Phase of water changes. During these periods, temperature stays constant and the phase change proceeds as heat is added: Q = +mL.



Temperature of water changes. During these periods, temperature rises as heat is added: $Q = mc \Delta T$.

Figure 131 – Graph of temperature versus time for a specimen of water initially in the solid phase (ice). Heat is added to the specimen at a constant rate. The temperature remains constant during each change of phase, provided that the pressure remains constant

The temperature-control mechanisms of many warm-blooded animals make use of heat of vaporization, removing heat from the body by using it to evaporate water from the tongue (panting) or from the skin (sweating). Evaporative cooling enables humans to maintain normal body temperature in hot, dry desert climates where the air temperature may reach 55°C. The skin temperature may be as much as 30°C cooler than the surrounding air. Under these conditions a normal person may perspire several liters per day, and this lost water must be replaced. Old-time desert rats state that in the desert, any canteen that holds less than a gallon should be viewed as a toy! Evaporative cooling also explains why you feel cold when you first step out of a swimming pool (see fig. 132).

Evaporative cooling is also used to cool buildings in hot, dry climates and to condense and recirculate "used" steam in coal-fired or nuclear-powered electricgenerating plants. That's what goes on in the large, tapered concrete towers that you see at such plants.



Figure 132 – The water may be warm and it may be a hot day, but these children will feel cold when they first step out of the swimming pool. That's because as water evaporates from their skin, it removes the heat of vaporization from their bodies. To stay warm, they will need to dry of immediately

Chemical reactions such as combustion are analogous to phase changes in that they involve definite quantities of heat. Complete combustion of 1 gram of gasoline produces about 46,000 J or about 11,000 cal, so the heat of combustion of gasoline is

$$L_c = 46000 J/g = 4.6 \times 10^7 \frac{J}{kg}$$

Energy values of foods are defined similarly. When we say that a gram of peanut butter "contains 6 calories," we mean that 6 kcal of heat (6000 cal or 25000 J) is released when the carbon and hydrogen atoms in the peanut butter react with oxygen (with the help of enzymes) and are completely converted CO_2 to and H_2O . Not all of this energy is directly useful for mechanical work.

2.1.4 Mechanisms of heat transfer

We have talked about *conductors* and *insulators*, materials that permit or prevent heat transfer between bodies. Now let's look in more detail at *rates* of energy transfer. In the kitchen you use a metal or glass pot for good heat transfer from the stove to whatever you're cooking, but your refrigerator is insulated with a material that *prevents* heat from flowing into the food inside the refrigerator. How do we describe the difference between these two materials?

The three mechanisms of heat transfer are conduction, convection, and radiation. *Conduction* occurs within a body or between two bodies in contact. *Convection* depends on motion of mass from one region of space to another. *Radiation* is heat transfer by electromagnetic radiation, such as sunshine, with noneed for matter to be present in the space between bodies.

Conduction. If you hold one end of a copper rod and place the other end in a flame, the end you are holding gets hotter and hotter, even though it is not in direct contact with the flame. Heat reaches the cooler end by **conduction** through the material. On the atomic level, the atoms in the hotter regions have more kinetic energy, on the average, than their cooler neighbors. They jostle their neighbors, giving them some of their energy. The neighbors jostle *their* neighbors, and so on through the material. The atoms themselves do not move from one region of material to another, but their energy does.

Most metals also use another, more effective mechanism to conduct heat. Within the metal, some electrons can leave their parent atoms and wander through the crystal lattice. These "free" electrons can rapidly carry energy from the hotter to the cooler regions of the metal, so metals are generally good conductors of heat. A metal rod at $20^{\circ}C$ feels colder than a piece of wood at $20^{\circ}C$ because heat can flow more easily from your hand into the metal. The presence of "free" electrons also causes most metals to be good electrical conductors.

Heat transfer occurs only between regions that are at different temperatures, and the direction of heat flow is always from higher to lower temperature. Figure 133a shows a rod of conducting material with cross-sectional area A and length L. The left end of the rod is kept at a temperature T_H and the right end at a lower temperature T_C so heat flows from left to right. The sides of the rod are covered by an ideal insulator, so no heat transfer occurs at the sides.

When a quantity of heat dQ is transferred through the rod in a time dt, the rate of heat flow is dQ/dT. We call this rate the **heat current**, denoted by *H*. That is, H=dQ/dT. Experiments show that the heat current is proportional to the crosssectional area S of the rod (see fig. 133b) and to the temperature difference $(T_H - T_C)$ and is inversely proportional to the rod length *L* (see fig. 133c). Introducing a proportionality constant *k* called the **thermal conductivity** of the material, we have

$$H = \frac{dQ}{dT} = kS \frac{T_H - T_C}{L}$$
(219)

The quantity $(T_H - T_C)/L$ is the temperature difference *per unit length;* it is called the magnitude of the **temperature gradient.** The numerical value of *k* depends on the material of the rod. Materials with large *k* are good conductors of heat; materials with small *k* are poor conductors, or insulators. Equation (219) also gives the heat current through a slab or through *any* homogeneous body with uniform cross section S perpendicular to the direction of flow; *L* is the length of the heatflow path. The units of heat current *H* are units of energy per time, or power; the SI unit of heat current is the watt 1 W = 1 J/s. We can find the units of *k* by solving Eq. (219) for *k*; you can show that the SI units are $W/m \cdot K$. Some numerical values of *k* are given in Table 6.

Substance	$k(W/m \cdot K)$			
Metals				
Aluminium	205			
Brass	109			
Copper	385			
Lead	34.7			
Mercury	8.3			
Silver	406			
Steel	50.2			
Solids (representative values)				
Brick, insulating	0.15			
Brick, red	0.6			
Concrete	0.8			
Cork	0.04			
Felt	0.04			
Fiberglass	0.04			
Glass	0.8			
Ice	1.6			
Rock wool	0.04			
Styrofoam	0.0027			
Wood	0.12-0.14			
Gases				
Air	0.024			
Argon	0.016			
Helium	0.14			
Hydrogen	0.14			
Oxygen	0.0023			

 Table 6 - Thermal conductivities

The thermal conductivity of "dead" (that is, nonmoving) air is very small. A wool sweater keeps you warm because it traps air between the fibers. In fact, many insulating materials such as Styrofoam and fiberglass are mostly dead air.

If the temperature varies in a nonuniform way along the length of the conducting rod, we introduce a coordinate x along the length and generalize the temperature gradient to be dT/dx. The corresponding generalization of Eq. (219) is

$$H = \frac{dQ}{dT} = -kS\frac{dT}{dx}$$
(220)

The negative sign shows that heat always flows in the direction of *decreasing* temperature.

(a) Heat current H



(b) Doubling the cross-sectional area of the conductor doubles the heat current (*H* is proportional to *A*).



(c) Doubling the length of the conductor halves the heat current (*H* is inversely proportional to *L*).



Figure 133 – Steady-state heat flow due to conduction in a uniform rod

For thermal insulation in buildings, engineers use the concept of thermal resistance, denoted by R. The thermal resistance R of a slab of material with area S is defined so that the heat current H through the slab is

$$H = \frac{S(T_H - T_C)}{R} \tag{221}$$

where T_H and T_C are the temperatures on the two sides of the slab. Comparing this with Eq. (219), we see that R is given by

$$R = \frac{L}{k}$$
(222)

where *L* is the thickness of the slab. The SI unit of *R* is $1 m^2 \cdot K/W$.

Convection is the transfer of heat by mass motion of a fluid from one region of space to another. Familiar examples include hot-air and hot-water home heating systems, the cooling system of an automobile engine, and the flow of blood in the body. If the fluid is circulated by a blower or pump, the process is called *forced convection;* if the flow is caused by differences in density due to thermal expansion, such as hot air rising, the process is called *natural convection* or *free convection* (see fig. 134).



Figure 134 – A heating in the tip of this submerged tube warms the surrounding water, producing a complex pattern of free convection

Free convection in the atmosphere plays a dominant role in determining the daily weather, and convection in the oceans is an important global heat-transfer mechanism. On a smaller scale, soaring hawks and glider pilots make use of thermal updrafts from the warm earth. The most important mechanism for heat transfer within the human body (needed to maintain nearly constant temperature in various environments) is *forced* convection of blood, with the heart serving as the pump.

Convective heat transfer is a very complex process, and there is no simple equation to describe it. Here are a few experimental facts:

1. The heat current due to convection is directly proportional to the surface area. This is the reason for the large surface areas of radiators and cooling fins.

2. The viscosity of fluids slows natural convection near a stationary surface, giving a surface film that on a vertical surface typically has about the same insulating value as 1.3 cm of plywood Forced convection decreases the thickness of this film, increasing the rate of heat transfer. This is the reason for the "wind-chill factor"; you get cold faster in a cold wind than in still air with the same temperature.

3. The heat current due to convection is found to be approximately proportional to the $\frac{5}{4}$ power of the temperature difference between the surface and the main body of fluid.

Radiation is the transfer of heat by electromagnetic waves such as visible light, infrared, and ultraviolet radiation. Everyone has felt the warmth of the sun's radiation and the intense heat from a charcoal grill or the glowing coals in a fireplace. Most of the heat from these very hot bodies reaches you not by conduction or convection in the intervening air but by *radiation*. This heat transfer would occur even if there were nothing but vacuum between you and the source of heat.

Every body, even at ordinary temperatures, emits energy in the form of electromagnetic radiation. Around nearly all the energy is carried by infrared waves with wavelengths much longer than those of visible light (see Fig. 135). As the temperature rises, the wavelengths shift to shorter values. At 800°C, a body emits enough visible radiation to appear "red-hot," although even at this temperature most of the energy is carried by infrared waves. At 3000°C, the temperature of an incandescent lamp filament, the radiation contains enough visible light that the body appears "white-hot."



Figure 135 – This false-color infrared photograph reveals radiation emitted by various parts of the man's body. The strongest emission (colored red) comes from the warmest areas, while there is very little emission from the bottle of cold beverage

The rate of energy radiation from a surface is proportional to the surface area S and to the fourth power of the absolute (Kelvin) temperature T. The rate also depends on the nature of the surface; this dependence is described by a quantity e called the emissivity. A dimensionless number between 0 and 1, *e* represents the ratio of the rate of radiation from a particular surface to the rate of radiation from an equal area of an ideal radiating surface at the same temperature. Emissivity also depends somewhat on temperature. Thus the heat current H=dQ/dT due to radiation from a surface area A with emissivity e at absolute temperature T can be expressed as

$$H = Se\sigma T^4 \tag{223}$$

where σ is a fundamental physical constant called the Stefan–Boltzmann constant. This relationship is called the Stefan–Boltzmann law in honor of its late-19thcentury discoverers. The current best numerical value of σ is

$$\sigma = 5.670400(40) \cdot 10^{-8} W/m^2 \cdot K^4$$

We invite you to check unit consistency in Eq. (223). Emissivity is often larger for dark surfaces than for light ones. The emissivity of a smooth copper surface is about 0.3, but e for a dull black surface can be close to unity.

2.2 Thermal properties of matter

2.2.1 Equations of state

The conditions in which a particular material exists are described by physical quantities such as pressure, volume, temperature, and amount of substance. For example, a tank of oxygen in a welding outfit has a pressure gauge and a label stating its volume. We could add a thermometer and place the tank on a scale to determine its mass. These variables describe the *state* of the material and are called **state variables**.

The volume V of a substance is usually determined by its pressure p, temperature T, and amount of substance, described by the mass m_{total} or number of moles n. (We are calling the total mass of a substance because later in the chapter we will use m for the mass of one molecule.) Ordinarily, we can't change one of these variables without causing a change in another. When the tank of oxygen gets hotter, the pressure increases. If the tank gets too hot, it explodes.

In a few cases the relationship among p, V, T, and m (or n) is simple enough hat we can express it as an equation called the **equation of state.** When it's too complicated for that, we can use graphs or numerical tables. Even then, the relationship among the variables still exists; we call it an equation of state even when we don't know the actual equation.

Here's a simple (though approximate) equation of state for a solid material.

The temperature coefficient of volume expansion β is the fractional volume change $\Delta V/V_0$ per unit temperature change, and the compressibility *k* is the negative of the fractional volume change $\Delta V/V_0$ per unit pressure change. If a certain amount of material has volume V_0 when the pressure is p_0 and the temperature is p_0 the volume *V* at slightly differing pressure *p* and temperature *T* is approximately

$$V = V_0 [1 + \beta (T - T_0) - k(p - p_0)]$$
(224)

(There is a negative sign in front of the term $k(p - p_0)$ because an *increase* in pressure causes a *decrease* in volume.)



Figure 136 – A hypothetical setup for studying the behaviour of gases. By heating the gas, varying the volume with a movable piston, and adding more gas, we can control the gas pressure, volume, temperature and numbers of moles
Another simple equation of state is the one for an *ideal gas*. Figure 136 shows an experimental setup to study the behavior of a gas. The cylinder has a movable piston to vary the volume, the temperature can be varied by heating, and we can pump any desired amount of any gas into the cylinder. We then measure the pressure, volume, temperature, and amount of gas. Note that *pressure* refers both to the force per unit area exerted by the cylinder on the gas and to the force per unit area exerted by the cylinder; by Newton's third law, these must be equal.

It is usually easiest to describe the amount of gas in terms of the number of moles n, rather than the mass. The **molar mass** M of a compound (sometimes called *molecular weight*) is the mass per mole, and the total mass m_{total} of a given quantity of that compound is the number of moles n times the mass per mole M:

$$m_{total} = nM \tag{225}$$

Hence if we know the number of moles of gas in the cylinder, we can determine the mass of gas using Eq. (225).

Measurements of the behavior of various gases lead to three conclusions:

1. The volume V is proportional to the number of moles n. If we double the number of moles, keeping pressure and temperature constant, the volume doubles.

2. The volume varies *inversely* with the absolute pressure p. If we double the pressure while holding the temperature T and number of moles n constant, the gas compresses to one-half of its initial volume. In other words, pV = const when n and T are constant.

3. The pressure is proportional to the *absolute* temperature. If we double the absolute temperature, keeping the volume and number of moles constant, the pressure doubles. In other words, $\frac{p}{T} = constant$ when *n* and *V* are constant.

These three relationships can be combined neatly into a single equation, called the **ideal-gas equation:**

$$pV = nRT \tag{226}$$

where R is a proportionality constant. An **ideal gas** is one for which Eq. (226) holds precisely for *all* pressures and temperatures. This is an idealized model; it works best at very low pressures and high temperatures, when the gas molecules are far apart and in rapid motion. It is reasonably good (within a few percent) at moderate pressures (such as a few atmospheres) and at temperatures well above those at which the gas liquefies (see fig. 137).



Figure 137 – The ideal-gas equation pV = nRT gives a good description of the air inside an inflated vehicle tire, where the pressure is about 3 atmospheres and the temperature is much too high for nitrogen or oxygen to liquefy. As the tire warms (T increases), the volume V changes only slightly but the pressure p increases

We might expect that the constant *R* in the ideal-gas equation would have different values for different gases, but it turns out to have the same value for *all*gases, at least at sufficiently high temperature and low pressure. It is called the **gas constant** (or *ideal-gas constant*). The numerical value of *R* depends on the units of *p*, *V*, and *T*. In SI units, in which the unit of *p* is Pa $(1 Pa = 1 N/m^2)$ and the unit of *V* is m^3 , the current best numerical value of *R* is

$$R = 8.314472(15) \frac{J}{mol \cdot K}$$

or $R = 8.314 J/mol \cdot K$ to four significant figures. Note that the units of pressure times volume are the same as the units of work or energy (for example, N/m^2 times m^3) that's why R has units of energy per mole per unit of absolute temperature. In chemical calculations, volumes are often expressed in liters (L) and pressures in atmospheres (atm). In this system, to four significant figures,

$$R = 0.08206 \frac{L \cdot atm}{mol \cdot K}$$

We can express the ideal-gas equation, Eq. (226), in terms of the mass f gas, using $m_{total} = nM$ from Eq. (225):

$$pV = \frac{m_{total}}{M}RT$$
(227)

From this we can get an expression for the density $\rho = m_{total}/V$ of the gas:

$$\rho = \frac{pM}{RT} \tag{228}$$

For a *constant mass* (or constant number of moles) of an ideal gas the product *nR* is constant, so the quantity $\frac{pV}{T}$ is also constant. If the subscripts 1 and 2 refer to any two states of the same mass of a gas, then

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{229}$$

Notice that you don't need the value of *R* to use this equation.

That may make it seem that the pressure-temperature relationship in the ideal-gas equation, Eq. (226), is just a result of the way we define temperature. But the equation also tells us what happens when we change the volume or the amount of substance. For now, consider Eq. (229) as being based on this genuinely material-independent temperature scale.

2.2.2 Molecular properties of matter

We have studied several properties of matter in bulk, including elasticity, density, surface tension, heat capacities, and equations of state. Now we want to look in more detail at the relationship of bulk behavior to *molecular* structure. We begin with a general discussion of the molecular structure of matter. Then in the next two sections we develop the kinetic-molecular model of an ideal gas, obtaining from this molecular model the equation of state and an expression for heat capacity.

Any specific chemical compound is made up of identical **molecules.** The smallest molecules contain one atom each and are of 10^{-10} *m* the order of in size; the largest contain many atoms and are at least 10000 times larger. In gases the molecules move nearly independently; in liquids and solids they are held together by intermolecular forces. These forces arise from interactions among the electrically charged particles that make up the molecules. Gravitational forces between molecules are negligible in comparison with electrical forces.

The interaction of two *point* electric charges is described by a force (repulsive for like charges, attractive for unlike charges) with a magnitude proportional to $1/r^2$, where *r* is the distance between the points. We will study this relationship, called *Coulomb's law*. Molecules are *not* point charges but complex structures containing both positive and negative charge, and their interactions are more complex. The force between molecules in a gas varies with the distance *r*

between molecules somewhat as shown in Fig. 138, where a positive F_r corresponds to a repulsive force and a negative F_r to an attractive force. When molecules are far apart, the intermolecular forces are very small and usually attractive. As a gas is compressed and its molecules are brought closer together, the attractive forces increase. The intermolecular force becomes zero at an equilibrium spacing r_0 corresponding roughly to the spacing between molecules in the liquid and solid states. In liquids and solids, relatively large pressures are needed to compress the substance appreciably. This shows that at molecular distances slightly *less* than the equilibrium spacing, the forces become *repulsive* and relatively large.



Figure 138 – How the force between molecules and their energy of interaction depend on their r

Figure 138 also shows the potential energy as a function of r. This function has a *minimum* at where the force is zero. The two curves are related by $F_r(r) = -dU/dr$. Such a potential-energy function is often called a **potential well.** A molecule at rest at a distance from a second molecule would need an additional energy $|U_0|$, the "depth" of the potential well, to "escape" to an indefinitely large value of r.

Molecules are always in motion; their kinetic energies usually increase with temperature. At very low temperatures the average kinetic energy of a molecule may be much *less* than the depth of the potential well. The molecules then condense into the liquid or solid phase with average intermolecular spacings of about r_0 . But at higher temperatures the average kinetic energy becomes larger than the depth $|U_0|$ of the potential well. Molecules can then escape the intermolecular force and become free to move independently, as in the gaseous phase of matter.

In *solids*, molecules vibrate about more or less fixed points. In a crystalline solid these points are arranged in a *crystal lattice*. Figure 139 shows the cubic crystal structure of sodium chloride, and Fig. 140 shows a scanning tunnelling microscope image of individual silicon atoms on the surface of a crystal.



Figure 139 – Schematic representation of the cubic crystal structure of sodium chloride (ordinary salt)



Figure 140 – A scanning tunnelling microscope image of the surface of a silicon crystal. The area shown is only 9 nm across. Each blue "bead" is an individual silicon atom; you can clearly see how these atoms are arranged in a (nearly) perfect array of hexagons

The vibration of molecules in a solid about their equilibrium positions may be nearly simple harmonic if the potential well is approximately parabolic in shape at distances close to r_0 . But if the potential-energy curve rises more gradually $r > r_0$ for than for $r < r_0$ as in Fig. 138, the average position shifts to larger r with increasing amplitude. This is the basis of thermal expansion.

In a *liquid*, the intermolecular distances are usually only slightly greater than in the solid phase of the same substance, but the molecules have much greater freedom of movement. Liquids show regularity of structure only in the immediate neighborhood of a few molecules.

The molecules of a *gas* are usually widely separated and so have only very small attractive forces. A gas molecule moves in a straight line until it collides with another molecule or with a wall of the container. In molecular terms, an *ideal gas* is a gas whose molecules exert *no* attractive forces on each other (see Fig. 141a) and therefore have no *potential* energy.

(a) An idealized model of a gas

(b) A more realistic model of a gas



Figure 141 – A gas as model by (a) the ideal-gas equation and (b) the van der Waals equation

At low temperatures, most common substances are in the solid phase. As the temperature rises, a substance melts and then vaporizes. From a molecular point of view, these transitions are in the direction of increasing molecular kinetic energy. Thus temperature and molecular kinetic energy are closely related.

We have used the mole as a measure of quantity of substance. One **mole** of any pure chemical element or compound contains a definite number of molecules, the same number for all elements and compounds. The official SI definition is:

One mole is the amount of substance that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12.

In our discussion, the "elementary entities" are molecules. (In a monatomic substance such as carbon or helium, each molecule is a single atom.) Atoms of a given element may occur in any of several isotopes, which are chemically identical but have different atomic masses; "carbon-12" is a specific isotope of carbon.

The number of molecules in a mole is called **Avogadro's number**, denoted by N_A . The current best numerical value of N_A is

$$N_A = 6.02214179(30) \cdot 10^{23}$$
 molecules/mol

The *molar mass* M of a compound is the mass of 1 mole. It is equal to the mass m of a single molecule multiplied by Avogadro's number:

$$M = N_A m \tag{230}$$

When the molecule consists of a single atom, the term *atomic mass* is often used instead of molar mass or molecular weight.

2.2.3 Kinetic-molecular model of an ideal gas

The goal of any molecular theory of matter is to understand the *macroscopic* properties of matter in terms of its atomic or molecular structure and behavior. Once we have this understanding, we can design materials to have specific desired

properties. Theories have led to the development of high-strength steels, semiconductor materials for electronic devices, and countless other materials essential to contemporary technology.

In this and the following sections we will consider a simple molecular model of an ideal gas. This *kinetic-molecular model* represents the gas as a large number of particles bouncing around in a closed container. In this section we use the kinetic-molecular model to understand how the ideal-gas equation of state, Eq. (226), is related to Newton's laws. In the following section we use the kineticmolecular model to predict the molar heat capacity of an ideal gas. We'll go on to elaborate the model to include "particles" that are not points but have a finite size.

Our discussion of the kinetic-molecular model has several steps, and you may need to go over them several times. Don't get discouraged!

Here are the assumptions of our model:

1. A container with volume V contains a very large number N of identical molecules, each with mass m.

2. The molecules behave as point particles that are small compared to the size of the container and to the average distance between molecules.

3. The molecules are in constant motion. Each molecule collides occasionally

with a wall of the container. These collisions are perfectly elastic.

4. The container walls are rigid and infinitely massive and do not move.

During collisions the molecules exert *forces* on the walls of the container; this is the origin of the *pressure* that the gas exerts. In a typical collision (see fig. 142) the velocity component parallel to the wall is unchanged, and the component perpendicular to the wall reverses direction but does not change in magnitude.

Our program is first to determine the *number* of collisions that occur per unit time for a certain area A of wall. Then we find the total momentum change associated with these collisions and the force needed to cause this momentum change. From this we can determine the pressure, which is force per unit area, and compare the result to the ideal-gas equation. We'll find a direct connection between the temperature of the gas and the kinetic energy of the gas molecules.

To begin, we will assume that all molecules in the gas have the same *magnitude* of x-velocity, $|v_x|$. This isn't right, but making this temporary assumption helps to clarify the basic ideas. We will show later that this assumption isn't really necessary.

As shown in Fig. 142, for each collision the *x*-component of velocity changes from $-|v_x|$ to $+|v_x|$. So the *x*-component of momentum changes from $-m|v_x|$ to $+m|v_x|$ and the *change* in the *x*-component of momentum is $m|v_x| - (-m|v_x|) = 2m|v_x|$.



Figure 142 – Elastic collision of a molecule with an idealized container wall

If a molecule is going to collide with a given wall area A during a small time interval dt, then at the beginning of dt it must be within a distance from the wall (see fig. 143) and it must be headed toward the wall. So the number of molecules that collide with S during dt is equal to the number of molecules within a cylinder with base area S and length $|v_x|dt$ that have their x-velocity aimed toward the wall. The volume of such a cylinder is $S|v_x|dt$. Assuming that the number of molecules per unit volume N/V is uniform, the number of molecules in this cylinder is $(N/V)(S|v_x|dt)$. On the average, half of these molecules are moving toward the wall and half are moving away from it. So the number of collisions with S during dt is

$$\frac{1}{2} \left(\frac{N}{V}\right) \left(S|v_x|dt\right) \tag{231}$$



Figure 143 – For a molecule to strike the wall in area S during a time interval dt, the molecule must be headed for the wall and be within the shaded cylinder of length $|v_x|dt$ at the beginning of the interval

For the system of all molecules in the gas, the total momentum change dP_x during *dt* is the *number* of collisions multiplied by $2m|v_x|$:

$$dP_x = \frac{1}{2} \left(\frac{N}{V}\right) \left(S|v_x|dt\right) \left(2m|v_x|\right) = \frac{NSmv_x^2}{V}$$
(232)

(We are using capital *P* for total momentum and small *p* for pressure. Be careful!) We wrote v_x^2 rather than $|v_x^2|$ in the final expression because the square of the absolute value of a number is equal to the square of that number. The *rate* of change of momentum component P_x is

$$\frac{dP_x}{dt} = \frac{NSmv_x^2}{V}$$
(233)

According to Newton's second law, this rate of change of momentum equals the force exerted by the wall area *S* on the gas molecules. From Newton's *third* law this is equal and opposite to the force exerted *on* the wall *by* the molecules. Pressure*p* is the magnitude of the force exerted on the wall per unit area, and we obtain

$$p = \frac{F}{S} = \frac{Nmv_{\chi}^2}{V}$$
(234)

The pressure exerted by the gas depends on the number of molecules per volume N/V the mass *m* per molecule, and the speed of the molecules.

2.2.4 Pressure and molecular kinetic energies

We mentioned that $|v_x|$ is really *not* the same for all the molecules. But we could have sorted the molecules into groups having the same $|v_x|$ within each group, then added up the resulting contributions to the pressure. The net effect of all this is just to replace v_x^2 in Eq. (234) by the *average* value of v_x^2 which we denote by $(v_x^2)_{av}$. We can relate $(v_x^2)_{av}$ to the *speeds* v of the molecules. The speed of a molecule is related to the velocity components v_x , v_y and v_z by

$$v^2 = v_x^2 + v_y^2 + v_z^2 \tag{235}$$

We can average this relation over all molecules:

$$(v^{2})_{av} = (v_{x}^{2})_{av} + (v_{y}^{2})_{av} + (v_{z}^{2})_{av}$$
(236)

But there is no real difference in our model between the x-, y-, and z-directions. (Molecular speeds are very fast in a typical gas, so the effects of gravity are negligibly small.) It follows that $(v_x^2)_{av}$, $(v_y^2)_{av}$ and $(v_z^2)_{av}$ must all be *equal*. Hence $(v^2)_{av}$ is equal $3(v_x^2)_{av}$ to and

$$(v^2)_{av} = \frac{1}{3} (v_x^2)_{av} \tag{237}$$

so Eq. (234) becomes

$$pV = \frac{1}{3}Nm(v_x^2)_{av} = \frac{1}{3}N\left[\frac{1}{2}m(v^2)_{av}\right]$$
(238)

We notice that $\frac{1}{2}m(v^2)_{av}$ is the average translational kinetic energy of a single molecule. The product of this and the total number of molecules *N* equals the total random kinetic energy K_{tr} of translational motion of all the molecules. (The notation reminds us that this is the energy of *translational* motion. There may also be energies associated with molecular rotation and vibration.) The product pV equals two-thirds of the total translational kinetic energy:

$$pV = \frac{2}{3}K_{tr} \tag{239}$$

Now we compare this with the ideal-gas equation,

$$pV = nRT \tag{240}$$

which is based on experimental studies of gas behavior. For the two equations to agree, we must have

$$K_{tr} = \frac{3}{2}nRT\tag{241}$$

This remarkably simple result shows that K_{tr} is *directly proportional* to the absolute temperature T (see fig. 144).



Figure 144 – Summer air (left) is warmer than winter air (right); that is, the average translational kinetic energy of air molecules is greater in summer

The average translational kinetic energy of a single molecule is the total translational kinetic energy of all molecules divided by the number of molecules, N:

Also, the total number of molecules N is the number of moles n multiplied by Avogadro's number so

$$N = nN_A \tag{242}$$

$$\frac{n}{N} = \frac{1}{N_A} \tag{243}$$

and

$$\frac{K_{tr}}{N} = \frac{1}{2}m(v^2)_{av} = \frac{3}{2}\left(\frac{R}{N_A}\right)T$$
(244)

The ratio $\frac{R}{N_A}$ occurs frequently in molecular theory. It is called the **Boltzmann constant**, k:

$$k = \frac{R}{N_A} = \frac{8.314 \, J/mol \cdot K}{6.022 \cdot 10^{23} \, molecules/mol} = 1.381 \cdot 10^{23} \, J/molecule \cdot K$$

(The current best numerical value of k is $1.3806504(24) \cdot 10^{23}$ J/molecule · K In terms of k we can rewrite Eq. (244) as

$$\frac{1}{2}m(v^2)_{av} = \frac{3}{2}kT$$
(245)

This shows that the average translational kinetic energy *per molecule* depends only on the temperature, not on the pressure, volume, or kind of molecule. We can obtain the average translational kinetic energy *per mole* by multiplying Eq. (245) by Avogadro's number and using the relation $M = N_A m$:

$$N_A \frac{1}{2} m(v^2)_{av} = \frac{1}{2} M(v^2)_{av} = \frac{3}{2} RT$$
(246)

The translational kinetic energy of a mole of an ideal gas depends only on T.

Finally, it is sometimes convenient to rewrite the ideal-gas equation on a molecular basis. We use $N = N_A n$ and $R = N_A k$ to obtain this alternative form:

$$pV = NkT \tag{247}$$

This shows that we can think of the Boltzmann constant k as a gas constant on a "per-molecule" basis instead of the usual "per-mole" basis for R.

From Eqs. (245) and (246) we can obtain expressions for the square root of $(v^2)_{av}$, called the **root-mean-square speed** (or **rms speed**) v_{rms} :

$$v_{rms} = \sqrt{(v^2)_{av}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$
(248)

It might seem more natural to characterize molecular speeds by their *average* value rather than by v_{rms} but we see that follows more directly from Eqs. (245) and (246). To compute the rms speed, we square each molecular speed, add, divide by the number of molecules, and take the square root; v_{rms} is the *root* of the *mean* of the *squares*.

Equations (245) and (248) show that at a given temperature *T*, gas molecules of different mass *m* have the same average kinetic energy but different root-mean-square speeds. On average, the nitrogen molecules ($M = 28 \ g/mol$) in the air around you are moving faster than are the oxygen molecules ($M = 32 \ g/mol$). Hydrogen molecules are fastest of all; this is why there is hardly any hydrogen in the earth's atmosphere, despite its being the most common element in the universe (see fig. 145). A sizable fraction of any H_2 molecules in the atmosphere would have speeds greater than the earth's escape speed of $1.12 \cdot 10^4 \ m/s$ and would escape into space. The heavier, slower-moving gases cannot escape so easily, which is why they predominate in our atmosphere.



Figure 145 – While hydrogen is a desirable fuel for vehicles, it is only a trace constituent of our atmosphere (0.00005% by volume). Hence hydrogen fuel has to be generated by electrolysis of water, which is itself an energy-intensive process

The assumption that individual molecules undergo perfectly elastic collisions with the container wall is actually a little too simple. More detailed investigation has shown that in most cases, molecules actually adhere to the wall for a short time and then leave again with speeds that are characteristic of the temperature *of the wall*. However, the gas and the wall are ordinarily in thermal equilibrium and have the same temperature. So there is no net energy transfer between gas and

wall, and this discovery does not alter the validity of our conclusions.

2.2.5 Phases of matter

An ideal gas is the simplest system to analyze from a molecular viewpoint because we ignore the interactions between molecules. But those interactions are the very thing that makes matter condense into the liquid and solid phases under some conditions. So it's not surprising that theoretical analysis of liquid and solid structure and behavior is a lot more complicated than that for gases. We won't try to go far here with a microscopic picture, but we can talk in general about phases of matter, phase equilibrium, and phase transitions.

We learned that each phase is stable only in certain ranges of temperature and pressure. A transition from one phase to another ordinarily requires **phase equilibrium** between the two phases, and for a given pressure this occurs at only one specific temperature. We can represent these conditions on a graph with axes pand T, called a **phase diagram**; Fig. 146 shows an example. Each point on the diagram represents a pair of values of p and T.



Figure 146 – A typical pT phase diagram, showing regions of temperature and pressure at which the various phase exist and where phase changes occur

Only a single phase can exist at each point in Fig. 146, except for points on the solid lines, where two phases can coexist in phase equilibrium. The fusion curve separates the solid and liquid areas and represents possible conditions of solid-liquid phase equilibrium. The vaporization curve separates the liquid and vapor areas, and the sublimation curve separates the solid and vapor areas. All three curves meet at the **triple point**, the only condition under which all three phases can coexist (see fig. 147). We used the triple-point temperature of water to define the Kelvin temperature scale. Table 7 gives triple-point data for several substances.

If we add heat to a substance at a constant pressure it goes through a series of states represented by the horizontal line (a) in Fig. 146. The melting and boiling temperatures at this pressure are the temperatures at which the line intersects the fusion and vaporization curves, respectively. When the pressure is constantpressure heating transforms a substance from solid directly to vapor. This process is called *sublimation;* the intersection of line (s) with the sublimation curve gives the temperature T_s at which it occurs for a pressure p_s . At any pressure less than the triple-point pressure, no liquid phase is possible. The triplepoint pressure for carbon dioxide is 5.1 atm. At normal atmospheric pressure, solid carbon dioxide ("dry ice") undergoes sublimation; there is no liquid phase at this pressure.

Substance	Temperature (K)	Pressure (Pa)
Hydrogen	13.8	$0.0704 \cdot 10^5$
Deuterium	18.63	$0.171 \cdot 10^5$
Neon	24.56	$0.432 \cdot 10^5$
Nitrogen	63.28	$0.125 \cdot 10^5$
Oxygen	54.36	$0.00152 \cdot 10^5$
Ammonia	195.4	$0.0607 \cdot 10^5$
Carbon dioxide	216.55	$5.17 \cdot 10^5$
Sulfur dioxide	197.68	$0.00167 \cdot 10^5$
Water	273.26	$0.0061 \cdot 10^5$



Figure 147 – Atmospheric pressure on earth is higher than the triple-point pressure of water. Depending on the temperature, water can exist as a vapour (in atmosphere), as a liquid (in ocean), or as a solid (like the iceberg shown here)

Line (b) in Fig. 146 represents compression at a constant temperature T_b . The material passes from vapor to liquid and then to solid at the points where line (b) crosses the vaporization curve and fusion curve, respectively. Line (d) shows constant-temperature compression at a lower temperature T_d ; the material passes from vapor to solid at the point where line (d) crosses the sublimation curve.

We saw in the *pV*-diagram of Fig. 148 that a liquid-vapor phase transition occurs only when the temperature and pressure are less than those at the point lying at the top of the green shaded area labeled "Liquid-vapor phase equilibrium region." This point corresponds to the endpoint at the top of the vaporization curve in Fig. 146. It is called the **critical point**, and the corresponding values of *p* and *T* are called the critical pressure and temperature, p_c and T_c . A gas at a pressure *above* the critical pressure does not separate into two phases when it is cooled at constant pressure (along a horizontal line above the critical point in Fig. 146). Instead, its properties change gradually and continuously from those we ordinarily associate with a gas (low density, large compressibility) to those of a liquid (high density, small compressibility) without a phase transition.

You can understand this by thinking about liquid-phase transitions at successively higher points on the vaporization curve. As we approach the critical point, the *differences* in physical properties (such as density and compressibility) between the liquid and vapor phases become smaller. Exactly *at* the critical point they all become zero, and at this point the distinction between liquid and vapour disappears. The heat of vaporization also grows smaller as we approach the critical point, and it too becomes zero at the critical point.

For nearly all familiar materials the critical pressures are much greater than atmospheric pressure, so we don't observe this behavior in everyday life. For example, the critical point for water is at 647.4 K and (about 218 atm or 3210 psi). But high-pressure steam boilers in electric generating plants regularly run at pressures and temperatures well above the critical point.

Many substances can exist in more than one solid phase. A familiar example is carbon, which exists as noncrystalline soot and crystalline graphite and diamond. Water is another example; at least eight types of ice, differing in crystal structure and physical properties, have been observed at very high pressures.

We remarked early that the equation of state of any material can be represented graphically as a surface in a three-dimensional space with coordinates p, V, and T. Visualizing such a surface can add to our understanding of the behaviour of materials at various temperatures and pressures. Figure 148 shows a typical pVT-surface. The light lines represent pV-isotherms; projecting them onto the pV-plane gives a diagram similar to Fig. 148. The pV-isotherms represent contour lines on the pVT-surface, just as contour lines on a topographic map represent the elevation (the third dimension) at each point. The projections of the edges of the surface onto the pT-plane give the pT phase diagram of Fig. 146.

 $T_4 > T_3 > T_c > T_2 > T_1$



Figure 148 – A pV-diagram fo a nonideal gas, showing isoterms for temperatures above and below the critical temperature. The liquid-vapor equilibrium region is shown as a green shaded area. At still lower temperatures the material might undegro phase translations from liquid to solid or from gas to solid; these are not shown In this diagram

Line *abcdef* in Fig. 149 represents constant-pressure heating, with melting along *bc* and vaporization along *de*. Note the volume changes that occur as *T* increases along this line. Line *ghjklm* corresponds to an isothermal (constant temperature) compression, with liquefaction along *hj* and solidification along *kl*. Between these, segments *gh* and *jk* represent isothermal compression with increase in pressure; the pressure increases are much greater in the liquid region *jk* and the solid region *lm* than in the vapor region *gh*. Finally, line *nopq* represents isothermal solidification directly from vapor, as in the formation of snowflakes or frost.

Figure 150 shows the much simpler pVT-surface for a substance that obeys the ideal-gas equation of state under all conditions. The projections of the constanttemperature curves onto the pV-plane correspond to the curves of Fig. 151, and the projections of the constant-volume curves onto the pT-plane show that pressure is directly proportional to absolute temperature.



Figure 149 pVT-surface for a substance that expands on melting. Projections of the boundaries on the surface onto the pT- and pV-planes are also shown



Figure 150 - A pVT-surface for an ideal gas. At the left, each red line corresponds to a certain constant volume; at the right, each green line corresponds to a certain constant temperature

Each curve represents pressure as a function of volume for an ideal gas at a single temperature.



Figure 151 – Isotherms, or constant-temperature curves, for a constant amount of an ideal gas. The highest temperature is T_4 ; the lowest is T_1 . This is a graphical representation of the ideal-gas equation of state

2.3 The first law of thermodynamics

2.3.1 Thermodynamics systems

Every time you drive a car, turn on an air conditioner, or cook a meal, you reap the practical benefits of *thermodynamics*, the study of relationships involving heat, mechanical work, and other aspects of energy and energy transfer. For example, in a car engine heat is generated by the chemical reaction of oxygen and vaporized gasoline in the engine's cylinders. The heated gas pushes on the pistons within the cylinders, doing mechanical work that is used to propel the car. This is an example of a *thermodynamic process*.

The first law of thermodynamics, central to the understanding of such processes, is an extension of the principle of conservation of energy. It broadens this principle to include energy exchange by both heat transfer and mechanical work and introduces the concept of the *internal energy* of a system. Conservation of energy plays a vital role in every area of physical science, and the first law has extremely broad usefulness. To state energy relationships precisely, we need the concept of a *thermodynamic system*. We'll discuss *heat* and *work* as two means of transferring energy into or out of such a system.

We have studied energy transfer through mechanical work and through heat transfer. Now we are ready to combine and generalize these principles.

We always talk about energy transfer to or from some specific *system*. The system might be a mechanical device, a biological organism, or a specified quantity of material, such as the refrigerant in an air conditioner or steam expanding in a turbine. In general, a **thermodynamic system** is any collection of objects that is convenient to regard as a unit, and that may have the potential to exchange energy with its surroundings. A familiar example is a quantity of popcorn kernels in a pot with a lid. When the pot is placed on a stove, energy is added to the popcorn by conduction of heat. As the popcorn pops and expands, it does work as it exerts an upward force on the lid and moves it through a displacement. The *state* of the popcorn all change as it pops. A process such as this one, in which there are changes in the state of a thermodynamic system, is called a **thermodynamic process**.

(a)

(b)



Figure 152 - (a) A rocket engine uses the heat of combustion of its fuel to do work propelling the launch vehicle. (b) Humans and other biological organisms are more complicated system than we can analyze fully in this book, but the same basic principles of thermodynamics apply to them

In mechanics we used the concept of *system* with free-body diagrams and with conservation of energy and momentum. For *thermodynamic* systems, as for all others, it is essential to define clearly at the start exactly what is and is not included in the system. Only then can we describe unambiguously the energy transfers into and out of that system. For instance, in our popcorn example we defined the system to include the popcorn but not the pot, lid, or stove.

Thermodynamics has its roots in many practical problems other than popping popcorn (see fig. 152). The gasoline engine in an automobile, the jet engines in an airplane, and the rocket engines in a launch vehicle use the heat of combustion of their fuel to perform mechanical work in propelling the vehicle. Muscle tissue in living organisms metabolizes chemical energy in food and performs mechanical work on the organism's surroundings. A steam engine or steam turbine uses the heat of combustion of coal or other fuel to perform mechanical work such as driving an electric generator or pulling a train.



Figure 153 – A thermodynamics system may exchange energy with its surroundings (environment) by means of heat, work or both. Note the sign conventions for Q and W

Signs for heat and work in thermodynamics. We describe the energy relationships in any thermodynamic process in terms of the quantity of heat Q added to the system and the work W done by the system. Both Q and W may be positive, negative, or zero (see fig. 153). A positive value of Q represents heat flow *into* the system, with a corresponding input of energy to it; negative Q represents heat flow *out of* the system. A positive value of W represents work done by the system against its surroundings, such as work done by an expanding gas, and hence corresponds to energy *leaving* the system. Negative W, such as work done during compression of a gas in which work is done *on the gas* by its surroundings, represents energy *entering* the system. We will use these conventions consistently in the examples in this chapter and the next.

2.3.2 Work of gas

A simple but common example of a thermodynamic system is a quantity of gas enclosed in a cylinder with a movable piston. Internal-combustion engines, steam engines, and compressors in refrigerators and air conditioners all use some version of such a system. In the next several sections we will use the gas-incylinder system to explore several kinds of processes involving energy transformations.

We'll use a microscopic viewpoint, based on the kinetic and potential energies of individual molecules in a material, to develop intuition about thermodynamic quantities. But it is important to understand that the central principles of thermodynamics can be treated in a completely *macroscopic* way, without reference to microscopic models. Indeed, part of the great power and generality of thermodynamics is that it does *not* depend on details of the structure of matter.

First we consider the work done by the system during a volume change. When a gas expands, it pushes outward on its boundary surfaces as they move outward. Hence an expanding gas always does positive work. The same thing is true of any solid or fluid material that expands under pressure.

We can understand the work done by a gas in a volume change by considering the molecules that make up the gas. When one such molecule collides with a stationary surface, it exerts a momentary force on the wall but does no work because the wall does not move. But if the surface is moving, like a piston in a gasoline engine, the molecule does do work on the surface during the collision. If the piston in Fig. 154a moves to the right, so that the volume of the gas increases, the molecules that strike the piston exert a force through a distance and do positive work on the piston. If the piston moves toward the left as in Fig. 154b, so the volume of the gas decreases, then positive work is done on the molecule during the collision. Hence the gas molecules do negative work on the piston.



Figure 154 – A molecule striking a piston (a) does positive work if the piston is moving away from the molecule and (b) does negative work if the piston is moving toward the molecule. Hence a gas does positive work when it expands as in (a) but does negative work when it compresses as in (b)



Figure 155 – The infinitesimal work done by the system during the small expansion dx is dW = pSdx

Figure 155 shows a system whose volume can change (a gas, liquid, or solid) in a cylinder with a movable piston. Suppose that the cylinder has cross-

sectional area A and that the pressure exerted by the system at the piston face is p. The total force F exerted by the system on the piston is F = pS. When the piston moves out an infinitesimal distance dx, the work dW done by this force is

$$dW = Fdx = pSdx \tag{249}$$

but

$$Sdx = dV \tag{250}$$

where dV is the infinitesimal change of volume of the system. Thus we can express the work done by the system in this infinitesimal volume change as

$$dW = pdV \tag{251}$$

In a finite change of volume from V_1 to V_2

$$W = \int_{V_1}^{V_2} p dV \tag{252}$$

In general, the pressure of the system may vary during the volume change. For example, this is the case in the cylinders of an automobile engine as the pistons move back and forth. To evaluate the integral in Eq. (252), we have to know how the pressure varies as a function of volume. We can represent this relationship as a graph of p as a function of V. Figure 156 a shows a simple example. In this figure, Eq. (252) is represented graphically as the area under the curve of p versus V between the limits V_1 and V_2 .

According to the rule, work is positive when a system expands. In an expansion from state 1 to state 2 in Fig. 156a, the area under the curve and the work are positive. A compression from 1 to 2 in Fig. 156b gives a negative area; when a system is compressed, its volume decreases and it does negative work on its surroundings (see also Fig. 154b).

If the pressure p remains constant while the volume changes from V_1 to V_2 (see fig. 156c), the work done by the system is

$$W = p(V_2 - V_1)$$
(253)

In any process in which the volume is constant, the system does no work because there is no displacement.



Figure 156 – The work done equals the area under the curve on a pV-diagram

We've seen that if a thermodynamic process involves a change in volume, the system undergoing the process does work (either positive or negative) on its surroundings. Heat also flows into or out of the system during the process if there is a temperature difference between the system and its surroundings. Let's now examine how the work done by and the heat added to the system during a thermodynamic process depend on the details of how the process takes place.

When a thermodynamic system changes from an initial state to a final state, it passes through a series of intermediate states. We call this series of states a **path**.

There are always infinitely many different possibilities for these intermediate states. When they are all equilibrium states, the path can be plotted on a pV-diagram (see fig. 157a). Point 1 represents an initial state with pressure p_1 and volume V_1 and point 2 represents a final state with pressure p_2 and volume V_2 . To pass from state 1 to state 2, we could keep the pressure constant at p_1 while the system expands to volume V_2 (point 3 in Fig. 157b), then reduce the pressure to p_2 (probably by decreasing the temperature) while keeping the volume constant at V_2 (to point 2 on the diagram). The work done by the system during this process is the area under the line $1 \rightarrow 3$; no work is done during the constant-volume process $3 \rightarrow 2$. Or the system might traverse the path $1 \rightarrow 4 \rightarrow 2$ (see fig. 157c); in that case the work is the area under the line $4 \rightarrow 2$, since no work is done during the constant-volume process $1 \rightarrow 4$. The smooth curve from 1 to 2 is another possibility (see fig. 157d), and the work for this path is different from that for either of the other paths.



Figure 157 – The work done by a system during a transition between two states depends on the path chosen

We conclude that the work done by the system depends not only on the initial and final states, but also on the intermediate states—that is, on the path. Furthermore, we can take the system through a series of states forming a closed loop, such as $1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$. In this case the final state is the same as the initial state, but the total work done by the system is *not* zero. (In fact, it is represented on the graph by the area enclosed by the loop; can you prove that?) It follows that it doesn't make sense to talk about the amount of work *contained in* a system. In a particular state, a system may have definite values of the state coordinates p, V, and T, but it wouldn't make sense to say that it has a definite value of W.

Like work, the *heat* added to a thermodynamic system when it undergoes a change of state depends on the path from the initial state to the final state. Here's an example. Suppose we want to change the volume of a certain quantity of an ideal gas from 2.0 L to 5.0 L while keeping the temperature constant at Figure 158 shows two different ways in which we can do this. In Fig. 158a the gas is contained in a cylinder with a piston, with an initial volume of 2.0 L. We let the gas expand slowly, supplying heat from the electric heater to keep the temperature at 300 K. After expanding in this slow, controlled, isothermal manner, the gas reaches its final volume of 5.0 L; it absorbs a definite amount of heat in the process.



Figure 19.8 – (a) Slow, controlled isothermal expansion of a gas from an initial state 1 to a final state 2 with the same temperature bet lower pressure. (b) Rapid, uncontrolled expansion of the same gas starting at the same state 1 and ending at the same state

Figure 158b shows a different process leading to the same final state. The container is surrounded by insulating walls and is divided by a thin, breakable partition into two compartments. The lower part has volume 2.0 L and the upper part has volume 3.0 L. In the lower compartment we place the same amount of the same gas as in Fig. 158a, again at The initial state is the same as before. Now we break the partition; the gas undergoes a rapid, uncontrolled expansion, with no heat passing through the insulating walls. The final volume is 5.0 L, the same as in Fig. 158a. The gas does no work during this expansion because it doesn't push against

anything that moves. This uncontrolled expansion of a gas into vacuum is called a **free expansion.**

Experiments have shown that when an ideal gas undergoes a free expansion, there is no temperature change. Therefore the final state of the gas is the same as in Fig. 158a. The intermediate states (pressures and volumes) during the transition from state 1 to state 2 are entirely different in the two cases; Figs. 158a and 158b represent *two different paths* connecting the *same states* 1 and 2. For the path in Fig. 158b, *no* heat is transferred into the system, and the system does no work. Like work, *heat depends not only on the initial and final states but also on the path*.

Because of this path dependence, it would not make sense to say that a system "contains" a certain quantity of heat. To see this, suppose we assign an arbitrary value to the "heat in a body" in some reference state. Then presumably the "heat in the body" in some other state would equal the heat in the reference state plus the heat added when the body goes to the second state. But that's ambiguous, as we have just seen; the heat added depends on the *path* we take from the reference state to the second state. We are forced to conclude that there is *no* consistent way to define "heat in a body"; it is not a useful concept.

While it doesn't make sense to talk about "work in a body" or "heat in a body," it *does* make sense to speak of the amount of *internal energy* in a body. This important concept is our next topic.

2.3.3 Internal energy. The first law of thermodynamics

Internal energy is one of the most important concepts in thermodynamics. When we discussed energy changes for a body sliding with friction, we stated that warming a body increased its internal energy and that cooling the body decreased its internal energy. But what *is* internal energy? We can look at it in various ways; let's start with one based on the ideas of mechanics. Matter consists of atoms and molecules, and these are made up of particles having kinetic and potential energies. We *tentatively* define the **internal energy** of a system as the sum of the kinetic energies of all of its constituent particles, plus the sum of all the potential energies of interaction among these particles.

We use the symbol U for internal energy. (We used this same symbol in our study of mechanics to represent potential energy. You may have to remind yourself occasionally that U has a different meaning in thermodynamics.) During a change of state of the system, the internal energy may change from an initial value U_1 to a final value U_2 . We denote the change in internal energy as $\Delta U = U_2 - U_1$.

When we add a quantity of heat Q to a system and the system does no work during the process (so W = 0), the internal energy increases by an amount equal to Q; that is, $\Delta U = Q$. When a system does work W by expanding against its surroundings and no heat is added during the process, energy leaves the system and the internal energy decreases: W is positive, Q is zero, and $\Delta U = -W$. When *both* heat transfer and work occur, the *total* change in internal energy is

$$U_2 - U_1 = \Delta U = Q - W \tag{254}$$

We can rearrange this to the form

$$Q = \Delta U + W \tag{255}$$

The message of Eq. (255) is that in general, when heat Q is added to a system, some of this added energy remains within the system, changing its internal energy by an amount the remainder leaves the system again as the system does work W against its surroundings. Because W and Q may be positive, negative, or zero, can be positive, negative, or zero ΔU for different processes (see fig. 159).







(c) Heat added to system equals work done by system: Internal energy of system unchanged.



Figure 159 – In a thermodynamic process, the internal energy U of a system may (a) increase ($\Delta U > 0$); (b) decrease ($\Delta U < 0$), or (c) remain the same ($\Delta U = 0$)

Equation (254) or (255) is the **first law of thermodynamics.** It is a generalization of the principle of conservation of energy to include energy transfer through heat as well as mechanical work. As you will see in later chapters, this principle can be extended to ever-broader classes of phenomena by identifying additional forms of energy and energy transfer. In every situation in which it seems that the total energy in all known forms is not conserved, it has been possible to identify a new form of energy such that the total energy, including the new form, *is* conserved. There is energy associated with electric fields, with magnetic fields, and, according to the theory of relativity, even with mass itself.

At the beginning of this discussion we tentatively defined internal energy in terms of microscopic kinetic and potential energies. This has drawbacks, however. Actually *calculating* internal energy in this way for any real system would be hopelessly complicated. Furthermore, this definition isn't an *operational* one because it doesn't describe how to determine internal energy from physical quantities that we can measure directly.

So let's look at internal energy in another way. Starting over, we define the *change* in internal energy ΔU during any change of a system as the quantity given by Eq. (254), $\Delta U = Q - W$. This *is* an operational definition because we can measure Q and W. It does not define U itself, only ΔU . This is not a shortcoming because we can *define* the internal energy of a system to have a specified value in some reference state, and then use Eq. (254) to define the internal energy in any other state. This is analogous to our treatment of potential energy in which we arbitrarily defined the potential energy of a mechanical system to be zero at a certain position.

This new definition trades one difficulty for another. If we define ΔU by Eq. (254), then when the system goes from state 1 to state 2 by two different paths, how do we know that ΔU is the same for the two paths? We have already seen that Q and W are, in general, *not* the same for different paths. If ΔU , which equals Q - W, is also path dependent, then ΔU is ambiguous. If so, the concept of internal energy of a system is subject to the same criticism as the erroneous concept of quantity of heat in a system.

The only way to answer this question is through *experiment*. For various materials we measure Q and W for various changes of state and various paths to learn whether ΔU is or is not path dependent. The results of many such investigations are clear and unambiguous: While Q and W depend on the path, $\Delta U = Q - W$ is independent of path. The change in internal energy of a system during any thermodynamic process depends only on the initial and final states, not on the path leading from one to the other.

Experiment, then, is the ultimate justification for believing that a thermodynamic system in a specific state has a unique internal energy that depends only on that state. An equivalent statement is that the internal energy U of a system is a function of the state coordinates p, V, and T (actually, any two of these, since the three variables are related by the equation of state).

To say that the first law of thermodynamics, given by Eq. (254) or (255), represents conservation of energy for thermodynamic processes is correct, as far as it goes. But an important *additional* aspect of the first law is the fact that internal energy depends only on the state of a system (see fig. 160). In changes of state, the change in internal energy is independent of the path.



Figure 160 – The internal energy of a cup of coffee depends on just its thermodynamic state – how much water and ground coffee it contains, and what its temperature is. It does not depend on the history of how the coffee was prepared – that is, the thermodynamic path that led to its current state

All this may seem a little abstract if you are satisfied to think of internal energy as microscopic mechanical energy. There's nothing wrong with that view, and we will make use of it at various times during our discussion. But in the interest of precise *operational* definitions, internal energy, like heat, can and must be defined in a way that is independent of the detailed microscopic structure of the material.

Two special cases of the first law of thermodynamics are worth mentioning. A process that eventually returns a system to its initial state is called a *cyclic process*. For such a process, the final state is the same as the initial state, and so the *total* internal energy change must be zero. Then

$$U_2 = U_1 \tag{256}$$

and

$$Q = W \tag{257}$$

If a net quantity of work W is done by the system during this process, an equal amount of energy must have flowed into the system as heat Q. But there is no reason either Q or W individually has to be zero (see fig. 161).



Figure 161 – Every day, your body (a thermodynamic system) goes through a cyclic thermodynamic process like this one. Heat Q is added by metabolizing food, and your body does work W in breathing, walking, and other activities. If you return to the same state at the end of the day, W = Q and the net change in your internal energy is zero

Another special case occurs in an *isolated system*, one that does no work on its surroundings and has no heat flow to or from its surroundings. For any process taking place in an isolated system,

$$W = Q = 0 \tag{258}$$

and therefore

$$U_2 = U_1 = \Delta U = 0$$
 (259)

In other words, the internal energy of an isolated system is constant.

We now show that for an ideal gas, the internal energy U depends only on temperature, not on pressure or volume. Let's think again about the free-expansion

experiment. A thermally insulated container with rigid walls is divided into two compartments by a partition (see fig. 162). One compartment has a quantity of an ideal gas and the other is evacuated.



Figure 162 – The partition is broken (or removed) to start the free expansion of gas into the vacuum region

When the partition is removed or broken, the gas expands to fill both parts of the container. The gas does no work on its surroundings because the walls of the container don't move, and there is no heat flow through the insulation. So both Q and W are zero and the internal energy U is constant. This is true of any substance, whether it is an ideal gas or not.

Does the *temperature* change during a free expansion? Suppose it *does* change, while the internal energy stays the same. In that case we have to conclude that the internal energy depends on both the temperature and the volume or on both the temperature and the pressure, but certainly not on the temperature alone. But if T is constant during a free expansion, for which we know that U is constant even though both p and V change, then we have to conclude that U depends only on T, not on p or V.

Many experiments have shown that when a low-density gas undergoes a free expansion, its temperature *does not* change. Such a gas is essentially an ideal gas. The conclusion is:

The internal energy of an ideal gas depends only on its temperature, not on its pressure or volume.

This property, in addition to the ideal-gas equation of state, is part of the ideal gas model. Make sure you understand that U depends only on T for an ideal gas, for we will make frequent use of this fact.

For nonideal gases, some temperature change occurs during free expansions, even though the internal energy is constant. This shows that the internal energy cannot depend *only* on temperature; it must depend on pressure as well. From the microscopic viewpoint, in which internal energy U is the sum of the kinetic and potential energies for all the particles that make up the system, this is not

surprising. Nonideal gases usually have attractive intermolecular forces, and when molecules move farther apart, the associated potential energies increase. If the total internal energy is constant, the kinetic energies must decrease. Temperature is directly related to molecular *kinetic* energy, and for such a gas a free expansion is usually accompanied by a *drop* in temperature.

2.3.4. Kinds of thermodynamic processes

In this section we describe four specific kinds of thermodynamic processes that occur often in practical situations. These can be summarized briefly as "no heat transfer" or *adiabatic*, "constant volume" or *isochoric*, "constant pressure" or *isobaric*, and "constant temperature" or *isothermal*. For some of these processes we can use a simplified form of the first law of thermodynamics.

An **adiabatic process** (pronounced "ay-dee-ah-*bat*-ic") is defined as one with no heat transfer into or out of a system; Q = 0. We can prevent heat flow eit er by surrounding the system with thermally insulating material or by carrying out the process so quickly that there is not enough time for appreciable heat flow. From the first law we find that for every adiabatic process,

$$U_2 - U_1 = \Delta U = -W \tag{260}$$

When a system expands adiabatically, W is positive (the system does work on its surroundings), so ΔU is negative and the internal energy decreases. When a system is *compressed* adiabatically, W is negative (work is done on the system by its surroundings) and U increases. In many (but not all) systems an increase of internal energy is accompanied by a rise in temperature, and a decrease in internal energy by a drop in temperature (see fig. 163).

The compression stroke in an internal-combustion engine is an approximately adiabatic process. The temperature rises as the air-fuel mixture in the cylinder is compressed. The expansion of the burned fuel during the power stroke is also an approximately adiabatic expansion with a drop in temperature.

An **isochoric process** (pronounced "eye-so-*kor*-ic") is a *constant-volume* process. When the volume of a thermodynamic system is constant, it does no work on its surroundings. Then W = 0 and

$$U_2 - U_1 = \Delta Q = 0 \tag{261}$$

In an isochoric process, all the energy added as heat remains in the system as an increase in internal energy. Heating a gas in a closed constant-volume container is an example of an isochoric process. The processes *ab* and *cd* are also examples of isochoric processes. (Note that there are types of work that do not involve a volume change. For example, we can do work on a fluid by stirring it. In some literature, "isochoric" is used to mean that no work of any kind is done.)



Figure 163 – When the cork is popped on a bottle of champagne, the pressurized gases inside the bottle expand rapidly and do work on the outside air (W > 0).

There is no time for the gases to exchange heat with their surroundings, so the expansion is adiabatic (Q = 0). Hence the internal energy of the expanding gases decrease ($\Delta U = -W < 0$) and their temperature drops. This makes water vapour condense and from a miniature cloud

An **isobaric process** (pronounced "eye-so-*bear*-ic") is a *constant-pressure* process. In general, none of the three quantities ΔU , Q, and W is zero in an isobaric process, but calculating W is easy nonetheless. From Eq. (253),

$$W = p(V_2 - V_1)$$
(262)

An **isothermal process** is a *constant-temperature* process. For a process to be isothermal, any heat flow into or out of the system must occur slowly enough that thermal equilibrium is maintained. In general, none of the quantities ΔU , Q, or W is zero in an isothermal process.

In some special cases the internal energy of a system depends *only* on its temperature, not on its pressure or volume. The most familiar system having this special property is an ideal gas, as we'll discuss in the next section. For such systems, if the temperature is constant, the internal energy is also constant; $\Delta U = 0$ and Q = W. That is, any energy entering the system as heat Q must leave it again as work W done by the system. Involving an ideal gas, is an example of an isothermal process in which U is also constant. For most systems other than ideal gases, the internal energy depends on pressure as well as temperature, so U may vary even when T is constant.

Figure 164 shows a pV-diagram for these four processes for a constant amount of an ideal gas. The path followed in an adiabatic process (a to 1) is
called an **adiabat.** A vertical line (constant volume) is an **isochor**, a horizontal line (constant pressure) is an **isobar**, and a curve of constant temperature (shown as light blue lines in Fig. 164) is an **isotherm**.



Figure 164 – Four different processes for a constant amount of an ideal gas, all starting in state a. For the adiabatic process, Q = 0; for the isochoric process, W = 0; and for the isothermal process, $\Delta U = 0$. The temperature increases only during the isobaric expansion

2.4 The second law of thermodynamics

2.4.1 Heat engines

The essence of our technological society is the ability to use sources of energy other than muscle power. Sometimes, mechanical energy is directly available; water power and wind power are examples. But most of our energy comes from the burning of fossil fuels (coal, oil, and gas) and from nuclear reactions. They supply energy that is transferred as *heat*. This is directly useful for heating buildings, for cooking, and for chemical processing, but to operate a machine or propel a vehicle, we need *mechanical* energy.

Thus it's important to know how to take heat from a source and convert as much of it as possible into mechanical energy or work. This is what happens in gasoline engines in automobiles, jet engines in airplanes, steam turbines in electric power plants, and many other systems. Closely related processes occur in the animal kingdom; food energy is "burned" (that is, carbohydrates combine with oxygen to yield water, carbon dioxide, and energy) and partly converted to mechanical energy as an animal's muscles do work on its surroundings.

Any device that transforms heat partly into work or mechanical energy is called a **heat engine** (see fig. 165). Usually, a quantity of matter inside the engine undergoes inflow and outflow of heat, expansion and compression, and sometimes change of phase. We call this matter the **working substance** of the engine. In internal-combustion engines, such as those used in automobiles, the working substance is a mixture of air and fuel; in a steam turbine it is water.



Figure 165 – All motorized vehicles other than purely electric vehicles use heat engines for propulsion. (Hybrid vehicles use their internal-combustion engine to help change the batteries for the electric motor)

The simplest kind of engine to analyze is one in which the working substance undergoes a **cyclic process**, a sequence of processes that eventually leaves the substance in the same state in which it started. In a steam turbine the water is recycled and used over and over. Internal-combustion engines do not use the same air over and over, but we can still analyze them in terms of cyclic processes that approximate their actual operation.

All heat engines *absorb* heat from a source at a relatively high temperature, perform some mechanical work, and *discard* or *reject* some heat at a lower temperature. As far as the engine is concerned, the discarded heat is wasted. In internal-combustion engines the waste heat is that discarded in the hot exhaust gases and the cooling system; in a steam turbine it is the heat that must flow out of the used steam to condense and recycle the water.

When a system is carried through a cyclic process, its initial and final internal energies are equal. For any cyclic process, the first law of thermodynamics requires that

$$U_2 - U_1 = 0 = Q - W \tag{263}$$

so

$$Q = W \tag{264}$$

That is, the net heat flowing into the engine in a cyclic process equals the net work done by the engine.

When we analyze heat engines, it helps to think of two bodies with which the working substance of the engine can interact. One of these, called the *hot reservoir*, represents the heat source; it can give the working substance large amounts of heat at a constant temperature T_H without appreciably changing its own temperature. The other body, called the cold reservoir, can absorb large amounts of discarded heat from the engine at a constant lower temperature T_C . In a steamturbine system the flames and hot gases in the boiler are the hot reservoir, and the cold water and air used to condense and cool the used steam are the cold reservoir.

We denote the quantities of heat transferred from the hot and cold reservoirs as Q_H and Q_C respectively. A quantity of heat Q is positive when heat is transferred into the working substance and is negative when heat leaves the working substance. Thus in a heat engine, Q_H is positive but Q_C is negative, representing heat leaving the working substance. This sign convention is consistent with the rules; we will continue to use those rules here. For clarity, we'll often state the relationships in terms of the absolute values of the Q's and W's because absolute values are always positive.

We can represent the energy transformations in a heat engine by the energyflow diagram of Fig. 166. The engine itself is represented by the circle. The amount of heat Q_H supplied to the engine by the hot reservoir is proportional to the width of the incoming "pipeline" at the top of the diagram. The width of the outgoing pipeline at the bottom is proportional to the magnitude $|Q_C|$ of the heat rejected in the exhaust. The branch line to the right represents the portion of the heat supplied that the engine converts to mechanical work, W.



Figure 166 – Schematic energy-flow diagram for a heat engine

When an engine repeats the same cycle over and over, Q_H and Q_C represent the quantities of heat absorbed and rejected by the engine during one cycle; Q_H is positive, and Q_C is negative. The net heat Q absorbed per cycle is

$$Q = Q_H + Q_C = |Q_H| - |Q_C|$$
(265)

The useful output of the engine is the net work W done by the working substance. From the first law,

$$W = Q = Q_H + Q_C = |Q_H| - |Q_C|$$
(266)

Ideally, we would like to convert all the heat Q_H into work; in that case we would have $Q_H = W$ and $Q_C = 0$. Experience shows that this is impossible; there is always some heat wasted, and Q_C is never zero. We define the thermal efficiency of an engine, denoted by e, as the quotient

$$e = \frac{W}{Q_H} \tag{267}$$

The thermal efficiency e represents the fraction of Q_H that is converted to work. To put it another way, e is what you get divided by what you pay for. This is always less than unity, an all-too-familiar experience! In terms of the flow diagram of Fig. 166, the most efficient engine is one for which the branch pipeline representing the work output is as wide as possible and the exhaust pipeline representing the heat thrown away is as narrow as possible.

When we substitute the two expressions for W given by Eq. (266) into Eq. (267), we get the following equivalent expressions for e:

$$e = \frac{W}{Q_H} = 1 + \frac{Q_C}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$$
(268)

Note that e is a quotient of two energy quantities and thus is a pure number, without units. Of course, we must always express W, Q_H and Q_C in the same units.

2.4.2 The Second law of thermodynamics

Experimental evidence suggests strongly that it is *impossible* to build a heat engine that converts heat completely to work—that is, an engine with 100% thermal efficiency. This impossibility is the basis of one statement of the **second law of thermodynamics**, as follows:

It is impossible for any system to undergo a process in which it absorbs heat from a reservoir at a single temperature and converts the heat completely into mechanical work, with the system ending in the same state in which it began. We will call this the "engine" statement of the second law. (It is also known to

physicists as the *Kelvin–Planck statement* of this law.)

The basis of the second law of thermodynamics is the difference between the nature of internal energy and that of macroscopic mechanical energy. In a moving body the molecules have random motion, but superimposed on this is a coordinated motion of every molecule in the direction of the body's velocity. The kinetic energy associated with this *coordinated* macroscopic motion is what we call the kinetic energy of the moving body. The kinetic and potential energies associated with the *random* motion constitute the internal energy.

When a body sliding on a surface comes to rest as a result of friction, the organized motion of the body is converted to random motion of molecules in the body and in the surface. Since we cannot control the motions of individual molecules, we cannot convert this random motion completely back to organized motion. We can convert *part* of it, and this is what a heat engine does.

If the second law were *not* true, we could power an automobile or run a power plant by cooling the surrounding air. Neither of these impossibilities violates the *first* law of thermodynamics. The second law, therefore, is not a deduction from the first but stands by itself as a separate law of nature. The first law denies the possibility of creating or destroying energy; the second law limits the *availability* of energy and the ways in which it can be used and converted.

Heat flows spontaneously from hotter to colder bodies, never the reverse. A refrigerator does take heat from a colder to a hotter body, but its operation requires an input of mechanical energy or work. Generalizing this observation, we state:

It is impossible for any process to have as its sole result the transfer of heat from a cooler to a hotter body.

We'll call this the "refrigerator" statement of the second law. (It is also known as the *Clausius statement*.) It may not seem to be very closely related to the "engine" statement. In fact, though, the two statements are completely equivalent. For example, if we could build a workless refrigerator, violating the second or "refrigerator" statement of the second law, we could use it in conjunction with a heat engine, pumping the heat rejected by the engine back to the hot reservoir to be reused. This composite machine (see fig. 167a) would violate the "engine" statement of the second law because its net effect would be to take a net quantity of heat $Q_H - |Q_C|$ from the hot reservoir and convert it completely to work W.

Alternatively, if we could make an engine with 100% thermal efficiency, in violation of the first statement, we could run it using heat from the hot reservoir and use the work output to drive a refrigerator that pumps heat from the cold reservoir to the hot (see fig. 167b). This composite device would violate the "refrigerator" statement because its net effect would be to take heat Q_C from the cold reservoir and deliver it to the hot reservoir without requiring any input of work. Thus any device that violates one form of the second law can be used tomake a device that violates the other form. If violations of the first form are impossible, so are violations of the second!

The conversion of work to heat and the heat flow from hot to cold across a finite temperature gradient are *irreversible* processes. The "engine" and "refrigerator" statements of the second law state that these processes can be only partially reversed. We could cite other examples. Gases naturally flow from a region of high pressure to a region of low pressure; gases and miscible liquids left by themselves always tend to mix, not to unmix. The second law of thermodynamics is an expression of the inherent one-way aspect of these and many other irreversible processes. Energy conversion is an essential aspect of all plant and animal life and of human technology, so the second law of thermodynamics is of fundamental importance.

The second law of thermodynamics, as we have stated it, is not an equation or a quantitative relationship but rather a statement of impossibility. However, the second law can be stated as a quantitative relationship with the concept of entropy, the subject of this section. (a) The "engine" statement of the second law of thermodynamics



If a workless refrigerator were possible, it could be used in conjunction with an ordinary heat engine to form a 100%-efficient engine, converting heat $Q_{\rm H} = |Q_{\rm C}|$ completely to work.



(b) The "refrigerator" statement of the second law of thermodynamics

Figure 167 – Energy-flow diagrams showing that the two forms of the second law are equivalent

refrigerator to form a workless refrigerator, transferring heat O_{Γ} from the cold to the hot

reservoir with no input of work.

We have talked about several processes that proceed naturally in the direction of increasing disorder. Irreversible heat flow increases disorder because the molecules are initially sorted into hotter and cooler regions; this sorting is lost when the system comes to thermal equilibrium. Adding heat to a body increases its disorder because it increases average molecular speeds and therefore the randomness of molecular motion. Free expansion of a gas increases its disorder because the molecules have greater randomness of position after the expansion than before. Figure 168 shows another process in which disorder increases.



Figure 168 – When firecrackers explode, disorder increases. The neatly packaged chemicals within each firecracker are dispersed in all directions and the stored chemical energy is converted to random kinetic energy of the fragments

Entropy provides a quantitative measure of disorder. To introduce this concept, let's consider an infinitesimal isothermal expansion of an ideal gas. We add heat dQ and let the gas expand just enough to keep the temperature constant. Because the internal energy of an ideal gas depends only on its temperature, the internal energy is also constant; thus from the first law, the work dW done by the gas is equal to the heat dQ added. That is,

$$dQ = dW = pdV = \frac{nRT}{V}dV$$
(269)

$$\frac{dV}{V} = \frac{dQ}{nRT}$$
(270)

The gas is more disordered after the expansion than before: The molecules are moving in a larger volume and have more randomness of position. Thus the fractional volume change dV/V is a measure of the increase in disorder, and the above equation shows that it is proportional to the quantity dQ/T We introduce the symbol S for the entropy of the system, and we define the infinitesimal entropy change dS during an infinitesimal reversible process at absolute temperature T as

$$dS = \frac{dQ}{T} \tag{271}$$

If a total amount of heat Q is added during a reversible isothermal process at absolute temperature T, the total entropy change $\Delta S = S_2 - S_1$ is given by

$$\Delta S = S_2 - S_1 = \frac{Q}{T} \tag{272}$$

Entropy has units of energy divided by temperature; the SI unit of entropy is 1 J/K.

We can see how the quotient Q/T is related to the increase in disorder. Higher temperature means greater randomness of motion. If the substance is initially cold, with little molecular motion, adding heat Q causes a substantial fractional increase in molecular motion and randomness. But if the substance is already hot, the same quantity of heat adds relatively little to the greater molecular motion already present. So Q/T is an appropriate characterization of the increase in randomness or disorder when heat flows into a system.

We can generalize the definition of entropy change to include *any* reversible process leading from one state to another, whether it is isothermal or not. We represent the process as a series of infinitesimal reversible steps. During a typical step, an infinitesimal quantity of heat dQ is added to the system at absolute temperature *T*. Then we sum (integrate) the quotients dQ/T for the entire process; that is,

$$\Delta S = \int_{1}^{2} \frac{dQ}{T}$$
(273)

The limits 1 and 2 refer to the initial and final states.

Because entropy is a measure of the disorder of a system in any specific state, it must depend only on the current state of the system, not on its past history. (We will verify this later.) When a system proceeds from an initial state with entropy S_1 to a final state with entropy S_2 , the change in entropy $\Delta S = S_2 - S_1$ defined by Eq. (273) does not depend on the path leading from the initial to the final state but is the same for *all possible* processes leading from state 1 to state 2. Thus the entropy of a system must also have a definite value for any given state of the system. *Internal energy* also has this property, although entropy and internal energy are very different quantities.

Since entropy is a function only of the state of a system, we can also compute entropy changes in *irreversible* (nonequilibrium) processes for which Eqs. (271) and (273) are not applicable. We simply invent a path connecting the given initial and final states that *does* consist entirely of reversible equilibrium processes and compute the total entropy change for that path. It is not the actual path, but the entropy change must be the same as for the actual path.

As with internal energy, the above discussion does not tell us how to calculate entropy itself, but only the change in entropy in any given process. Just as with internal energy, we may arbitrarily assign a value to the entropy of a system in a specified reference state and then calculate the entropy of any other state with reference to this.

Total entropy change for a cycle of a particular Carnot engine, which uses an ideal gas as its working substance, is zero. This result follows directly from Eq. (283), which we can rewrite as

$$\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = 0 \tag{274}$$

The quotient $\frac{Q_H}{T_H}$ equals ΔS_H , the entropy change of the engine that occurs at $T = T_H$. Likewise, $\frac{Q_C}{T_C}$ equals ΔS_C the (negative) entropy change of the engine that occurs at $T = T_C$. Hence Eq. (274) says that that $\Delta S_H + \Delta S_C = 0$; is, there is zero net entropy change in one cycle.

What about Carnot engines that use a different working substance? According to the second law, any Carnot engine operating between given temperatures T_H and T_C has the same efficiency $e = 1 - T_C/T_H$ [Eq. (284)]. Combining this expression for e with Eq. (268), $e = 1 - Q_C/Q_H$ just reproduces Eq. (274). So Eq. (274) is valid for any Carnot engine working between these temperatures, whether its working substance is an ideal gas or not. We conclude that *the total entropy change in one cycle of any Carnot engine is zero*.

This result can be generalized to show that the total entropy change during *any* reversible cyclic process is zero. A reversible cyclic process appears on a pVdiagram as a closed path (see fig. 20.19a). We can approximate such a path as closely as we like by a sequence of isothermal and adiabatic processes forming parts of many long, thin Carnot cycles (see fig. 20.19b). The total entropy change for the full cycle is the sum of the entropy changes for each small Carnot cycle, each of which is zero. So **the total entropy change during** *any* **reversible cycle is zero:**

$$\int \frac{dQ}{T} = 0 \tag{275}$$

It follows that when a system undergoes a reversible process leading from any state *a* to any other state *b*, the entropy change of the system is independent of the path (see fig. 20.19c). If the entropy change for path 1 were different from the change for path 2, the system could be taken along path 1 and then backward along path 2 to the starting point, with a nonzero net change in entropy. This would violate the conclusion that the total entropy change in such a cyclic process must be zero. Because the entropy change in such processes is independent of path, we conclude that in any given state, the system has a definite value of entropy that depends only on the state, not on the processes that led to that state.



Figure 169 – (a) A reversible cyclic process for an ideal gas a red closed path on a pV-diagram. Several ideal-gas isotherms are shown in blue. (b) We can approximate the path in (a) by a series of long, thin Carnot cycles; one of these in highlighted in gold. The total entropy change is zero for each Carnot cycle and for the actual cyclic process. (c) The entropy change between points a and b is independent of the path

In an idealized, reversible process involving only equilibrium states, the total entropy change of the system and its surroundings is zero. But all *irreversible* processes involve an increase in entropy. Unlike energy, *entropy is not a conserved quantity*. The entropy of an isolated system *can* change, but as we shall see, it can never decrease. The free expansion of a gas is an irreversible process in an isolated system in which there is an entropy increase.

2.4.3 The Carnot cycle

According to the second law, no heat engine can have 100% efficiency. How great an efficiency *can* an engine have, given two heat reservoirs at temperatures T_H and T_C ? This question was answered in 1824 by the French engineer Sadi Carnot (1796–1832), who developed a hypothetical, idealized heat engine that has the maximum possible efficiency consistent with the second law. The cycle of this engine is called the **Carnot cycle**.

To understand the rationale of the Carnot cycle, we return to *reversibility* and its relationship to directions of thermodynamic processes. Conversion of work to heat is an irreversible process; the purpose of a heat engine is a *partial* reversal of this process, the conversion of heat to work with as great an efficiency as possible. For maximum heat-engine efficiency, therefore, *we must avoid all irreversible processes* (see fig. 20.12).



Figure 170 – The temperature of the firebox of a steam engine is much higher than the temperature of water in the boiler, so heat flows irreversibly from firebox to water. Carnot's quest to understand the efficiency of steam engines led him to the idea that an ideal engine would involve only reversible processes

Heat flow through a finite temperature drop is an irreversible process. Therefore, during heat transfer in the Carnot cycle there must be *no* finite temperature difference. When the engine takes heat from the hot reservoir at temperature T_H , the working substance of the engine must also be at T_H otherwise, irreversible heat flow would occur. Similarly, when the engine discards heat to the

cold reservoir at T_C the engine itself must be at T_C . That is, every process that involves heat transfer must be *isothermal* at either T_H or T_C .

Conversely, in any process in which the temperature of the working substance of the engine is intermediate between T_H and T_C there must be *no* heat transfer between the engine and either reservoir because such heat transfer could not be reversible. Therefore any process in which the temperature *T* of the working substance changes must be *adiabatic*.

The bottom line is that every process in our idealized cycle must be either isothermal or adiabatic. In addition, thermal and mechanical equilibrium must be maintained at all times so that each process is completely reversible.

The Carnot cycle consists of two reversible isothermal and two reversible adiabatic processes. Figure 171 shows a Carnot cycle using as its working substance an ideal gas in a cylinder with a piston. It consists of the following steps:

1. The gas expands isothermally at temperature T_H , absorbing heat $Q_H(ab)$

2. It expands adiabatically until its temperature drops to $T_{C}(bc)$

3. It is compressed isothermally at T_c , rejecting heat $|Q_c|$ (cd)

4. It is compressed adiabatically back to its initial state at temperature T_H (*da*).

We can calculate the thermal efficiency e of a Carnot engine in the special case shown in Fig. 20.13 in which the working substance is an *ideal gas*. To carry out this calculation, we will first find the ratio Q_C/Q_H of the quantities of heat transferred in the two isothermal processes and then use Eq. (268) to find e.



Figure 171 – The Carnot cycle for an ideal gas. The light blue lines in the pVdiagram are isotherms (curves of constant temperature) and the dark blue lines are adiabats (curves of zero heat flow)

For an ideal gas the internal energy U depends only on temperature and is thus constant in any isothermal process. For the isothermal expansion ab, $\Delta U_{ab} =$ 0 and Q_H is equal to the work W_{ab} done by the gas during its isothermal expansion at temperature T_H . We calculated this work; using that result, we have

$$Q_H = W_{ab} = \nu R T_H \ln \frac{V_b}{V_a} \tag{276}$$

Similarly,

$$Q_C = W_{cd} = \nu R T_C \ln \frac{V_d}{V_c} = -\nu R T_C \ln \frac{V_c}{V_d}$$
(277)

Because V_d is less than V_c , Q_c is negative ($Q_c = -|Q_c|$); heat flows out of the gas during the isothermal compression at temperature T_c .

The ratio of the two quantities of heat is thus

$$\frac{Q_C}{Q_H} = -\left(\frac{T_C}{T_H}\right) \frac{\ln \frac{V_C}{V_d}}{\ln \frac{V_b}{V_a}}$$
(278)

This can be simplified further by use of the temperature–volume relationship for an adiabatic process, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$. We find for the two adiabatic processes:

$$T_H V_b^{\gamma - 1} = T_C V_c^{\gamma - 1} \tag{279}$$

and

$$T_H V_a^{\gamma - 1} = T_C V_d^{\gamma - 1} \tag{280}$$

Dividing the first of these by the second, we find

$$\frac{V_b^{\gamma-1}}{V_a^{\gamma-1}} = \frac{V_c^{\gamma-1}}{V_d^{\gamma-1}}$$
(281)

and

$$\frac{V_b}{V_a} = \frac{V_c}{V_d} \tag{282}$$

Thus the two logarithms in Eq. (278) are equal, and that equation reduces to

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$$

or

$$\frac{|Q_C|}{|Q_H|} = -\frac{T_C}{T_H} \tag{283}$$

The ratio of the heat rejected at T_C to the heat absorbed at T_H is just equal to the ratio T_C/T_H . Then from Eq. (268) the efficiency of the Carnot engine is

$$e_{Carnot} = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H}$$
(284)

This simple result says that the efficiency of a Carnot engine depends only on the temperatures of the two heat reservoirs. The efficiency is large when the temperature difference is large, and it is very small when the temperatures are nearly equal. The efficiency can never be exactly unity unless $T_c = 0$; we'll see later that this, too, is impossible.

We can prove that no engine can be more efficient than a Carnot engine operating between the same two temperatures. The key to the proof is the above observation that since each step in the Carnot cycle is reversible, the *entire cycle* may be reversed. Run backward, the engine becomes a refrigerator. Suppose we have an engine that is more efficient than a Carnot engine (see fig. 20.15). Let the Carnot engine, run backward as a refrigerator by negative work -|W|, take in heat Q_{C} from the cold reservoir and expel heat $|Q_{H}|$ to the hot reservoir. The superefficient engine expels heat $|Q_c|$, but to do this, it takes in a greater amount of heat $Q_H + \Delta$. Its work output is then $W + \Delta$ and the net effect of the two machines together is to take a quantity of heat and convert it completely into work. This violates the engine statement of the second law. We could construct a similar argument that a superefficient engine could be used to violate the refrigerator statement of the second law. Note that we don't have to assume that the superefficient engine is reversible. In a similar way we can show that no refrigerator can have a greater coefficient of performance than a Carnot refrigerator operating between the same two temperatures.

If a superefficient engine were possible, it could be used in conjunction with a Carnot refrigerator to convert the heat Δ completely to work, with no net transfer to the cold reservoir.



Figure 172 – Proving that the Carnot engine has the highest possible efficiency. A "superefficient" engine (more efficient than a Carnot engine) combined with a Carnot refrigerator could convert heat completely into work with no net heat transfer to the cold reservoir. This would violate the second law of thermodynamics

Thus the statement that no engine can be more efficient than a Carnot engine is yet another equivalent statement of the second law of thermodynamics. It also follows directly that **all Carnot engines operating between the same two temperatures have the same efficiency, irrespective of the nature of the working substance.** Although we derived Eq. (284) for a Carnot engine using an ideal gas as its working substance, it is in fact valid for *any* Carnot engine, no matter what its working substance.

Equation (284), the expression for the efficiency of a Carnot engine, sets an upper limit to the efficiency of a real engine such as a steam turbine. To maximize this upper limit and the actual efficiency of the real engine, the designer must make the intake temperature T_H as high as possible and the exhaust temperature T_C as low as possible (see fig. 20.16).



Figure 173 – To maximize efficiency, the temperatures inside a jet engine are made as high as possible. Exotic ceramic materials are used that can withstand temperatures in excess of 1000°C without melting or becoming soft

The exhaust temperature cannot be lower than the lowest temperature available for cooling the exhaust. For a steam turbine at an electric power plant, T_C may be the temperature of river or lake water; then we want the boiler temperature T_H to be as high as possible. The vapor pressures of all liquids increase rapidly with temperature, so we are limited by the mechanical strength of the boiler. At 500°C the vapor pressure of water is about 235 atm; this is about the maximum practical pressure in large present-day steam boilers.

TOPIC 3 ELECTRICITY

3.1 Electric charge and electric field

3.1.1 Electric charge

Electromagnetic interactions involve particles that have a property called *electric charge*, an attribute that is as fundamental as mass. Just as objects with mass are accelerated by gravitational forces, so electrically charged objects are accelerated by electric forces. The shock you feel when you scuff your shoes across a carpet and then reach for a metal doorknob is due to charged particles leaping between your finger and the doorknob. Electric currents are simply streams of charged particles flowing within wires in response to electric forces. Even the forces that hold atoms together to form solid matter, and that keep the atoms of solid objects from passing through each other, are fundamentally due to electric interactions between the charged particles within atoms.

We begin our study of electromagnetism in this chapter by examining the nature of electric charge. We'll find that charge is quantized and obeys a conservation principle. When charges are at rest in our frame of reference, they exert *electrostatic* forces on each other. These forces are of tremendous importance in chemistry and biology and have many technological applications. Electrostatic forces are governed by a simple relationship known as *Coulomb's law* and are most conveniently described by using the concept of *electric field*. In later chapters we'll expand our discussion to include electric charges in motion. This will lead us to an understanding of magnetism and, remarkably, of the nature of light.

While the key ideas of electromagnetism are conceptually simple, applying them to practical problems will make use of many of your mathematical skills, especially your knowledge of geometry and integral calculus. For this reason you may find this chapter and those that follow to be more mathematically demanding than earlier chapters. The reward for your extra effort will be a deeper understanding of principles that are at the heart of modern physics and technology.

The ancient Greeks discovered as early as 600 B.C. that after they rubbed amber with wool, the amber could attract other objects. Today we say that the amber has acquired a net **electric charge**, or has become *charged*. The word "electric" is derived from the Greek word *elektron*, meaning amber. When you scuff your shoes across a nylon carpet, you become electrically charged, and you can charge a comb by passing it through dry hair.

Plastic rods and fur (real or fake) are particularly good for demonstrating **electrostatics**, the interactions between electric charges that are at rest (or nearly so). After we charge both plastic rods in Fig. 174a by rubbing them with the piece of fur, we find that the rods repel each other.

When we rub glass rods with silk, the glass rods also become charged and repel each other (see fig. 174b). But a charged plastic rod *attracts* a charged glass

rod; furthermore, the plastic rod and the fur attract each other, and the glass rod and the silk attract each other (see fig. 174c).



Figure 174 – Experiments in electrostatics. (a) Negatively charged objects repel each other. (b) Positively charged objects repel each other. (c) Positively charged objects and negatively charged objects attract each other

These experiments and many others like them have shown that there are exactly two kinds of electric charge: the kind on the plastic rod rubbed with fur and the kind on the glass rod rubbed with silk. Benjamin Franklin (1706–1790) suggested calling these two kinds of charge *negative* and *positive*, respectively, and these names are still used. The plastic rod and the silk have negative charge; the glass rod and the fur have positive charge.

Two positive charges or two negative charges repel each other. A positive charge and a negative charge attract each other.

One application of forces between charged bodies is in a laser printer (see fig. 175). The printer's light-sensitive imaging drum is given a positive charge. As the drum rotates, a laser beam shines on selected areas of the drum, leaving those areas with a *negative* charge. Positively charged particles of toner adhere only to the areas of the drum "written" by the laser. When a piece of paper is placed in contact with the drum, the toner particles stick to the paper and form an image.



Figure 175 – Schematic diagram of the operation of a laser printer

When you charge a rod by rubbing it with fur or silk as in fig. 174, there is no visible change in the appearance of the rod. What, then, actually happens to the rod when you charge it? To answer this question, we must look more closely at the structure of atoms, the building blocks of ordinary matter.

The structure of atoms can be described in terms of three particles: the negatively charged **electron**, the positively charged **proton**, and the uncharged **neutron** (see fig. 176). The proton and neutron are combinations of other entities called *quarks*, which have charges of and times the electron charge. Isolated quarks have not been observed, and there are theoretical reasons to believe that it is impossible in principle to observe a quark in isolation.



Figure 176 – The structure of an atom. The particular atom depicted here is lithium

The protons and neutrons in an atom make up a small, very dense core called the **nucleus**, with dimensions of the order of Surrounding the nucleus are the electrons, extending out to distances of the order of from the nucleus. If an atom were a few kilometers across, its nucleus would be the size of a tennis ball. The negatively charged electrons are held within the atom by the attractive electric forces exerted on them by the positively charged nucleus. (The protons and neutrons are held within stable atomic nuclei by an attractive interaction, called the *strong nuclear force*, that overcomes the electric repulsion of the protons. The strong nuclear force has a short range, and its effects do not extend far beyond the nucleus.)

The masses of the individual particles, to the precision that they are presently known, are

Mass of electron = me = 9.109382151452 * 10-31 kgMass of proton = mp = 1.6726216371832 * 10-27 kgMass of neutron = mn = 1.6749272111842 * 10-27 kg

The numbers in parentheses are the uncertainties in the last two digits. Note that the masses of the proton and neutron are nearly equal and are roughly 2000 times the mass of the electron. Over 99.9% of the mass of any atom is concentrated in its nucleus.

The negative charge of the electron has (within experimental error) *exactly* the same magnitude as the positive charge of the proton. In a neutral atom the number of electrons equals the number of protons in the nucleus, and the net electric charge (the algebraic sum of all the charges) is exactly zero (see fig. 177a). The number of protons or electrons in a neutral atom of an element is called the **atomic number** of the element. If one or more electrons are removed from an atom, what remains is called a **positive ion** (see fig. 177b). A **negative ion** is an atom that has *gained* one or more electrons (see fig. 177c). This gain or loss of electrons is called **ionization**.

When the total number of protons in a macroscopic body equals the total number of electrons, the total charge is zero and the body as a whole is electrically neutral. To give a body an excess negative charge, we may either *add negative* charges to a neutral body or *remove positive* charges from that body. Similarly, we can create an excess positive charge by either *adding positive* charge or *removing negative* charge. In most cases, negatively charged (and highly mobile) electrons are added or removed, and a "positively charged body" is one that has lost some of its normal complement of electrons. When we speak of the charge of a body, we always mean its *net* charge. The net charge is always a very small fraction (typically no more than) of the total positive charge or negative charge in the body.



Figure 177 – (a) A neutral atom has as many electrons as it does protons. (b) A positive ion has s deficit of electrons. (c) A negative ion has an excess of electrons. (The electron "shells" are a schematic representation of the actual electron distribution, a diffuse cloud many times larger than the nucleus)

Implicit in the foregoing discussion are two very important principles. First is the **principle of conservation of charge:**

The algebraic sum of all the electric charges in any closed system is constant. If we rub together a plastic rod and a piece of fur, both initially uncharged, the rod acquires a negative charge (since it takes electrons from the fur) and the fur acquires a positive charge of the *same* magnitude (since it has lost as many electrons as the rod has gained). Hence the total electric charge on the two bodies together does not change. In any charging process, charge is not created or destroyed; it is merely *transferred* from one body to another.

Conservation of charge is thought to be a *universal* conservation law. No experimental evidence for any violation of this principle has ever been observed. Even in high-energy interactions in which particles are created and destroyed, such as the creation of electron–positron pairs, the total charge of any closed system is exactly constant.

The second important principle is:

The magnitude of charge of the electron or proton is a natural unit of charge. Every observable amount of electric charge is always an integer multiple of this basic unit. We say that charge is *quantized*. A familiar example of quantization is money. When you pay cash for an item in a store, you have to do it in one-cent increments. Cash can't be divided into amounts smaller than one cent, and electric charge can't be divided into amounts smaller than the charge of one electron or proton. (The quark charges, and of the electron charge, are probably not observable as isolated charges.) Thus the charge on any macroscopic body is always either zero or an integer multiple (negative or positive) of the electron charge.

Understanding the electric nature of matter gives us insight into many aspects of the physical world. The chemical bonds that hold atoms together to form molecules are due to electric interactions between the atoms. They include the strong ionic bonds that hold sodium and chlorine atoms together to make table salt and the relatively weak bonds between the strands of DNA that record your body's genetic code. The normal force exerted on you by the chair in which you're sitting arises from electric forces between charged particles in the atoms of your seat and in the atoms of your chair. The tension force in a stretched string and the adhesive force of glue are likewise due to the electric interactions of atoms.

3.1.2 Coulomb's law

Charles Augustin de Coulomb (1736–1806) studied the interaction forces of charged particles in detail in 1784. He used a torsion balance similar to the one used 13 years later by Cavendish to study the much weaker gravitational interaction. For **point charges**, chargedbodies that are very small in comparison with the distance between them, Coulomb found that the electric force is proportional to That is, when the distance doubles, the force decreases to one-quarter of its initial value; when the distance is halved, the force increases to four times its initial value.

The electric force between two point charges also depends on the quantity of charge on each body, which we will denote by q or Q. To explore this dependence, Coulomb divided a charge into two equal parts by placing a small charged spherical conductor into contact with an identical but uncharged sphere; by symmetry, the charge is shared equally between the two spheres. (Note the essential role of the principle of conservation of charge in this procedure.) Thus he could obtain one-half, one-quarter, and so on, of any initial charge. He found that the forces that two point charges q_1 and q_2 exert on each other are proportional to each charge and therefore are proportional to the *product* q_1q_2 of the two charges.

Thus Coulomb established what we now call **Coulomb's law:**

The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

In mathematical terms, the magnitude F of the force that each of two point charges q_1 and q_2 a distance apart exerts on the other can be expressed as

$$F = k \frac{|q_1 q_2|}{r^2}$$
(285)

where k is a proportionality constant whose numerical value depends on the system of units used. The absolute value bars are used in Eq. (285) because the

charges q_1 and q_2 can be either positive or negative, while the force magnitude F is always positive.

The directions of the forces the two charges exert on each other are always along the line joining them. When the charges q_1 and q_2 have the same sign, either both positive or both negative, the forces are repulsive; when the charges have opposite signs, the forces are attractive. The two forces obey Newton's third law; they are always equal in magnitude and opposite in direction, even when the charges are not equal in magnitude.

The proportionality of the electric force to $1/r^2$ has been verified with great precision. There is no reason to suspect that the exponent is different from precisely 2. Thus the form of Eq. (285) is the same as that of the law of gravitation. But electric and gravitational interactions are two distinct classes of phenomena. Electric interactions depend on electric charges and can be either attractive or repulsive, while gravitational interactions depend on mass and are always attractive (because there is no such thing as negative mass).

The value of the proportionality constant k in Coulomb's law depends on the system of units used. In our study of electricity and magnetism we will use SI units exclusively. The SI electric units include most of the familiar units such as the volt, the ampere, the ohm, and the watt. (There is *no* British system of electric units.) The SI unit of electric charge is called one **coulomb** (1 C). In SI units the constant k in Eq. (285) is

$$k = 8.987551787 \times 10^9 \, N \cdot \frac{m^2}{C^2} \cong 8.988 \times 10^9 N \cdot \frac{m^2}{C^2}$$

The value of k is known to such a large number of significant figures because this value is closely related to the speed of light in vacuum. As we discussed this speed is *defined* to be exactly $c = 2.99792458 \times 10^8 \frac{m}{s}$. The numerical value of is defined in terms of to be precisely

$$k = \left(10^{-7} N \cdot \frac{m^2}{C^2}\right) c^2$$

You should check this expression to confirm that *k* has the right units.

In principle we can measure the electric force F between two equal charges at a measured distance r and use Coulomb's law to determine the charge. Thus we could regard the value of k as an operational definition of the coulomb. For reasons of experimental precision it is better to define the coulomb instead in terms of a unit of electric *current* (charge per unit time), the *ampere*, equal to 1 coulomb per second.

In SI units we usually write the constant in Eq. (285) as $1/4\pi\varepsilon_0$, where ε_0 ("epsilon-nought" or "epsilon-zero") is another constant. This appears to

complicate matters, but it actually simplifies many formulas that we will encounter in later chapters. From now on, we will usually write Coulomb's law as

$$F = \frac{|q_1 q_2|}{4\pi\varepsilon_0 r^2} \tag{286}$$

The constants in Eq. (286) are approximately

$$\varepsilon_0 = 8.854 \times 10^{-12} \ \frac{C^2}{N \cdot m^2}$$

and

$$\frac{1}{4\pi\varepsilon_0} = k = 8.988 \times 10^9 \, N \cdot \frac{m^2}{C^2}$$

In examples and problems we will often use the approximate value

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \, N \cdot \frac{m^2}{C^2}$$

which is within about 0.1% of the correct value.

As we mentioned early, the most fundamental unit of charge is the magnitude of the charge of an electron or a proton, which is denoted by e. The most precise value available as of the writing of this book is

$$e = 1.602176487(40) \times 10^{-19} C$$

One coulomb represents the negative of the total charge of about 6×10^{18} electrons. For comparison, a copper cube 1 cm on a side contains about 2.4×10^{24} electrons. About 10^{19} electrons pass through the glowing filament of a flashlight bulb every second.

In electrostatics problems (that is, problems that involve charges at rest), it's very unusual to encounter charges as large as 1 coulomb. Two 1-C charges separated by 1 m would exert forces on each other of magnitude $9 \times 10^9 N$ (about 1 million tons)! The total charge of all the electrons in a copper one-cent coin is even greater, about $1.4 \times 10^5 C$, which shows that we can't disturb electric neutrality very much without using enormous forces. More typical values of charge range from about 10^{-9} to about $10^{-6} C$. The microcoulomb ($1 \ \mu C = 10^{-6} C$) and the nanocoulomb ($1 \ nC = 10^{-9} C$) are often used as practical units of charge.

Coulomb's law as we have stated it describes only the interaction of two *point* charges. Experiments show that when two charges exert forces imultaneously

on a third charge, the total force acting on that charge is the *vector sum* of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces**, holds for any number of charges. By using this principle, we can apply Coulomb's law to *any* collection of charges. Two of the examples at the end of this section use the superposition principle.

Strictly speaking, Coulomb's law as we have stated it should be used only for point charges *in vacuum*. If matter is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening material. We will describe this effect later. As a practical matter, though, we can use Coulomb's law unaltered for point charges in air. At normal atmospheric pressure, the presence of air changes the electric force from its vacuum value by only about one part in 2000.

3.1.3 Electric field and electric forces

When two electrically charged particles in empty space interact, how does each one know the other is there? We can begin to answer this question, and at the same time reformulate Coulomb's law in a very useful way, by using the concept of electric field.

To introduce this concept, let's look at the mutual repulsion of two positively charged bodies A and B (see fig. 178a). Suppose B has charge q_0 and let \vec{F}_0 be the electric force A of on B. One way to think about this force is as an "action-at-adistance" force—that is, as a force that acts across empty space without needing any matter (such as a push rod or a rope) to transmit it through the intervening space. (Gravity can also be thought of as an "action-at-adistance" force.) But a more fruitful way to visualize the repulsion between A and B is as a two-stage process. We first envision that body A, as a result of the charge that it carries, somehow *modifies the properties of the space around it*. Then body B, as a result of the charge that *it* carries, senses how space has been modified at its position. The response of body is to experience the force \vec{F}_0 .

To elaborate how this two-stage process occurs, we first consider body A by itself: We remove body B and label its former position as point P (see fig. 178b). We say that the charged body A produces or causes an **electric field** at point P (and at all other points in the neighborhood). This electric field is present at P even if there is no charge at P; it is a consequence of the charge on body A only. If a point charge q_0 is then placed at point P, it experiences the force \vec{F}_0 . We take the point of view that this force is exerted on q_0 by the field at P (see fig. 178c). Thus the electric field is the intermediary through which A communicates its presence to q_0 . Because the point charge q_0 would experience a force at any point in the neighborhood of A, the electric field that A produces exists at all points in the region around A. (a) A and B exert electric forces on each other.





(c) Body A sets up an electric field \vec{E} at point P.



Figure 178 – A charged body creates an electric field in the space around it

We can likewise say that the point charge q_0 produces an electric field in the space around it and that this electric field exerts the force $-\vec{F}_0$ on body A. For each force (the force of A on q_0 and the force of q_0 on A), one charge sets up an electric field that exerts a force on the second charge. We emphasize that this is an *interaction* between *two* charged bodies. A single charge produces an electric field in the surrounding space, but this electric field cannot exert a net force on the charge that created it a body cannot exert a net force on itself. (If this wasn't true, you would be able to lift yourself to the ceiling by pulling up on your belt!)

The electric force on a charged body is exerted by the electric field created by *other* charged bodies.

To find out experimentally whether there is an electric field at a particular point, we place a small charged body, which we call a **test charge**, at the point (see fig. 178c). If the test charge experiences an electric force, then there is an electric field at that point. This field is produced by charges other than q_0 .

Force is a vector quantity, so electric field is also a vector quantity. (Note the use of vector signs as well as boldface letters and plus, minus, and equals signs in the following discussion.) We define the *electric field* \vec{E} at a point as the electric force \vec{F}_0 experienced by a test charge q_0 at the point, divided by the charge q_0 . That is, the electric field at a certain point is equal to the *electric force per unit charge* experienced by a charge at that point:

$$\vec{E} = \frac{\vec{F}_0}{q_0} \tag{287}$$

In SI units, in which the unit of force is 1 N and the unit of charge is 1 C, the unit of electric field magnitude is 1 newton per coulomb (1 N/C).

If the field \vec{E} at a certain point is known, rearranging Eq. (287) gives the force \vec{F}_0 experienced by a point charge q_0 placed at that point. This force is just equal to the electric field \vec{E} produced at that point by charges other than q_0 , multiplied by the charge q_0 :

$$\vec{F}_0 = q_0 \vec{E} \tag{288}$$

The charge q_0 can be either positive or negative. If q_0 is *positive*, the force \vec{F}_0 experienced by the charge is the same direction as \vec{E} ; if q_0 is *negative*, \vec{F}_0 and \vec{E} are in opposite directions (see fig. 179).

 $\vec{P}_{q_{1}} \neq \vec{F}_{q_{1}} \neq \vec{F}$

The force on a negative test charge q_0 points opposite to the electric field.

Figure 179 The force $\vec{F}_0 = q_0 \vec{E}$ exerted on a point charge q_0 placed in an electric

field
$$\overline{E}$$

While the electric field concept may be new to you, the basic idea - that one body sets up a field in the space around it and a second body responds to that field - is one that you've actually used before. Compare Eq. (288) to the familiar expression for the gravitational force \vec{F}_g that the earth exerts on a mass m_0 :

$$\vec{F}_g = m_0 \vec{g} \tag{289}$$

In this expression, \vec{g} is the acceleration due to gravity. If we divide both sides of Eq. (289) by the mass we obtain

$$\vec{g} = \frac{\vec{F}_g}{m_0} \tag{290}$$

Thus \vec{g} can be regarded as the gravitational force per unit mass. By analogy to Eq. (287), we can interpret \vec{g} as the gravitational field. Thus we treat the gravitational interaction between the earth and the mass m_0 as a two-stage process: The earth sets up a gravitational field in the space around it, and this gravitational field exerts a force given by Eq. (289) on the mass m_0 (which we can regard as a *test mass*). The gravitational field \vec{g} or gravitational force per unit mass, is a useful concept because it does not depend on the mass of the body on which the gravitational force is exerted; likewise, the electric field \vec{E} or electric force per unit charge, is useful because it does not depend on the charge of the body on which the electric force is exerted.

If the source distribution is a point charge q, it is easy to find the electric field that it produces. We call the location of the charge the **source point**, and we call the point *P* where we are determining the field the **field point**. It is also useful to introduce a *unit vector* \hat{r} that points along the line from source point to field point (see fig. 180a). This unit vector is equal to the displacement vector \vec{r} from the source point to the field point, divided by the distance $r = |\vec{r}|$ between these two points; that is, $\hat{r} = \vec{r}/r$. If we place a small test charge q_0 at the field point *P*, at a distance *r* from the source point, the magnitude F_0 of the force is given by Coulomb's law, Eq. (286):

$$F_0 = \frac{|qq_0|}{4\pi\varepsilon_0 r^2} \tag{291}$$

From Eq. (287) the magnitude E of the electric field at P is

$$E = \frac{|q|}{4\pi\varepsilon_0 r^2} \tag{292}$$



Figure 180 – The electric field \vec{E} produced at point *P* by an isolated point charge *q* at *S*. Note that in both (b) and (c), \vec{E} is produced by q_0 but acts on the charge at point *P*

Using the unit vector \hat{r} we can write a *vector* equation that gives both the magnitude and direction of the electric field \vec{E} :

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \tag{293}$$

By definition, the electric field of a point charge always points *away from* a positive charge (that is, in the same direction as \hat{r} ; see Fig. 180b) but *toward* a negative charge (that is, in the direction opposite \hat{r} , see Fig. 180c).

We have emphasized calculating the electric field \vec{E} at a certain point. But since \vec{E} can vary from point to point, it is not a single vector quantity but rather an *infinite* set of vector quantities, one associated with each point in space. This is an example of a **vector field.** Figure 181 shows a number of the field vectors produced by a positive or negative point charge. If we use a rectangular (x, y, z)coordinate system, each component of \vec{E} at any point is in general a function of the coordinates (x, y, z) of the point. We can represent the functions as $E_x(x, y, z)$, $E_y(x, y, z)$ and $E_z(x, y, z)$. Vector fields are an important part of the language of physics, not just in electricity and magnetism. One everyday example of a vector field is the velocity \vec{v} of wind currents; the magnitude and direction of \vec{v} and hence its vector components, vary from point to point in the atmosphere.

(a) The field produced by a positive point charge points away from the charge.



Figure 181 – A point charge q produces an electric field \vec{E} at all points in space. The field strength decreases with increasing distance

In some situations the magnitude and direction of the field (and hence its vector components) have the same values everywhere throughout a certain region; we then say that the field is *uniform* in this region. An important example of this is the electric field inside a *conductor*. If there is an electric field within a conductor, the field exerts a force on every charge in the conductor, giving the free charges a net motion. By definition an electrostatic situation is one in which the charges have *no* net motion. We conclude that *in electrostatics the electric field at every point within the material of conductor must be zero*. (Note that we are not saying that the field is necessarily zero in a *hole* inside a conductor.)

In summary, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in the field. Our analysis often has two corresponding steps: first, calculating the field caused by a source charge distribution; second, looking at the effect of the field in terms of force and motion. The second step often involves Newton's laws as well as the principles of electric interactions. In the next section we show how to calculate fields caused by various source distributions, but first here are three examples of calculating the field due to a point charge and of finding the force on a charge due to a given field \vec{E} .

Equation (293) gives the electric field caused by a single point charge. But in most realistic situations that involve electric fields and forces, we encounter charge that is *distributed* over space. The charged plastic and glass rods in Fig. 174 have electric charge distributed over their surfaces, as does the imaging drum of a laser printer (see fig. 175). In this section we'll learn to calculate electric fields caused by various distributions of electric charge. Calculations of this kind are of

tremendous importance for technological applications of electric forces. To determine the trajectories of atomic nuclei in an accelerator for cancer radiotherapy or of charged particles in a semiconductor electronic device, you have to know the detailed nature of the electric field acting on the charges.

The Superposition of Electric Fields. To find the field caused by a charge distribution, we imagine the distribution to be made up of many point charges q_1, q_2, q_3, \ldots (This is actually quite a realistic description, since we have seen that charge is carried by electrons and protons that are so small as to be almost pointlike.) At any given point each point charge produces its own electric field $\vec{E}_1, \vec{E}_2, \vec{E}_3, \ldots$, so a test charge placed at *P* experiences a force $\vec{F}_1 = q_0 \vec{E}_1$ from charge q_1 , a force $\vec{F}_2 = q_0 \vec{E}_2$ from charge q_2 and so on. From the principle of superposition of forces \vec{F}_0 the *total* force that the charge distribution exerts on is the vector sum of these individual forces:

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots = q_0 \vec{E}_1 + q_0 \vec{E}_2 + q_0 \vec{E}_3 + \dots$$
(294)

The combined effect of all the charges in the distribution is described by the *total* electric field \vec{E} at point *P*. From the definition of electric field, Eq. (287), this is

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$
⁽²⁹⁵⁾

The total electric field at P is the vector sum of the fields at P due to each point charge in the charge distribution. This is the **principle of superposition of electric fields.**

When charge is distributed along a line, over a surface, or through a volume, a few additional terms are useful. For a line charge distribution (such as a long, thin, charged plastic rod), we use λ (the Greek letter lambda) to represent the **linear charge density** (charge per unit length, measured in C/m). When charge is distributed over a surface (such as the surface of the imaging drum of a laser printer), we use σ (sigma) to represent the **surface charge density** (charge per unit area, measured in C/m²). And when charge is distributed through a volume, we use ρ (rho) to represent the **volume charge density** (charge per unit volume, C/m³).

Some of the calculations in the following examples may look fairly intricate. After you've worked through the examples one step at a time, the process will seem less formidable.

3.2 Electric potential

3.2.1 Electric potential energy

The concepts of work, potential energy, and conservation of energy proved to be extremely useful in our study of mechanics. In this section we'll show that these concepts are just as useful for understanding and analyzing electrical interactions.

Let's begin by reviewing three essential points. First, when a force \vec{F} acts on a particle that moves from point *a* to point *b*, the work $W_{a\to b}$ done by the force is given by a line integral:

$$W_{a \to b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = \int_{a}^{b} F \cos \varphi \, dl \tag{296}$$

where $d\vec{l}$ is an infinitesimal displacement along the particle's path and φ is the angle between \vec{F} and $d\vec{l}$ at each point along the path.

Second, if the force \vec{F} is conservative the work done by \vec{F} can always be expressed in terms of a **potential energy** U. When the particle moves from a point where the potential energy is U_a to a point where it is U_b , the change in potential energy $\Delta U = U_a - U_a$ is and the work $W_{a \to b}$ done by the force is

$$W_{a \to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$
 (297)

When $W_{a\to b}$ is positive, U_a is greater than U_b , ΔU is negative, and the potential energy *decreases*. That's what happens when a baseball falls from a high point (*a*) to a lower point (*b*) under the influence of the earth's gravity; the force of gravity does positive work, and the gravitational potential energy decreases (see fig. 182). When a tossed ball is moving upward, the gravitational force does negative work during the ascent, and the potential energy increases.





Figure 182 – The work done on a baseball moving in a uniform gravitational field

Third, the work-energy theorem says that the change in kinetic energy $\Delta K = K_b - K_a$ during a displacement equals the *total* work done on the particle. If only conservative forces do work, then Eq. (297) gives the total work, and $K_b - K_a = -(U_b - U_a)$. We usually write this as

$$K_a + U_a = K_b + U_b \tag{298}$$

That is, the total mechanical energy (kinetic plus potential) is conserved under these circumstances.

Let's look at an electrical example of these basic concepts. In Fig. 183 a pair of charged parallel metal plates sets up a uniform, downward electric field with magnitude E. The field exerts a downward force with magnitude $F = q_0 E$ on a positive test charge q_0 . As the charge moves downward a distance d from point a to point b, the force on the test charge is constant and independent of its location. So the work done by the electric field is the product of the force magnitude and the component of displacement in the (downward) direction of the force:

$$W_{a \to b} = Fd = q_0 Ed \tag{299}$$

This work is positive, since the force is in the same direction as the net displacement of the test charge.



Figure 183 – The work done on a point charge moving in a uniform electric field

The y-component of the electric force, $F_y = -q_0 E$ is constant, and there is no x- or z-component. This is exactly analogous to the gravitational force on a mass m near the earth's surface; for this force, there is a constant y-component $F_y = -mg$ and the x- and z-components are zero. Because of this analogy, we can conclude that the force exerted on by the uniform electric field in Fig. 183 is *conservative*, just as is the gravitational force. This means that the work $W_{a\to b}$ done by the field is independent of the path the particle takes from a to b. We can represent this work with a potential-energy function U, just as we did for gravitational potential energy. The potential energy for the gravitational force $F_y = -mg$ was U = mgy hence the potential energy for the electric force $F_y = -q_0 E$ is

$$U = q_0 E y \tag{300}$$

When the test charge moves from height y_a to height y_b , the work done on the charge by the field is given by

$$W_{a \to b} = -\Delta U = -(U_b - U_a) = -(q_0 E y_b - q_0 E y_a) = q_0 E(y_b - y_a)$$
(301)

When y_b is greater than y_a (see fig. 184a), the positive test charge q_0 moves downward, in the same direction as \vec{E} ; the displacement is in the same direction as the force $\vec{F} = q_0 \vec{E}$, so the field does positive work and U decreases. [In particular, if $y_a - y_b = d$ as in Fig. 183, Eq. (301) gives $W_{a\to b} = q_0 E d$ in agreement with Eq. (299).] When y_a is less than y_b (see fig. 184b), the positive test charge q_0 moves upward, in the opposite direction to \vec{E} the displacement is opposite the force, the field does negative work, and U increases.



Figure 184 – A positive charge moving (a) in the direction of the electric field \vec{E} and (b) in the direction opposite \vec{E}

If the test charge q_0 is negative, the potential energy increases when it moves with the field and decreases when it moves against the field (see fig. 185).



Figure 185 – A negative charge moving (a) in the directional of the electric field \vec{E} and (b) in the direction opposite \vec{E}

Whether the test charge is positive or negative, the following general rules apply: U increases if the test charge moves in the direction opposite the electric Force $\vec{F} = q_0 \vec{E}$ (Figs. 184b and 185a); U decreases if moves in the same direction as $\vec{F} = q_0 \vec{E}$ (Figs. 184a and 185b). This is the same behavior as for gravitational potential energy, which increases if a mass m moves upward (opposite the direction of the gravitational force) and decreases if m moves downward (in the same direction as the gravitational force).

The idea of electric potential energy isn't restricted to the special case of a uniform electric field. Indeed, we can apply this concept to a point charge in any electric field caused by a static charge distribution. We can represent any charge distribution as a collection of point charges. Therefore it's useful to calculate the work done on a test charge q_0 moving in the electric field caused by a single, stationary point charge q.

We'll consider first a displacement along the radial line in Fig. 186. The force on q_0 is given by Coulomb's law, and its radial component is

$$F_r = \frac{qq_0}{4\pi\varepsilon_0 r^2} \tag{302}$$

If q and q_0 have the same sign (+ or -) the force is repulsive and F_r is positive; if the two charges have opposite signs, the force is attractive and F_r is negative. The force is not constant during the displacement, and we have to integrate to calculate the work $W_{a\to b}$ done on q_0 by this force as q_0 moves from a to b:
$$W_{a \to b} = \int_{r_a}^{r_b} F_r \cdot dr = \int_{r_a}^{r_b} \frac{qq_0}{4\pi\varepsilon_0 r^2} \cdot dr = \frac{qq_0}{4\pi\varepsilon_0} \left(\frac{1}{r_a} + \frac{1}{r_b}\right)$$
(303)

The work done by the electric force for this particular path depends only on the endpoints.



Figure 186 – Test charge q_0 moves along a straight line extending radially from charge q. As it moves from a to b, the distance varies from r_a to r_b

Now let's consider a more general displacement (see fig. 187) in which a and b do not lie on the same radial line. From Eq. (296) the work done on q_0 during this displacement is given by

$$W_{a \to b} = \int_{r_a}^{r_b} F \cos \varphi \, dl = \int_{r_a}^{r_b} \frac{qq_0}{4\pi\varepsilon_0 r^2} \cos \varphi \, dl \tag{304}$$

But Fig. 187 shows that $\cos \varphi \, dl = dr$. That is, the work done during a small displacement $d\vec{l}$ depends only on the change dr in the distance r between the charges, which is the *radial component* of the displacement. Thus Eq. (303) is valid even for this more general displacement; the work done on q_0 by the electric field \vec{E} produced by q depends only on r_a and r_b , not on the details of the path.

Also, if q_0 returns to its starting point *a* by a different path, the total work done in the round-trip displacement is zero (the integral in Eq. (303) is from r_a back to r_a). These are the needed characteristics for a conservative force. Thus the force on is a conservative force.



Figure 187 – The work done on charge q_0 by the electric field of charge q does not depend on the path taken, but only on the distances r_a and r_b

We see that Eqs. (297) and (303) are consistent if we define the potential energy to be $U_a = \frac{qq_0}{4\pi\varepsilon_0 r_a}$ when q_0 is a distance r_a from q, and to be $U_b = \frac{qq_0}{4\pi\varepsilon_0 r_b}$ when q_0 is a distance r_b from q. Thus the potential energy U when the test charge q_0 is at any distance r from charge q is

$$U = \frac{qq_0}{4\pi\varepsilon_0 r} \tag{305}$$

Equation (305) is valid no matter what the signs of the charges q and q_0 . The potential energy is positive if the charges q and have the same sign (see fig. 188a) and negative if they have opposite signs (see fig. 188b).

(a) q and q₀ have the same sign.

(b) q and q₀ have opposite signs.



Figure 188 – Graphs of the potential energy U of two point charges q and q_0 versus their separation r

Potential energy is always defined relative to some reference point where U = 0. In Eq. (305), U is zero when q and q_0 are infinitely far apart and $r = \infty$. Therefore U represents the work that would be done on the test charge q_0 by the field of q if q_0 moved from an initial distance r to infinity. If q and q_0 have the same sign, the interaction is repulsive, this work is positive, and U is positive at any finite separation (see fig. 188a). If the charges have opposite signs, the interactive, the work done is negative, and U is negative (see fig. 188b).

We emphasize that the potential energy U given by Eq. (305) is a *shared* property of the two charges. If the distance between q and q_0 is changed from r_a to r_b the change in potential energy is the same whether q is held fixed and q_0 is moved or q_0 is held fixed and q is moved. For this reason, we never use the phrase "the electric potential energy of a point charge." (Likewise, if a mass m is at a height h above the earth's surface, the gravitational potential energy is a shared property of the mass m and the earth.

Equation (305) also holds if the charge q_0 is outside a spherically symmetric charge distribution with total charge q; the distance r is from to the center of the distribution. That's because Gauss's law tells us that the electric field outside such a distribution is the same as if all of its charge q were concentrated at its center.

Suppose the electric field \vec{E} in which charge q_0 moves is caused by *several* point charges $q_1, q_2, q_3, ...$ at distances $r_1, r_2, r_3, ...$ from q_0 as in Fig. 189. For example, q_0 could be a positive ion moving in the presence of other ions. The total electric field at each point is the *vector sum* of the fields due to the individual charges, and the total work done on q_0 during any displacement is the sum of the contributions from the individual charges. From Eq. (305) we conclude that the

potential energy associated with the test charge q_0 at point *a* in Fig. 189 is the *algebraic* sum (*not* a vector sum):

$$U = \frac{q_0}{4\pi\varepsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots \right) = \frac{q_0}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$
(306)

When q_0 is at a different point *b* the potential energy is given by the same expression, but $r_1, r_2, ...$ are the distances from $q_1, q_2, ...$ to point The work done on charge q_0 when it moves from *a* to *b* along any path is equal to the difference $U_a - U_b$ between the potential energies when q_0 is at *a* and at *b*.



Figure 189 – The potential energy associated with a charge q_0 at point *a* depends on the other charges q_1, q_2 and q_3 and on their distances r_1, r_2, r_3 from point *a*

We can represent *any* charge distribution as a collection of point charges, so Eq. (306) shows that we can always find a potential-energy function for *any* static electric field. It follows that **for every electric field due to a static charge distribution, the force exerted by that field is conservative.**

Equations (305) and (306) define U to be zero when all the distances $r_1, r_2, ...$ are infinite - that is, when the test charge q_0 is very far away from all the charges that produce the field. As with any potential-energy function, the point where U = 0 is arbitrary; we can always add a constant to make U equal zero at any point we choose. In electrostatics problems it's usually simplest to choose this point to be at infinity.

Equation (306) gives the potential energy associated with the presence of the test charge q_0 in the field \vec{E} produced by q_1, q_2, q_3, \dots But there is also potential energy involved in assembling these charges. If we start with charges q_1, q_2, q_3, \dots all separated from each other by infinite distances and then bring them together so that the distance between q_i and q_j is the *total* potential energy U is the sum of the potential energies of interaction for each pair of charges. We can write this as

$$U = \frac{q_0}{4\pi\varepsilon_0} \sum_{i< j} \frac{q_i q_j}{r_{ij}}$$
(307)

This sum extends over all *pairs* of charges; we don't let i = j (because that would be an interaction of a charge with itself), and we include only terms with i < j to make sure that we count each pair only once. Thus, to account for the interaction between and we include a term with i = 3 and j = 4 but not a term with i = 4 and j = 3.

As a final comment, here are two viewpoints on electric potential energy. We have defined it in terms of the work done by the electric field on a charged particle moving in the field, we defined potential energy in terms of the work done by gravity or by a spring. When a particle moves from point *a* to point the work done on it by the electric field is $W_{a\rightarrow b} = U_a - U_b$. Thus the potential-energy difference $U_a - U_b$ equals the work that is done by the electric force when the particle moves from a to b. When is greater than the field does positive work on the particle as it "falls" from a point of higher potential energy to a point of lower potential energy

An alternative but equivalent viewpoint is to consider how much work we would have to do to "raise" a particle from a point *b* where the potential energy is U_b to a point *a* where it has a greater value U_b (pushing two positive charges closer together, for example). To move the particle slowly (so as not to give it any kinetic energy), we need to exert an additional external force \vec{F}_{ext} that is equal and opposite to the electric-field force and does positive work. The potential energy difference $U_a - U_b$ is then defined as *the work that must be done by an external force to move the particle slowly from b to a against the electric force*. Because \vec{F}_{ext} is the negative of the electric-field force and the displacement is in the opposite direction, this definition of the potential difference $U_a - U_b$ is equivalent to that given above. This alternative viewpoint also works U_a if is less than corresponding to "lowering" the particle; an example is moving two positive charges away from each other. In this case, $U_a - U_b$ is again equal to the work done by the external force, but now this work is negative.

We will use both of these viewpoints in the next section to interpret what is meant by electric *potential*, or potential energy per unit charge.

3.2.2 Electric potential

Early we looked at the potential energy U associated with a test charge q_0 in an electric field. Now we want to describe this potential energy on a "per unit charge" basis, just as electric field describes the force per unit charge on a charged particle in the field. This leads us to the concept of *electric potential*, often called simply *potential*. This concept is very useful in calculations involving energies of charged particles. It also facilitates many electric-field calculations because electric potential is closely related to the electric field \vec{E} . When we need to determine an electric field, it is often easier to determine the potential first and then find the field from it.

Potential is potential energy per unit charge. We define the potential φ at any point in an electric field as the potential energy U per unit charge associated with a test charge q_0 at that point:

$$\varphi = \frac{U}{q_0} \tag{308}$$

or

$$U = q_0 \varphi \tag{309}$$

Potential energy and charge are both scalars, so potential is a scalar. From Eq. (309) its units are the units of energy divided by those of charge. The SI unit of potential, called one volt (1 V) in honor of the Italian electrical experimenter Alessandro Volta (1745–1827), equals 1 joule per coulomb:

$$1 V = 1 volt = 1 \frac{J}{C} = 1 joule/coulomb$$

Let's put Eq. (297), which equates the work done by the electric force during a displacement from *a* to *b* to the quantity $-\Delta U = -(U_b - U_a)$ on a "work per unit charge" basis. We divide this equation by q_0 obtaining

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = \varphi_b - \varphi_a = \varphi_a - \varphi_b \tag{310}$$

Where $\varphi_a = \frac{U_a}{q_0}$ is the potential energy per unit charge at point *a* and similarly for φ_b . We call φ_a and φ_b the *potential at point a and potential at point b* respectively. Thus the work done per unit charge by the electric force when a charged body moves from *a* to *b* b is equal to the potential at *a* minus the potential at *b*.

The difference $\varphi_a - \varphi_b$ is called the *potential of a with respect to b*; we sometimes abbreviate this difference as $\varphi_{ab} = \varphi_a - \varphi_b$ (note the order of the subscripts).

This is often called the potential difference between a and b but that's ambiguous unless we specify which is the reference point. In electric circuits, which we will analyze in later chapters, the potential difference between two points is often called voltage (see fig. 190). Equation (310) then states: φ_{ab} , the potential of with respect to equals the work done by the electric force when a UNIT charge moves from a to b.

Another way to interpret the potential difference φ_{ab} in Eq. (310) is to use the alternative viewpoint mentioned. In that viewpoint, $U_b - U_a$ is the amount of work that must be done by an *external* force to move a particle of charge q_0 slowly from *b* to *a* against the electric force. The work that must be done *per unit charge* by the external force is then $\frac{(U_b - U_a)}{q_0} = \varphi_a - \varphi_b = \varphi_{ab}$. In other words: φ_{ab} the potential *a* of with respect to *b*, equals the work that must be done to move a UNIT charge slowly from *b* to *a*gainst the electric force.



Figure 190 – The voltage of this battery equals the difference in potential $\varphi_{ab} = \varphi_a - \varphi_b$ between its positive terminal (point *a*) and its negative terminal (points *b*)

An instrument that measures the difference of potential between two points is called a *voltmeter*. Voltmeters that can measure a potential difference of $1 \mu V$ are common, and sensitivities down to $10^{-12} V$ can be attained.

To find the potential φ due to a single point charge q we divide Eq. (305) by q_0 :

$$\varphi = \frac{U}{q_0} = \frac{q}{4\pi\varepsilon_0} \tag{311}$$

where r is the distance from the point charge q to the point at which the potential is evaluated. If q is positive, the potential that it produces is positive at all points; if q is negative, it produces a potential that is negative everywhere. In either case, φ is equal to zero at $r = \infty$ an infinite distance from the point charge. Note that potential, like electric field, is independent of the test charge q_0 that we use to define it.

Similarly, we divide Eq. (306) by q_0 to find the potential due to a collection of point charges:

$$\varphi = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$
(312)

In this expression, r_i is the distance from the charge q_0 , to the point at which φ is evaluated. Just as the electric field due to a collection of point charges is the *vector* sum of the fields produced by each charge, the electric potential due to a collection of point charges is the *scalar* sum of the potentials due to each charge. When we have a continuous distribution of charge along a line, over a surface, or through a volume, we divide the charge into elements dq and the sum in Eq. (312) becomes an integral:

$$\varphi = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} \tag{313}$$

where *r* is the distance from the charge element dq to the field point where we are finding φ . We'll work out several examples of such cases. The potential defined by Eqs. (312) and (23.16) is zero at points that are infinitely far away from all the charges. Later we'll encounter cases in which the charge distribution itself extends to infinity. We'll find that in such cases we cannot set $\varphi = 0$ at infinity, and we'll need to exercise care in using and interpreting Eqs. (312) and (23.16).

When we are given a collection of point charges, Eq. (312) is usually the easiest way to calculate the potential φ . But in some problems in which the electric field is known or can be found easily, it is easier to determine φ from \vec{E} . The force \vec{F} on a test charge q_0 can be written as $\vec{F} = q_0 \vec{E}$ so from Eq. (23.1) the work done by the electric force as the test charge moves from *a* to *b* is given by

$$W_{a \to b} = \int_{a}^{b} \vec{F} d\vec{l} = \int_{a}^{b} q_{0} \vec{E} d\vec{l}$$
(314)

If we divide this by q_0 and compare the result with Eq. (310), we find

$$\varphi_a - \varphi_b = \int_a^b \vec{E} d\vec{l} = \int_a^b E \cos \varphi \, dl \tag{315}$$

The value $\varphi_a - \varphi_b$ of is independent of the path taken from *a* to *b*, just as the value of $W_{a \to b}$ is independent of the path. To interpret Eq. (315), remember that \vec{E} is the electric force per unit charge on a test charge. If the line integral $\int_a^b \vec{E} d\vec{l}$ is positive, the electric field does positive work on a positive test charge as it moves from *a* to *b*. In this case the electric potential energy decreases as the test charge

moves, so the potential energy per unit charge decreases as well; hence φ_b is less than φ_a and $\varphi_a - \varphi_b$ is positive.

As an illustration, consider a positive point charge (see fig. 191a). The electric field is directed away from the charge, and $\varphi = q/4\pi\varepsilon_0 r$ is positive at any finite distance from the charge. If you move away from the charge, in the direction of \vec{E} , you move toward lower values of φ ; if you move toward the charge, in the direction opposite \vec{E} , you move toward greater values of φ . For the negative point charge in Fig. 191b, \vec{E} is directed toward the charge and $\varphi = q/4\pi\varepsilon_0 r$ is negative at any finite distance from the charge. In this case, if you move toward the charge, you are moving in the direction \vec{E} of and in the direction of decreasing (more negative) φ . Moving away from the charge, in the direction opposite \vec{E} moves you toward increasing (less negative) values of φ . The general rule, valid for *any* electric field, is: Moving *with* the direction of \vec{E} means moving in the direction of *decreasing* φ .



Figure 191 – If you move in the direction of \vec{E} , electric potential φ decreases, if you move in the direction opposite \vec{E} , φ increases

Also, a positive test charge q_0 experiences an electric force in the direction of \vec{E} toward lower values of φ ; a negative test charge experiences a force opposite \vec{E} , toward higher values of φ . Thus a positive charge tends to "fall" from a highpotential region to a lower-potential region. The opposite is true for a negative charge.

Notice that Eq. (315) can be rewritten as

$$\varphi_a - \varphi_b = -\int_b^a \vec{E} d\vec{l}$$
(316)

This has a negative sign compared to the integral in Eq. (315), and the limits are reversed; hence Eqs. (315) and (316) are equivalent. But Eq. (316) has a slightly different interpretation. To move a unit charge slowly against the electric force, we must apply an *external* force per unit charge equal to $-\vec{E}$, equal and opposite to the electric force per unit charge \vec{E} . Equation (316) says that $\varphi_a - \varphi_b = \varphi_{ab}$, the potential of *a* with respect to *b*, equals the work done per unit charge by this external force to move a unit charge from *b* to *a*. This is the same alternative interpretation we discussed under Eq. (310).

Equations (315) and (316) show that the unit of potential difference (1 V) is equal to the unit of electric field $(1\frac{N}{c})$ multiplied by the unit of distance (1 m). Hence the unit of electric field can be expressed as 1 *volt per meter* 1 V/m, as well as 1 N/C:

$$1\frac{V}{m} = 1\frac{volt}{meter} = 1\frac{N}{C} = 1$$
 newton/coulomb

In practice, the volt per meter is the usual unit of electric-field magnitude.

3.2.3 Equipotential surfaces

Field lines help us visualize electric fields. In a similar way, the potential at various points in an electric field can be represented graphically by *equipotential surfaces*. These use the same fundamental idea as topographic maps like those used by hikers and mountain climbers (see fig. 192). On a topographic map, contour lines are drawn through points that are all at the same elevation. Any number of these could be drawn, but typically only a few contour lines are shown at equal spacings of elevation. If a mass *m* is moved over the terrain along such a contour line, the gravitational potential energy *mgy* does not change because the elevation *y* is constant. Thus contour lines on a topographic map are really curves of constant gravitational potential energy. Contour lines are close together where the terrain is steep and there are large changes in elevation over a small horizontal distance; the contour lines are farther apart where the terrain is gently sloping. A ball allowed to roll downhill will experience the greatest downhill gravitational force where contour lines are closest together.



Figure 192 – Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy

By analogy to contour lines on a topographic map, an **equipotential surface** is a three-dimensional surface on which the *electric potential* φ is the same at every point. If a test charge q_0 is moved from point to point on such a surface, the *electric* potential energy $q_0\varphi$ remains constant. In a region where an electric field is present, we can construct an equipotential surface through any point. In diagrams we usually show only a few representative equipotentials, often with equal potential differences between adjacent surfaces. No point can be at two different potentials, so equipotential surfaces for different potentials can never touch or intersect.

Because potential energy does not change as a test charge moves over an equipotential surface, the electric field can do no work on such a charge. It follows that \vec{E} must be perpendicular to the surface at every point so that the electric force $q_0\vec{E}$ is always perpendicular to the displacement of a charge moving on the surface.

Field lines and equipotential surfaces are always mutually perpendicular. In general, field lines are curves, and equipotentials are curved surfaces. For the special case of a *uniform* field, in which the field lines are straight, parallel, and equally spaced, the equipotentials are parallel *planes* perpendicular to the field lines.

Figure 193 shows three arrangements of charges. The field lines in the plane of the charges are represented by red lines, and the intersections of the equipotential surfaces with this plane (that is, cross sections of these surfaces) are shown as blue lines. The actual equipotential surfaces are three-dimensional. At each crossing of an equipotential and a field line, the two are perpendicular.



Figure 193 – Cross section of equipotential surface (blue lines) and electric field lines (red lines) for assemblies of point charge. There are equal potential difference between adjacent surfaces.

In Fig. 193 we have drawn equipotentials so that there are equal potential differences between adjacent surfaces. In regions where the magnitude of is large, the equipotential surfaces are close together because the field does a relatively large amount of work on a test charge in a relatively small displacement. This is the case near the point charge in Fig. 193a or between the two point charges in Fig. 193b; note that in these regions the field lines are also closer together. This is directly analogous to the downhill force of gravity being greatest in regions on a topographic map where contour lines are close together. Conversely, in regions where the field is weaker, the equipotential surfaces are farther apart; this happens at larger radii in Fig. 193a, to the left of the negative charge or the right of the

positive charge in Fig. 193b, and at greater distances from both charges in Fig. 193c. (It may appear that two equipotential surfaces intersect at the center of Fig. 193c, in violation of the rule that this can never happen. In fact this is a single figure-8–shaped equipotential surface.)

Here's an important statement about equipotential surfaces: When all charges are at rest, the surface of a conductor is always an equipotential surface. Since the electric field \vec{E} is always perpendicular to an equipotential surface, we can prove this statement by proving that when all charges are at rest, the electric field just outside a conductor must be perpendicular to the surface at every point (see fig. 194). We know that $\vec{E} = 0$ everywhere inside the conductor; otherwise, charges would move. In particular, at any point just inside the surface the component of \vec{E} tangent to the surface is zero. It follows that thetangential component of \vec{E} is also zero just outside the surface. If it were not, a charge could move around a rectangular path partly inside and partly outside (see fig. 195) and return to its starting point with a net amount of work having been done on it. This would violate the conservative nature of electrostatic fields, so the tangential component of \vec{E} just outside the surface must be zero at every point on the surface. Thus \vec{E} is perpendicular to the surface at each point, proving our statement.



Electric field lines

Figure 194 – When charges are at rest, a conducting surface is always an equipotential surface. Field lines are perpendicular to a conducting surface

An impossible electric field If the electric field just outside a conductor had a tangential component E_{\parallel} , a charge could move in a loop with net work done.



Figure 195 – At all points on the surface of a conductor, the electric field must be perpendicular to the surface. If \vec{E} had a tangential component, a net amount of work would be done on a test charge by moving in around a loop as shown here – which is impossible because the electric force is conservative

It also follows that when all charges are at rest, the entire solid volume of a conductor is at the same potential. Equation (315) states that the potential difference between two points *a* and *b* within the conductor's solid volume, $\varphi_a - \varphi_b$, is equal to the line integral $\int_b^a \vec{E} d\vec{l}$ of the electric field from *a* to *b*. Since everywhere inside the conductor, the integral is guaranteed to be zero for any two such points *a* and *b*. Hence the potential is the same for any two points within the solid volume of the conductor. We describe this by saying that the solid volume of the conductor is an equipotential volume.

Finally, we can now prove a theorem. The theorem is as follows: In an electrostatic situation, if a conductor contains a cavity and if no charge is present inside the cavity, then there can be no net charge *anywhere* on the surface of the cavity. This means that if you're inside a charged conducting box, you can safely touch any point on the inside walls of the box without being shocked. To prove this theorem, we first prove that *every point* in the cavity is at the *same potential*. In Fig. 196 the conducting surface A of the cavity is an equipotential surface, as we have just proved. Suppose point P in the cavity is at a different potential; then we can construct a different equipotential surface B including point P.

Now consider a Gaussian surface, shown in Fig. 196, between the two equipotential surfaces. Because of the relationship between and the equipotentials, we know that the field at every point between the equipotentials is from A toward or else at every point it is from B toward A, depending on which equipotential surface is at higher potential. In either case the flux through this Gaussian surface is certainly not zero. But then Gauss's law says that the charge enclosed by the Gaussian surface cannot be zero. This contradicts our initial assumption that there is no charge in the cavity. So the potential at P cannot be different from that at the cavity wall.



Figure 23.26 - A cavity in a conductor. If the cavity is at the same potential, the electric field is zero every where in the cavity, and there is no charge anywhere on the surface of the cavity

The entire region of the cavity must therefore be at the same potential. But for this to be true, the electric field inside the cavity must be zero everywhere. Finally, Gauss's law shows that the electric field at any point on the surface of a conductor is proportional to the surface charge density σ at that point. We conclude that the surface charge density on the wall of the cavity is zero at every point. This chain of reasoning may seem tortuous, but it is worth careful study.

3.3 Current, resistance and electromotive force

3.3.1 Current

A **current** is any motion of charge from one region to another. In this section we'll discuss currents in conducting materials. The vast majority of technological applications of charges in motion involve currents of this kind.

In electrostatic situations the electric field is zero everywhere within the conductor, and there is no current. However, this does not mean that all charges within the conductor are at rest. In an ordinary metal such as copper or alumium, some of the electrons are free to move within the conducting material. These free electrons move randomly in all directions, somewhat like the molecules of a gas but with much greater speeds, of the order of $10^6 m/s$. The electrons nonetheless do not escape from the conducting material, because they are attracted to the positive ions of the material. The motion of the electrons is random, so there is no net flow of charge in any direction and hence no current.

Now consider what happens if a constant, steady electric field \vec{E} is established inside a conductor. (We'll see later how this can be done.) A charged particle (such as a free electron) inside the conducting material is then subjected to a steady force $\vec{F} = q\vec{E}$. If the charged particle were moving in vacuum, this steady

force would cause a steady acceleration in the direction of \vec{E} and after a time the charged particle would be moving in that direction at high speed. But a charged particle moving in a conductor undergoes frequent collisions with the massive, nearly stationary ions of the material. In each such collision the particle's direction of motion undergoes a random change. The net effect of the electric field \vec{E} is that in addition to the random motion of the charged particles within the conductor, there is also a very slow net motion or drift of the moving charged particles as a group in the direction of the electric force $\vec{F} = q\vec{E}$ (see fig. 197). This motion is described in terms of the **drift velocity** \vec{v}_d of the particles. As a result, there is a net current in the conductor.



Figure 197 – If there is no electric field inside a conductor, an electron moves randomly from point P_1 to point P_2 in a time Δt . If an electric field \vec{E} is present, the electric force $\vec{F} = q\vec{E}$ impose a small drift (greatly exaggerated here) that takes the electron to point P'_2 , a distance $v_d\Delta t$ from P_2 in the direction of the force

While the random motion of the electrons has a very fast average speed of about $10^6 m/s$, the drift speed is very slow, often on the order of $10^{-4} m/s$. Given that the electrons move so slowly, you may wonder why the light comes on immediately when you turn on the switch of a flashlight. The reason is that the electric field is set up in the wire with a speed approaching the speed of light, and electrons start to move all along the wire at very nearly the same time. The time that it takes any individual electron to get from the switch to the light bulb isn't really relevant. A good analogy is a group of soldiers standing at attention when the sergeant orders them to start marching; the order reaches the soldiers' ears at the speed of sound, which is much faster than their marching speed, so all the soldiers start to march essentially in unison.

The drift of moving charges through a conductor can be interpreted in terms of work and energy. The electric field \vec{E} does work on the moving charges. The resulting kinetic energy is transferred to the material of the conductor by means of collisions with the ions, which vibrate about their equilibrium positions in the crystalline structure of the conductor. This energy transfer increases the average vibrational energy of the ions and therefore the temperature of the material. Thus much of the work done by the electric field goes into heating the conductor, not into making the moving charges move ever faster and faster. This heating is sometimes useful, as in an electric toaster, but in many situations is simply an unavoidable by-product of current flow.

In different current-carrying materials, the charges of the moving particles may be positive or negative. In metals the moving charges are always (negative) electrons, while in an ionized gas (plasma) or an ionic solution the moving charges may include both electrons and positively charged ions. In a semiconductor material such as germanium or silicon, conduction is partly by electrons and partly by motion of *vacancies*, also known as *holes*; these are sites of missing electrons and act like positive charges.

Figure 198 shows segments of two different current-carrying materials. In Fig. 198a the moving charges are positive, the electric force is in the same direction as \vec{E} , and the drift velocity \vec{v}_d is from left to right. In Fig. 198b the charges are negative, the electric force is opposite to \vec{E} , and the drift velocity \vec{v}_d is from right to left. In both cases there is a net flow of positive charge from left to right, and positive charges end up to the right of negative ones. We *define* the current, denoted by *I*, to be in the direction in which there is a flow of *positive* charge flow, even in cases in which we know that the actual current is due to electrons. Hence the current is to the right in both Figs. 25.2a and 25.2b. This choice or convention for the direction of current flow is called **conventional current**. While the direction of the conventional current is *not* necessarily the same as the direction in which charged particles are actually moving, we'll find that the sign of the moving charges is of little importance in analyzing electric circuits.



A conventional current is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.



Figure 198 – The same current can be produced by (a) positive charges moving in the direction of the electric filed \vec{E} or (b) the same number of negative charges moving as the same speed in the direction opposite to \vec{E}

Figure 199 shows a segment of a conductor in which a current is flowing. We consider the moving charges to be *positive*, so they are moving in the same direction as the current. We define the current through the cross-sectional area S to be *the net charge flowing through the area per unit time*. Thus, if a net charge dQ flows through an area in a time dt, the current I through the area is

$$I = \frac{dQ}{dt} \tag{317}$$

The SI unit of current is the ampere; one ampere is defined to be *one* coulomb per second 1A = 1C/s. This unit is named in honor of the French scientist André Marie Ampère (1775–1836). When an ordinary flashlight (D-cell size) is turned on, the current in the flashlight is about 0.5 to 1; the current in the wires of a car engine's starter motor is around 200 A. Currents in radio and

television circuits are usually expressed in *milliamperes* $(1 \ mA = 10^{-3} \ A)$ or *microamperes* $(1 \ \mu A = 10^{-6} \ A)$ and currents in computer circuits are expressed in *nanoamperes* $(1 \ nA = 10^{-9} \ A)$ or *picoamperes* $(1 \ pA = 10^{-12} \ A)$.

We can express current in terms of the drift velocity of the moving charges. Let's consider again the situation of Fig. 199 of a conductor with cross-sectional area S and an electric field \vec{E} directed from left to right. To begin with, we'll assume that the free charges in the conductor are positive; then the drift velocity is in the same direction as the field.



Figure 199 – The current *I* is the time rate of charge transfer through the crosssection area *A* (*S*). The random component of each moving charged particle's motion averages to zero, and the current is in the same direction as \vec{E} whether the moving charges are positive (as shown here) or negative

Suppose there are moving charged particles per unit volume. We call *n* the **concentration** of particles; its SI unit is 10^{-3} . Assume that all the particles move with the same drift velocity with magnitude v_d . In a time interval dt, each particle moves a distance $\vec{v}_d dt$. The particles that flow out of the right end of the shaded cylinder with length $\vec{v}_d dt$ during dt are the particles that were within this cylinder at the beginning of the interval dt. The volume of the cylinder is Sdt, and the

number of particles within it is $nSv_d dt$. If each particle has a charge q, the charge dQ that flows out of the end of the cylinder dt during time is

$$dQ = q(nSv_d dt) = nqSv_d dt \tag{318}$$

and the current is

$$I = \frac{dQ}{dt} = nqSv_d \tag{319}$$

The current *per unit cross-sectional area* is called the **current density** *j*

$$j = \frac{I}{S} = nqv_d \tag{320}$$

The units of current density are amperes per square meter A/m^2 .

If the moving charges are negative rather than positive, as in Fig. 198b, the drift velocity is opposite to But the *current* is still in the same direction as \vec{E} at each point in the conductor. Hence the current *I* and current density *j* don't depend on the sign of the charge, and so in the above expressions for *I* and *j* we replace the charge by its absolute value |q|:

$$j = \frac{dQ}{dt} = n|q|v_d S$$
(321)
$$j = \frac{I}{S} = n|q|v_d$$
(322)

The current in a conductor is the product of the concentration of moving charged particles, the magnitude of charge of each such particle, the magnitude of the drift velocity, and the cross-sectional area of the conductor.

We can also define a *vector* current density \vec{j} that includes the direction of the drift velocity:

$$\vec{j} = nq\vec{v}_d \tag{323}$$

There are *no* absolute value signs in Eq. (323). If is positive, \vec{v}_d is in the same direction as \vec{E} ; if q is negative, is opposite to \vec{E} . In either case, is in the same direction as \vec{E} . Equation (322) gives the *magnitude j* of the vector current density \vec{j} .

In general, a conductor may contain several different kinds of moving charged particles having charges $q_1, q_2, ...$, concentrations $n_1, n_2, ...$ and drift velocities with magnitudes $v_{d1}, v_{d2}, ...$. An example is current flow in an ionic solution (see fig. 200). In a sodium chloride solution, current can be carried by both positive sodium ions and negative chlorine ions; the total current I is found by

adding up the currents due to each kind of charged particle, using Eq. (321). Likewise, the total vector current density \vec{j} is found by using Eq. (323) for each kind of charged particle and adding the results.



Figure 200 Part of the electric circuit that includes this light bulb passes through a beaker with a solution of sodium chloride. The current in the solution is carried by both positive charges (Na^+ ions) and negative charges (Cl^- ions)

It is possible to have a current that is steady (that is, one that is constant in time) only if the conducting material forms a closed loop, called a complete circuit. In such a steady situation, the total charge in every segment of the conductor is constant. Hence the rate of flow of charge out at one end of a segment at any instant equals the rate of flow of charge in at the other end of the segment, and the current is the same at all cross sections of the circuit. We'll make use of this observation when we analyze electric circuits later in this chapter.

In many simple circuits, such as flashlights or cordless electric drills, the direction of the current is always the same; this is called direct current. But home appliances such as toasters, refrigerators, and televisions use alternating current, in which the current continuously changes direction. In this chapter we'll consider direct current only. Alternating current has many special features worthy of detailed study.

3.3.2 Resistivity and resistance

The current density \vec{j} in a conductor depends on the electric field \vec{E} and on the properties of the material. In general, this dependence can be quite complex. But for some materials, especially metals, at a given temperature, \vec{j} is nearly *directly proportional* to \vec{E} , and the ratio of the magnitudes of E and j is constant. This relationship, called Ohm's law, was discovered in 1826 by the German physicist Georg Simon Ohm (1787–1854). The word "law" should actually be in quotation marks, since Ohm's law, like the ideal-gas equation and Hooke's law, is an idealized model that describes the behavior of some materials quite well but is not a general description of all matter. In the following discussion we'll assume that Ohm's law is valid, even though there are many situations in which it is not. The situation is comparable to our representation of the behavior of the static and kinetic friction forces; we treated these friction forces as being directly proportional to the normal force, even though we knew that this was at best an approximate description.

We define the **resistivity** ρ of a material as the ratio of the magnitudes of electric field and current density:

$$\rho = \frac{E}{j} \tag{324}$$

The greater the resistivity, the greater the field needed to cause a given current density, or the smaller the current density caused by a given field. From Eq. (324) the units of ρ are $(V/m)/(A/m^2) = V \cdot m/A$. As we will discuss in the next section, 1 V/A is called one *ohm* (1); we use the Greek letter Ω , or omega, which is alliterative with "ohm"). So the SI units for ρ are Ω (ohm-meters). Table 25.1 lists some representative values of resistivity. A perfect conductor would have zero resistivity, and a perfect insulator would have an infinite resistivity. Metals and alloys have the smallest resistivities and are the best conductors. The resistivities of insulators are greater than those of the metals by an enormous factor, on the order of 10^{22} .

The reciprocal of resistivity is **conductivity.** Its units are $(\Omega \cdot m)^{-1}$. Good conductors of electricity have larger conductivity than insulators. Conductivity is the direct electrical analog of thermal conductivity. Comparing Table 8 with Table 6 (Thermal Conductivities), we note that good electrical conductors, such as metals, are usually also good conductors of heat. Poor electrical conductors, such as ceramic and plastic materials, are also poor thermal conductors. In a metal the free electrons that carry charge in electrical conduction also provide the principal mechanism for heat conductivity. Because of the enormous difference in conductivity between electrical conductors and insulators, it is easy to confine electric currents to well-defined paths or circuits (see fig. 201). The variation in *thermal* conductivity is much less, only a factor of 10^3 or so, and it is usually impossible to confine heat currents to that extent.

	Substance	$\rho (\Omega \cdot m)$
Conductors		
Metals	Silver	$1.47 \cdot 10^{-8}$
	Copper	$1.72 \cdot 10^{-8}$
	Gold	$2.44 \cdot 10^{-8}$
	Aluminium	$2.75 \cdot 10^{-8}$
	Tungsten	$5.25 \cdot 10^{-8}$
	Steel	$20 \cdot 10^{-8}$
	Lead	$22 \cdot 10^{-8}$
	Mercury	$95 \cdot 10^{-8}$
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	$44 \cdot 10^{-8}$
	Constantan (Cu 60%, Ni 40%)	$49 \cdot 10^{-8}$
	Nichrome (Ni 55-78%, Cr 13-23%,	$100 \cdot 10^{-8}$
	Mn, Si, Fe, Al)	
Semiconductors		
	Pure carbon (graphite)	$3.5 \cdot 10^{-5}$
	Pure germanium	0.6
	Pure silicon	2300
Insulators		
	Amber	$5 \cdot 10^{14}$
	Glass	$10^{10} - 10^{14}$
	Lucite	> 10 ¹³
	Mica	$10^{11} - 10^{15}$
	Quartz (fused)	$75 \cdot 10^{16}$
	Sulfur	101
	Teflon	> 10 ¹³
	Wood	$10^8 - 10^{11}$

Table 8 – Resistivities at room temperature 20°C

Semiconductors have resistivities intermediate between those of metals and those of insulators. These materials are important because of the way their resistivities are affected by temperature and by small amounts of impurities. Amaterial that obeys Ohm's law reasonably well is called an *ohmic* conductor or a *linear* conductor. For such materials, at a given temperature, ρ is a *constant* that does not depend on the value of *E*. Many materials show substantial departures from Ohm's-law behavior; they are *nonohmic*, or *nonlinear*. In these materials, depends on in a more complicated manner.

Analogies with fluid flow can be a big help in developing intuition about electric current and circuits. For example, in the making of wine or maple syrup, the product is sometimes filtered to remove sediments. A pump forces the fluid through the filter under pressure; if the flow rate (analogous to j) is proportional to the pressure difference between the upstream and downstream sides (analogous to E), the behavior is analogous to Ohm's law.



Figure 201 – The copper "wires" or traces, on this circuit board are printed directly onto the surface of the dark-colored insulating board. Even though the traces are very close to each other (only about a millimetre apart), the board has such a high resistivity (and low conductivity) that no current can flow between the traces

The resistivity of a metallic conductor nearly always increases with increasing temperature, as shown in Fig. 202a. As temperature increases, the ions of the conductor vibrate with greater amplitude, making it more likely that a moving electron will collide with an ion as in Fig. 197; this impedes the drift of electrons through the conductor and hence reduces the current. Over a small temperature range (up to 100°C or so), the resistivity of a metal can be represented approximately by the equation

$$\rho(T) = \rho_0 [1 + \alpha (T - T_0)]$$
(325)

where ρ_0 is the resistivity at a reference temperature T_0 (often taken as 0°C or 20°C and $\rho(T)$ is the resistivity at temperature T, which may be higher or lower than T_0 . The factor is called the temperature coefficient of resistivity. Some representative

values are given in Table 9. The resistivity of the alloy manganin is practically independent of temperature.

Material	$\alpha[(^{\circ}C)^{-1}]$
Aluminium	0.0039
Brass	0.002
Carbon (graphite)	-0.0005
Constantan	0.00001
Copper	0.00393
Iron	0.005
Lead	0.0043
Manganin	0.00000
Mercury	0.00088
Nochrome	0.0004
Silver	0.0038
Tungsten	0.0045

Table 9 – Temperature coefficients of resistivity (approximate values near room temperature)

The resistivity of graphite (a nonmetal) *decreases* with increasing temperature, since at higher temperatures, more electrons are "shaken loose" from the atoms and become mobile; hence the temperature coefficient of resistivity of graphite is negative. This same behavior occurs for semiconductors (see fig. 202b). Measuring the resistivity of a small semiconductor crystal is therefore a sensitive measure of temperature; this is the principle of a type of thermometer called a *thermistor*.

Some materials, including several metallic alloys and oxides, show a phenomenon called superconductivity. As the temperature decreases, the resistivity at first decreases smoothly, like that of any metal. But then at a certain critical temperature T_c a phase transition occurs and the resistivity suddenly drops to zero, as shown in Fig. 202c. Once a current has been established in a superconducting ring, it continues indefinitely without the presence of any driving field.

Superconductivity was discovered in 1911 by the Dutch physicist Heike Kamerlingh Onnes (1853–1926). He discovered that at very low temperatures, below 4.2 K, the resistivity of mercury suddenly dropped to zero. For the next 75 years, the highest T_C attained was about 20 K. This meant that superconductivity occurred only when the material was cooled using expensive liquid helium, with a boiling-point temperature 4.2 K, or explosive liquid hydrogen, with a boiling point of 20.3 K. But in 1986 Karl Müller and Johannes Bednorz discovered an oxide of barium, lanthanum, and copper with a T_C of nearly 40 K, and the race was on to develop "high-temperature" superconducting materials.



Figure 202 – Variation of resistivity ρ with absolute temperature T for (a) a normal metal, (b) a semiconductor, and (c) a sperconductor. In (a) the linear approximation to ρ as a function of T is shown as a green line; the approximation agrees exactly at $T = T_0$, where $\rho = \rho_0$

By 1987 a complex oxide of yttrium, copper, and barium had been found that has a value T_c of well above the 77 K boiling temperature of liquid nitrogen, a refrigerant that is both inexpensive and safe. The current (2010) record for T_c at atmospheric pressure is 138 K, and materials that are superconductors at room temperature may become a reality. The implications of these discoveries for power-distribution systems, computer design, and transportation are enormous. Meanwhile, superconducting electromagnets cooled by liquid helium are used in particle accelerators and some experimental magnetic-levitation railroads. For a conductor with resistivity the current density \vec{j} at a point where the electric field is \vec{E} is given by Eq. (324), which we can write as

$$\vec{E} = \rho \vec{j} \tag{326}$$

When Ohm's law is obeyed ρ , is constant and independent of the magnitude of the electric field, so \vec{E} is directly proportional to \vec{j} . Often, however, we are more interested in the total current in a conductor than in \vec{j} and more interested in the potential difference between the ends of the conductor than in \vec{E} . This is so largely because current and potential difference are much easier to measure than \vec{j} are and \vec{E} .

Suppose our conductor is a wire with uniform cross-sectional area S and length L, as shown in Fig. 203. Let U be the potential difference between the higher-potential and lower-potential ends of the conductor, so that U is positive. The *direction* of the current is always from the higher-potential end to the lowerpotential end. That's because current in a conductor flows in the direction of \vec{E} , no matter what the sign of the moving charges (see fig. 198), and because \vec{E} points in the direction of *decreasing* electric potential. As the current flows through the potential difference, electric potential energy is lost; this energy is transferred to the ions of the conducting material during collisions.



Figure 203 – A conductor with uniform cross section. The current density uniform over any cross section, and the electric field is constant along the length

We can also relate the value of the current I to the potential difference between the ends of the conductor. If the magnitudes of the current density \vec{j} and the electric field \vec{E} are uniform throughout the conductor, the total current I is given by I = jS, and the potential difference U between the ends is U = EL. When we solve these equations for j and E, respectively, and substitute the results in Eq. (326), we obtain

$$\frac{U}{L} = \frac{\rho I}{S} \tag{327}$$

or

$$U = \frac{\rho L}{S} I \tag{328}$$

This shows that when ρ is constant, the total current *I* is proportional to the potential difference *U*.

The ratio of to for a particular conductor is called its **resistance** *R*:

$$R = \frac{U}{I} \tag{329}$$

Comparing this definition of to Eq. (328), we see that the resistance R of a particular conductor is related to the resistivity ρ of its material by

$$R = \frac{\rho L}{S} \tag{330}$$

If ρ is constant, as is the case for ohmic materials, then so is *R*.

The equation

$$U = IR \tag{331}$$

is often called Ohm's law, but it is important to understand that the real content of Ohm's law is the direct proportionality (for some materials) of U to I or of j to E. Equation (329) or (331) *defines* resistance R for *any* conductor, whether or not it obeys Ohm's law, but only when R is constant can we correctly call this relationship Ohm's law.

Equation (330) shows that the resistance of a wire or other conductor of uniform cross section is directly proportional to its length and inversely proportional to its cross-sectional area. It is also proportional to the resistivity of the material of which the conductor is made.

The flowing-fluid analogy is again useful. In analogy to Eq. (330), a narrow water hose offers more resistance to flow than a fat one, and a long hose has more resistance than a short one (see fig. 25.8). We can increase the resistance to flow by stuffing the hose with cotton or sand; this corresponds to increasing the resistivity. The flow rate is approximately proportional to the pressure difference between the ends. Flow rate is analogous to current, and pressure difference is analogous to potential difference ("voltage"). Let's not stretch this analogy too far,

though; the water flow rate in a pipe is usually *not* proportional to its crosssectional area.

The SI unit of resistance is the **ohm**, equal to one volt per ampere $(1 \Omega = 1 V/A)$. The kilohm $1 k\Omega = 10^3 \Omega$ and the megohm $1 M\Omega = 10^6 \Omega$ are also in common use. A 100-m length of 12-gauge copper wire, the size usually used in household wiring, has a resistance at room temperature of about 0.5 Ω . A 100 –W, 12-V light bulb has a resistance (at operating temperature) of . 140 Ω If the same current *I* flows in both the copper wire and the light bulb, the potential difference U = IR is much greater across the light bulb, and much more potential energy is lost per charge in the light bulb. This lost energy is converted by the light bulb filament into light and heat. You don't want your household wiring to glow whitehot, so its resistance is kept low by using wire of low resistivity and large cross-sectional area.

Because the resistivity of a material varies with temperature, the resistance of a specific conductor also varies with temperature. For temperature ranges that are not too great, this variation is approximately a linear relationship, analogous to Eq. (325):

$$R(T) = R_0 [1 + \alpha (T - T_0)]$$
(332)

In this equation, R(T) is the resistance at temperature T and R_0 is the resistance at temperature T_0 often taken to be 0°C or 100°C. The *temperature coefficient of resistance* α is the same constant that appears in Eq. (325) if the dimensions L and S in Eq. (330) do not change appreciably with temperature; this is indeed the case for most conducting materials. Within the limits of validity of Eq. (332), the *change* in resistance resulting from a temperature change $T - T_0$ is given by $R\alpha(T - T_0)$.

A circuit device made to have a specific value of resistance between its ends is called a **resistor**. Resistors in the range 0.01 to $10^7 \Omega$ can be bought off the shelf. Individual resistors used in electronic circuitry are often cylindrical, a few millimeters in diameter and length, wit h wires coming out of the ends. The resistance may be marked with a standard code using three or four color bands near one end (see fig. 204), according to the scheme shown in Table 10. The first two bands (starting with the band nearest an end) are digits, and the third is a power-of-10 multiplier, as shown in Fig. 204. For example, green–violet–red means $57 \cdot 10^2 \Omega$, or 5.7 k Ω . The fourth band, if present, indicates the precision (tolerance) of the value; no band means $\pm 20\%$, a silver band $\pm 10\%$ and $\pm 5\%$ a gold band Another important characteristic of a resistor is the maximum *power* it can dissipate without damage.



Figure 204 – This resistor has a resistance of 5.7 $k\Omega$ with a precision (tolerance) of $\pm 10\%$

Color	Value as digit	Value as muliplier		
Black	0	1		
Brown	1	10		
Red	2	10^{2}		
Orange	3	10^{3}		
Yellow	4	10^{4}		
Green	5	10^{5}		
Blue	6	10^{6}		
Violet	7	10 ⁷		
Gray	8	10 ⁸		
White	9	10 ⁹		

Table 10 – Color codes for resistors

For a resistor that obeys Ohm's law, a graph of current as a function of potential difference (voltage) is a straight line (see fig. 205a). The slope of the line is 1/R. If the sign of the potential difference changes, so does the sign of the current produced; in Fig. 203 this corresponds to interchanging the higherand lower-potential ends of the conductor, so the electric field, current density, and current all reverse direction. In devices that do not obey Ohm's law, the relationship of voltage to current may not be a direct proportion, and it may be different for the two directions of current. Figure 205b shows the behavior of a semiconductor diode, a device used to convert alternating current to direct current and to perform a wide variety of logic functions in computer circuitry. For positive potentials of the anode (one of two terminals of the diode) with respect to the cathode (the other terminal), increases exponentially with increasing for negative potentials the current is extremely small. Thus a positive *U* causes a current to flow in the positive direction, but a potential difference of the other sign causes little or no current. Hence a diode acts like a one-way valve in a circuit.





3.3.3 Electromotive force and circuits

For a conductor to have a steady current, it must be part of a path that forms a closed loop or **complete circuit.** Here's why. If you establish an electric field \vec{E}_1 inside an isolated conductor with resistivity ρ that is *not* part of a complete circuit, a current begins to flow with current density $\vec{j} = \vec{E}_1/\rho$ (Fig 206a). As a result a net positive charge quickly accumulates at one end of the conductor and a net negative charge accumulates at the other end (Fig 206b). These charges themselves produce an electric field \vec{E}_2 in the direction opposite to \vec{E}_1 , causing the total electric field and hence the current to decrease. Within a very small fraction of a second, enough charge builds up on the conductor ends that the total electric field $\vec{E} = \vec{E}_1 + \vec{E}_2$ inside the conductor. Then $\vec{j} = 0$ as well, and the current stops altogether (Fig 206c). So there can be no steady motion of charge in such an *incomplete* circuit.

To see how to maintain a steady current in a *complete* circuit, we recall a basic fact about electric potential energy: If a charge goes around a complete circuit and returns to its starting point, the potential energy must be the same at the end of the round trip as at the beginning. As described early, there is always a *decrease* in potential energy when charges move through an ordinary conducting material with resistance. So there must be some part of the circuit in which the potential energy *increases*.

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(a) An electric field E₁ produced inside an isolated conductor causes a current.







The charge buildup produces an opposing field \vec{E}_2 , thus reducing the current.

(c) After a very short time \vec{E}_2 has the same magnitude as \vec{E}_1 ; then the total field is $\vec{E}_{total} = 0$ and the current stops completely.

$$\vec{E}_1 = 0 \quad \vec{E}_1 \longrightarrow \vec{E}_2$$

$$\vec{J} = 0 \quad \vec{E}_{\text{total}} = 0$$

Figure 206 – If an electric field is produced inside a conductor that is not part of a complete circuit, current flows for only a very short time

The problem is analogous to an ornamental water fountain that recycles its water. The water pours out of openings at the top, cascades down over the terraces and spouts (moving in the direction of decreasing gravitational potential energy), and collects in a basin in the bottom. A pump then lifts it back to the top (increasing the potential energy) for another trip. Without the pump, the water would just fall to the bottom and stay there.

In an electric circuit there must be a device somewhere in the loop that acts like the water pump in a water fountain. In this device a charge travels "uphill," from lower to higher potential energy, even though the electrostatic force is trying to push it from higher to lower potential energy. The direction of current in such a device is from lower to higher potential, just the opposite of what happens in an ordinary conductor. The influence that makes current flow from lower to higher potential is called **electromotive force** (abbreviated **emf** and pronounced "ee-emeff"). This is a poor term because emf is *not* a force but an energy-per-unit-charge quantity, like potential. The SI unit of emf is the same as that for potential, the volt (1 V = 1 J/C). A typical flashlight battery has an emf of 1.5 V; this means that the battery does 1.5 J of work on every coulomb of charge that passes through it. We'll use the symbol \mathscr{E} (a script capital E) for emf.

Every complete circuit with a steady current must include some device that provides emf. Such a device is called a **source of emf.** Batteries, electric generators, solar cells, thermocouples, and fuel cells are all examples of sources of emf. All such devices convert energy of some form (mechanical, chemical, thermal, and so on) into electric potential energy and transfer it into the circuit to which the device is connected. An *ideal* source of emf maintains a constant potential difference between its terminals, independent of the current through it. We define electromotive force quantitatively as the magnitude of this potential difference. As we will see, such an ideal source is a mythical beast, like the frictionless plane and the massless rope. We will discuss later how real-life sources of emf differ in their behavior from this idealized model.

Figure 207 is a schematic diagram of an ideal source of emf that maintains a potential difference between conductors *a* and *b*, called the *terminals* of the device. Terminal *a*, marked +, is maintained at *higher* potential than terminal *b*, marked -. Associated with this potential difference is an electric field \vec{E} in the region around the terminals, both inside and outside the source. The electric field inside the device is directed from *a* to *b*, as shown. A charge *q* within the source experiences an electric force $\vec{F_e} = q\vec{E}$. But the source also provides an additional influence, which we represent as a nonelectrostatic force $\vec{F_n}$. This force, operating inside the device, pushes charge from to in an "uphill" direction against the electric force $\vec{F_e}$. Thus $\vec{F_n}$ maintains the potential difference between the terminals. If were not present, charge would flow between the terminals until the potential difference was zero. The origin of the additional influence $\vec{F_n}$ depends on the kind of source. In a generator it results from magnetic-field forces on moving charges. In a battery or fuel cell it is associated with diffusion processes and varying electrolyte concentrations resulting from chemical reactions. In an electrostatic machine such as a Van de Graaff generator, an actual mechanical force is applied by a moving belt or wheel.



circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

Figure 207 – Schematic diagram of a source of emf in an "open-circuit" situation. The electric-field force $\vec{F}_e = q\vec{E}$ and the nonelectrostatic force \vec{F}_n are shown for a positive charge q

If a positive charge q is moved from to inside the source, the nonelectrostatic force \vec{F}_n does a positive amount of work $W_n = q\mathcal{E}$ on the charge. This displacement is *opposite* to the electrostatic force \vec{F}_e , so the potential energy associated with the charge *increases* by an amount equal to qU, where $U = \varphi_a - \varphi_b$ is the (positive) potential of point a with respect to point b. For the ideal source of emf that we've described, \vec{F}_e and \vec{F}_n are equal in magnitude but opposite in direction, so the total work done on the charge q is zero; there is an increase in potential energy but *no* change in the kinetic energy of the charge. It's like lifting a book from the floor to a high shelf at constant speed. The increase in potential energy is just equal to the nonelectrostatic work W_n , so $q\mathcal{E} = qU$, or

$$U = \mathcal{E} \tag{333}$$

Now let's make a complete circuit by connecting a wire with resistance R to the terminals of a source (see fig. 208). The potential difference between terminals a and b sets up an electric field within the wire; this causes current to flow around the loop from a toward b, from higher to lower potential. Where the wire bends, equal amounts of positive and negative charge persist on the "inside" and "outside" of the bend. These charges exert the forces that cause the current to follow the bends in the wire.

From Eq. (331) the potential difference between the ends of the wire in Fig. 208 is given by U = IR. Combining with Eq. (333), we have

$$\mathcal{E} = U = IR \tag{334}$$

That is, when a positive charge q flows around the circuit, the potential *rise* \mathscr{E} as it passes through the ideal source is numerically equal to the potential *drop* U = IR as it passes through the remainder of the circuit. Once \mathscr{E} and R are known, this relationship determines the current in the circuit.



Figure 208 - Schematic diagram of an ideal source of emf in a complete circuit. The electric-field force $\vec{F}_e = q\vec{E}$ and the nonelectrostatic force \vec{F}_n are shown for a positive charge q. The current is in the direction from a to b in the external circuit and from b to a within the source

Real sources of emf in a circuit don't behave in exactly the way we have described; the potential difference across a real source in a circuit is *not* equal to the emf as in Eq. (334). The reason is that charge moving through the material of any real source encounters *resistance*. We call this the **internal resistance** of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant and independent of the current I. As the current moves through , it

experiences an associated drop in potential equal to Ir. Thus, when a current is flowing through a source from the negative terminal b to the positive terminal a, the potential difference U between the terminals is

$$U = \mathcal{E} - Ir \tag{335}$$

The potential φ_{ab} , called the **terminal voltage**, is less than the emf \mathscr{E} because of the term Ir representing the potential drop across the internal resistance r. Expressed another way, the increase in potential energy qU as a charge q moves from b to a within the source is now less than the work $q\mathcal{E}$ done by the nonelectrostatic force \vec{F}_n since some potential energy is lost in traversing the internal resistance.

A 1.5-battery has an emf of 1.5 V, but the terminal voltage U of the battery is equal to 1.5 V only if no current is flowing through it so that I = 0 in Eq. (335). If the battery is part of a complete circuit through which current is flowing, the terminal voltage will be less than 1.5 V. For a real source of emf, the terminal voltage equals the emf only if no current is flowing through the source. Thus we can describe the behavior of a source in terms of two properties: an emf & which supplies a constant potential difference independent of current, in series with an internal resistance r.

The current in the external circuit connected to the source terminals *a* and *b* is still determined by U = IR. Combining this with Eq. (335), we find

$$\mathcal{E} - Ir = IR \tag{336}$$

or

$$I = \frac{\mathcal{E}}{R+r}$$
(337)

That is, the current equals the source emf divided by *total* circuit resistance (R + r).

The net change in potential energy for a charge making a round trip around a complete circuit must be zero. Hence the net change in *potential* around the circuit must also be zero; in other words, the algebraic sum of the potential differences and emfs around the loop is zero. We can see this by rewriting Eq. (337) in the form

$$\mathcal{E} - Ir - IR = 0 \tag{338}$$

A potential gain \mathscr{E} of is associated with the emf, and potential drops of Ir and IR are associated with the internal resistance of the source and the external circuit, respectively. Figure 209 is a graph showing how the potential varies as we go around the complete circuit. The horizontal axis doesn't necessarily represent actual distances, but rather various points in the loop. If we take the potential to be zero at the negative terminal of the battery, then we have a rise \mathscr{E} and a drop Ir in
the battery and an additional drop in the external resistor, and as we finish our trip around the loop, the potential is back where it started.



Figure 209 – Potential rise and drops in a circuit

In this section we have considered only situations in which the resistances are ohmic. If the circuit includes a nonlinear device such as a diode (see Fig. 205b), Eq. (337) is still valid but cannot be solved algebraically because R is not a constant. In such a situation, the current I can be found by using numerical techniques.

Finally, we remark that Eq. (335) is not always an adequate representation of the behavior of a source. The emf may not be constant, and what we have described as an internal resistance may actually be a more complex voltage–current relationship that doesn't obey Ohm's law. Nevertheless, the concept of internal resistance frequently provides an adequate description of batteries, generators, and other energy converters. The principal difference between a fresh flashlight battery and an old one is not in the emf, which decreases only slightly with use, but in the internal resistance, which may increase from less than an ohm when the battery is fresh to as much as 1000Ω or more after long use. Similarly, a car battery can deliver less current to the starter motor on a cold morning than when the battery is warm, not because the emf is appreciably less but because the internal resistance increases with decreasing temperature.

3.3.4 Energy and power in electric circuits

Let's now look at some energy and power relationships in electric circuits. The box in Fig. 210 represents a circuit element with potential difference $\varphi_a - \varphi_b = U$ between its terminals and current *I* passing through it in the direction *a* from *b* toward. This element might be a resistor, a battery, or something else; the details don't matter. As charge passes through the circuit element, the electric field does work on the charge. In a source of emf, additional work is done by the force \vec{F}_n .



Figure 210 – The power input to the circuit element between *a* and *b* $P = (\varphi_a - \varphi_b)I = UI$

As an amount of charge q passes through the circuit element, there is a change in potential energy equal to qU. For example, if q > 0 and $U = \varphi_a - \varphi_b$ is positive, potential energy decreases as the charge "falls" from potential φ_a to lower potential φ_b . The moving charges don't gain *kinetic* energy, because the current (the rate of charge flow) out of the circuit element must be the same as the current into the element. Instead, the quantity qU represents energy transferred into the circuit element. This situation occurs in the coils of a toaster or electric oven, in which electrical energy is converted to thermal energy.

If the potential at a is lower than at b, then U is negative and there is a net transfer of energy *out* of the circuit element. The element then acts as a source, delivering electrical energy into the circuit to which it is attached. This is the usual situation for a battery, which converts chemical energy into electrical energy and delivers it to the external circuit. Thus U can denote either a quantity of energy delivered to a circuit element or a quantity of energy extracted from that element.

In electric circuits we are most often interested in the *rate* at which energy is either delivered to or extracted from a circuit element. If the current through the element is I, then in a time interval dt an amount of charge dQ = Idt passes through the element. The potential energy change for this amount of charge is UdQ = UIdt. Dividing this expression by dt, we obtain the *rate* at which energy is transferred either into or out of the circuit element. The time rate of energy transfer is *power*, denoted by P, so we write

$$P = UI \tag{339}$$

(rate at which energy is delivered to or extracted from a circuit)

The unit of *U* is one volt, or one joule per coulomb, and the unit of *I* is one ampere, or one coulomb per second. Hence the unit of P = UI is one watt, as it should be:

$$(1 J/C)(1 C/s) = 1 J/s = 1 W$$

Let's consider a few special cases.

If the circuit element in Fig. 210 is a resistor, the potential difference is U = IR From Eq. (339) the electrical power delivered to the resistor by the circuit is

$$P = UI = I^2 R = \frac{U^2}{R}$$
(340)

In this case the potential at a (where the current enters the resistor) is always higher than that at b (where the current exits). Current enters the higher-potential terminal of the device, and Eq. (340) represents the rate of transfer of electric potential energy *into* the circuit element.

What becomes of this energy? The moving charges collide with atoms in the resistor and transfer some of their energy to these atoms, increasing the internal energy of the material. Either the temperature of the resistor increases or there is a flow of heat out of it, or both. In any of these cases we say that energy is *dissipated* in the resistor at a rate I^2R . Every resistor has a *power rating*, the maximum power the device can dissipate without becoming overheated and damaged. Some devices, such as electric heaters, are designed to get hot and transfer heat to their surroundings. But if the power rating is exceeded, even such a device may melt or even explode.

The upper rectangle in Fig. 211a represents a source with emf \mathscr{E} and internal resistance r, connected by ideal (resistanceless) conductors to an external circuit represented by the lower box. This could describe a car battery connected to one of the car's headlights (see fig. 211b). Point a is at higher potential than point b, so $\varphi_a > \varphi_b$ and U is positive. Note that the current I is *leaving* the source at the higher-potential terminal (rather than entering there). Energy is being delivered to the external circuit, at a rate given by Eq. (339):

$$P = UI \tag{341}$$

For a source that can be described by an emf \mathscr{E} and an internal resistance r, we may use Eq. (335):

$$U = \mathcal{E} - Ir \tag{342}$$

Multiplying this equation by *I*, we find

$$P = UI = \mathcal{E}I - I^2 r \tag{343}$$

What do the terms $\mathcal{E}I$ and I^2r mean? We defined the emf \mathscr{E} as the work per unit charge performed on the charges by the nonelectrostatic force as the charges are pushed "uphill" from b to a in the source. In a time dt, a charge dQ = Idtflows through the source; the work done on it by this nonelectrostatic force is $\mathcal{E}dQ = \mathcal{E}Idt$. Thus $\mathcal{E}I$ is the *rate* at which work is done on the circulating charges by whatever agency causes the nonelectrostatic force in the source. This term represents the rate of conversion of nonelectrical energy to electrical energy within the source. The term I^2r is the rate at which electrical energy is dissipated in the internal resistance of the source. The difference $\mathcal{E}I - I^2r$ is the *net* electrical energy to the remainder of the circuit.

(a) Diagrammatic circuit



(b) A real circuit of the type shown in (a)



Figure 211 – Energy conservation in a simple circuit

Suppose that the lower rectangle in Fig. 211a is itself a source, with an emf *larger* than that of the upper source and with its emf opposite to that of the upper source. Figure 212 shows a practical example, an automobile battery (the upper circuit element) being charged by the car's alternator (the lower element). The current I in the circuit is then *opposite* to that shown in Fig. 211; the lower source is pushing current backward through the upper source. Because of this reversal of current, instead of Eq. (335) we have for the upper source

$$U = \mathcal{E} + Ir \tag{344}$$

and instead of Eq. (341), we have

$$P = UI = \mathcal{E}I + I^2 r \tag{345}$$

Work is being done *on*, rather than by, the agent that causes the nonelectrostatic force in the upper source. There is a conversion of electrical energy into nonelectrical energy in the upper source at a rate $\mathcal{E}I$. The term I^2r in Eq. (345) is again the rate of dissipation of energy in the internal resistance of the upper source, and the sum $\mathcal{E}I + I^2r$ is the total electrical power input to the upper source. This is what happens when a rechargeable battery (a storage battery) is connected to a charger. The charger supplies electrical energy to the battery; part of it is converted to chemical energy, to be reconverted later, and the remainder is dissipated (wasted) in the battery's internal resistance, warming the battery and causing a heat flow out of it. If you have a power tool or laptop computer with a rechargeable battery, you may have noticed that it gets warm while it is charging.



Figure 212 – When two sources are connected in a simple loop circuit, the source with the larger emf delivers energy to the other source

3.4 Capacitance and dielectrics

3.4.1 Capacitors and Capacitance

Any two conductors separated by an insulator (or a vacuum) form a capacitor (see fig. 213). In most practical applications, each conductor initially has zero net charge and electrons are transferred from one conductor to the other; this is called charging the capacitor. Then the two conductors have charges with equal magnitude and opposite sign, and the net charge on the capacitor as a whole remains zero. We will assume throughout this chapter that this is the case. When we say that a capacitor has charge Q, or that a charge Q is stored on the capacitor, we mean that the conductor at higher potential has charge +Q and the conductor at lower potential has charge -Q (assuming that Q is positive). Keep this in mind in the following discussion and examples.



Figure 213 – Any two conductors a and b insulated from each other form a capacitor

In circuit diagrams a capacitor is represented by either of these symbols: $\neg \vdash \neg \leftarrow$. In either symbol the vertical lines (straight or curved) represent the conductors and the horizontal lines represent wires connected to either conductor. One common way to charge a capacitor is to connect these two wires to opposite terminals of a battery. Once the charges Q and -Q are established on the conductors, the battery is disconnected. This gives a fixed *potential difference U* between the conductors (that is, the potential of the positively charged conductor a with respect to the negatively charged conductor b) that is just equal to the voltage of the battery.

The electric field at any point in the region between the conductors is proportional to the magnitude Q of charge on each conductor. It follows that the *potential difference U* between the conductors is also proportional to Q. If we double the magnitude of charge on each conductor, the charge density at each point doubles, the electric field at each point doubles, and the potential difference between conductors doubles; however, the *ratio* of charge to potential difference does not change. This ratio is called the **capacitance** C of the capacitor:

$$C = \frac{Q}{U} \tag{346}$$

The SI unit of capacitance is called one **farad** (1 F), in honor of the 19th-century English physicist Michael Faraday. From Eq. (346), one farad is equal to one *coulomb per volt* (1 C/V):

The greater the capacitance C of a capacitor, the greater the magnitude Q of charge on either conductor for a given potential difference U and hence the greater the amount of stored energy. (Remember that potential is potential energy per unit charge.) Thus *capacitance is a measure of the ability of a capacitor to store energy*. We will see that the value of the capacitance depends only on the shapes and sizes of the conductors and on the nature of the insulating material between them. (The above remarks about capacitance being independent of Q and U do not apply to certain special types of insulating materials. We won't discuss these materials in this book, however.)

We can calculate the capacitance C of a given capacitor by finding the potential difference U between the conductors for a given magnitude of charge Q and then using Eq. (346). For now we'll consider only *capacitors in vacuum*; that is, we'll assume that the conductors that make up the capacitor are separated by empty space.

The simplest form of capacitor consists of two parallel conducting plates, each with area S, separated by a distance d that is small in comparison with their dimensions (see fig. 214a). When the plates are charged, the electric field is almost completely localized in the region between the plates (see fig. 214b). As we discussed the field between such plates is essentially *uniform*, and the charges on the plates are uniformly distributed over their opposing surfaces. We call this arrangement a **parallel-plate capacitor**.

(a) Arrangement of the capacitor plates



(b) Side view of the electric field \vec{E}



Figure 214- A charged parallel-plate capacitor

We worked out the electric-field magnitude for this arrangement using the principle of superposition of electric fields and again using Gauss's law. It would be a good idea to review those examples. We found that $E = \sigma/\varepsilon_0$, where σ is the magnitude (absolute value) of the surface charge density on each plate. This is equal to the magnitude of the total charge Q on each plate divided by the area S of the plate, or $\sigma = Q/S$ so the field magnitude E can be expressed as

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 S} \tag{347}$$

The field is uniform and the distance between the plates is d, so the potential difference (voltage) between the two plates is

$$U = Ed = \frac{Qd}{\varepsilon_0 S} \tag{348}$$

From this we see that the capacitance C of a parallel-plate capacitor in vacuum is

$$C = \frac{Q}{U} = \varepsilon_0 \frac{S}{d} \tag{349}$$

The capacitance depends only on the geometry of the capacitor; it is directly proportional to the area *S* of each plate and inversely proportional to their separation *d*. The quantities *S* and *d* are constants for a given capacitor, and is ε_0 a universal constant. Thus in vacuum the capacitance *C* is a constant independent of the charge on the capacitor or the potential difference between the plates. If one of the capacitor plates is flexible, the capacitance *C* changes as the plate separation *d* changes. This is the operating principle of a condenser microphone.

When matter is present between the plates, its properties affect the capacitance. Meanwhile, we remark that if the space contains air at atmospheric pressure instead of vacuum, the capacitance differs from the prediction of Eq. (349) by less than 0.06%.

In Eq. (349), if S is in square meters and d in meters, C is in farads. The units ε_0 of are $C^2/N^2 \cdot m^2$ so we see that

$$1 F = \frac{C^2}{N^2 \cdot m} = 1 \frac{C^2}{J}$$

Because 1 V = 1 J/C (energy per unit charge), this is consistent with our definition 1 F = 1 C/V. Finally, the units of ε_0 can be expressed as $1 \frac{C^2}{N^2 \cdot m} = 1 F/m$, so

$$\varepsilon_0 = 8.85 \cdot 10^{-12} \, F/m$$

This relationship is useful in capacitance calculations, and it also helps us to verify that Eq. (349) is dimensionally consistent.

One farad is a very large capacitance, as the following example shows. In many applications the most convenient units of capacitance are the *microfarad* $(1 \ \mu F = 10^{-6} F)$ and the *picofarad* $(1 \ pF = 10^{-12} F)$. For example, the flash unit in a point-and-shoot camera uses a capacitor of a few hundred microfarads(see fig. 215), while capacitances in a radio tuning circuit are typically from 10 to 100 picofarads.



Figure 215 – A commercial capacitor is labelled with value of its capacitance. For these capacitors, $C = 2200 \ \mu F$, 1000 μF , 470 μF

For any capacitor in vacuum, the capacitance C depends only on the shapes, dimensions, and separation of the conductors that make up the capacitor. If the conductor shapes are more complex than those of the parallel-plate capacitor, the expression for capacitance is more complicated than in Eq. (349). In the following examples we show how to calculate C for two other conductor geometries.

Capacitors are manufactured with certain standard capacitances and working voltages. However, these standard values may not be the ones you actually need in a particular application. You can obtain the values you need by combining capacitors; many combinations are possible, but the simplest combinations are a series connection and a parallel connection.

Figure 216a is a schematic diagram of a **series connection.** Two capacitors are connected in series (one after the other) by conducting wires between points *a* and *b*. Both capacitors are initially uncharged. When a constant positive potential difference *U* is applied between points *a* and *b* the capacitors become charged; the figure shows that the charge on *all* conducting plates has the same magnitude. To see why, note first that the top plate of C_1 acquires a positive charge *Q*. The electric field of this positive charge pulls negative charge up to the bottom plate of C_1 until all of the field lines that begin on the top plate end on the bottom plate. This requires that the bottom plate have charge -Q. These negative charges had to come from the top plate of C_2 , which becomes positively charged with charge +Q. This positive charge then pulls negative charge -Q from the connection at point *b* onto the bottom plate of C_2 . The total charge on the lower plate of C_1 and the upper plate of C_2 together must always be zero because these plates aren't connected to anything except each other. Thus *in a series connection the magnitude of charge on all plates is the same*.

Referring to Fig. 216a, we can write the potential differences between points a and c, c and b, and a and b as

$$U_{ac} = U_1 = \frac{Q}{C_1}$$
 $U_{cb} = U_2 = \frac{Q}{C_2}$ (350)

$$U_{ab} = U = U_1 + U_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$
(351)

and so

$$\frac{U}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$
(352)

Following a common convention, we use the symbols U_1, U_2 and U to denote the potential *differences* U_{ac} (across the first capacitor), U_{cb} (across the second capacitor), and U_{ab} (across the entire combination of capacitors), respectively.



The equivalent capacitance C_{eq} of the series combination is defined as the capacitance of a *single* capacitor for which the charge Q is the same as for the

combination, when the potential difference U is the same. In other words, the combination can be replaced by an *equivalent capacitor* of capacitance C_{eq} .For such a capacitor, shown in Fig. 216b,

$$C_{eq} = \frac{Q}{V} \tag{353}$$

or

$$\frac{1}{C_{eq}} = \frac{U}{Q} \tag{354}$$

Combining Eqs. (352) and (354), we find

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$
(355)

We can extend this analysis to any number of capacitors in series. We find the following result for the *reciprocal* of the equivalent capacitance:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$
(356)

The reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances. In a series connection the equivalent capacitance is always *less than* any individual capacitance.

The arrangement shown in Fig. 216a is called a **parallel connection.** Two capacitors are connected in parallel between points *a* and *b*. In this case the upper plates of the two capacitors are connected by conducting wires to form an equipotential surface, and the lower plates form another. Hence *in a parallel connection the potential difference for all individual capacitors is the same* and is equal to $U_{ab} = U$. The charges Q_1 and Q_2 are not necessarily equal, however, since charges can reach each capacitor independently from the source (such as a battery) of the voltage U_{ab} . The charges are

$$Q_1 = C_1 U \tag{357}$$

or

$$Q_2 = C_2 U \tag{368}$$

The *total* charge Q of the combination, and thus the total charge on the equivalent capacitor, is

$$Q = Q_1 + Q_2 = (C_1 + C_2)U \tag{359}$$

so

$$\frac{Q}{U} = C_1 + C_2 \tag{360}$$

The parallel combination is equivalent to a single capacitor with the same total charge $Q = Q_1 + Q_2$ and potential difference U as the combination (see fig. 216b). The equivalent capacitance of the combination, C_{eq} is the same as the capacitance Q/U of this single equivalent capacitor. So from Eq. (360),

$$C_{eq} = C_1 + C_2 (361)$$

In the same way we can show that for any number of capacitors in parallel,

$$C_{eq} = C_1 + C_2 + C_3 + \cdots. (362)$$

The equivalent capacitance of a parallel combination equals the sum of the individual capacitances. In a parallel connection the equivalent capacitance is always *greater than* any individual capacitance.

3.4.2 Energy storage in capacitors and electric-field energy

Many of the most important applications of capacitors depend on their ability to store energy. The electric potential energy stored in a charged capacitor is just equal to the amount of work required to charge it—that is, to separate opposite charges and place them on different conductors. When the capacitor is discharged, this stored energy is recovered as work done by electrical forces.

We can calculate the potential energy E_p of a charged capacitor by calculating the work W required to charge it. Suppose that when we are done charging the capacitor, the final charge Q is and the final potential difference is U, From Eq. (346) these quantities are related by

$$U = \frac{Q}{C} \tag{363}$$

Let q and v be the charge and potential difference, respectively, at an intermediate stage during the charging process; then v = q/C. At this stage the work dW required to transfer an additional element of charge dq is

$$dW = vdq = \frac{qdq}{C} \tag{364}$$

The total work W needed to increase the capacitor charge q from zero to a final value Q is

$$W = \int_{0}^{W} dW = \frac{1}{C} \int_{0}^{Q} q dq = \frac{Q^{2}}{2C}$$
(365)

This is also equal to the total work done by the electric field on the charge when the capacitor discharges. Then q decreases from an initial value Q to zero as the elements of charge dq "fall" through potential differences v that vary from V down to zero.

If we define the potential energy of an *uncharged* capacitor to be zero, then W in Eq. (365) is equal to the potential energy E_p of the charged capacitor. The final stored charge is Q = CU, so we can express E_p (which is equal to W) as

$$E_p = \frac{Q^2}{2C} = \frac{1}{2}CU^2 = \frac{1}{2}QU$$
(366)

When Q is in coulombs, C in farads (coulombs per volt), and U in volts (joules per coulomb), E_p is in joules.

The last form of Eq. (366), $E_p = \frac{1}{2}QU$, shows that the total work W required to charge the capacitor is equal to the total charge Q multiplied by the *average* potential difference $\frac{1}{2}U$ during the charging process.

The expression $E_p = \frac{Q^2}{2c}$ in Eq. (366) shows that a charged capacitor is the electrical analog of a stretched spring with elastic potential energy $E_p = \frac{kx^2}{2}$. The charge Q is analogous to the elongation x and the *reciprocal* of the capacitance, 1/C, is analogous to the force constant k. The energy supplied to a capacitor in the charging process is analogous to the work we do on a spring when we stretch it.

Equations (365) and (366) tell us that capacitance measures the ability of a capacitor to store both energy and charge. If a capacitor is charged by connecting it to a battery or other source that provides a fixed potential difference U, then increasing the value of C gives a greater charge Q = CU and a greater amount of stored energy $E_p = \frac{Q^2}{2C}$. If instead the goal is to transfer a given quantity of charge Q from one conductor to another, Eq. (365) shows that the work W required is inversely proportional to C; the greater the capacitance, the easier it is to give a capacitor a fixed amount of charge.

We can charge a capacitor by moving electrons directly from one plate to another. This requires doing work against the electric field between the plates. Thus we can think of the energy as being stored *in the field* in the region between the plates. To develop this relationship, let's find the energy *per unit volume* in the space between the plates of a parallel-plate capacitor with plate area S and separation d. We call this the **energy density**, denoted by u. From Eq. (366) the

total stored potential energy is $\frac{1}{2}CU^2$ and the volume between the plates is just hence the energy density is

$$u = \text{Energy density} = \frac{\frac{1}{2}CU^2}{Sd}$$
(367)

From Eq. (349) the capacitance C is given by $C = \varepsilon_0 S/d$. The potential difference U is related to the electric-field magnitude E by U = Ed. If we use these expressions in Eq. (367), the geometric factors S and d cancel, and we find

$$u = \frac{1}{2}\varepsilon_0 E^2 \tag{368}$$

Although we have derived this relationship only for a parallel-plate capacitor, it turns out to be valid for any capacitor in vacuum and indeed *for any electric field configuration in vacuum*. This result has an interesting implication. We think of vacuum as space with no matter in it, but vacuum can nevertheless have electric fields and therefore energy. Thus "empty" space need not be truly empty after all. We will use this idea and Eq. (368) in Chapter 32 in connection with the energy transported by electromagnetic waves.

3.4.3 Dielectrics

Most capacitors have a nonconducting material, or **dielectric**, between their conducting plates. A common type of capacitor uses long strips of metal foil for the plates, separated by strips of plastic sheet such as Mylar. A sandwich of these materials is rolled up, forming a unit that can provide a capacitance of several microfarads in a compact package (see fig. 217).



Figure 217 – A common type of capacitor uses dielectric sheets to separate the conductors

Placing a solid dielectric between the plates of a capacitor serves three functions. First, it solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contact. Second, using a dielectric increases the maximum possible potential difference between the capacitor plates. Any insulating material, when subjected to a sufficiently large electric field, experiences a partial ionization that permits conduction through it. This is called **dielectric breakdown**. Many dielectric materials can tolerate stronger electric fields without breakdown than can air. Thus using a dielectric allows a capacitor to sustain a higher potential difference U and so store greater amounts of charge and energy.

Third, the capacitance of a capacitor of given dimensions is greater when there is a dielectric material between the plates than when there is vacuum. We can demonstrate this effect with the aid of a sensitive electrometer, a device that measures the potential difference between two conductors without letting any appreciable charge flow from one to the other. Figure 218a shows an electrometer connected across a charged capacitor, with magnitude of charge Q on each plate and potential difference U_0 . When we insert an uncharged sheet of dielectric, such as glass, paraffin, or polystyrene, between the plates, experiment shows that the potential difference decreases to a smaller value U (see fig. 218b). When we remove the dielectric, the potential difference returns to its original value U_0 showing that the original charges on the plates have not changed.



Figure 218 – Effect of dielectric between the plates of a parallel-plate capacitor. (a) With a given charge, the potential difference is U_0 . (b) With the same charge but with a dielectric between the plates, the potential difference U is smaller than U_0

The original capacitance C_0 is given by $C_0 = Q/U_0$ and the capacitance C with the dielectric present is C = Q/U. The charge Q is the same in both cases, and

U is less than U_0 , so we conclude that the capacitance *C* with the dielectric present is *greater* than C_0 . When the space between plates is completely filled by the dielectric, the ratio of *C* to C_0 (equal to the ratio of U_0 to *U*) is called the **dielectric constant** of the material, ε :

$$\varepsilon = \frac{C}{C_0} \tag{369}$$

When the charge is constant, $Q = C_0 U_0 = CV$ and $C/C_0 = U_0/U$ In this case, Eq. (369) can be rewritten as

$$U = \frac{U_0}{\varepsilon} \tag{370}$$

With the dielectric present, the potential difference for a given charge Q is *reduced* by a factor ε .

The dielectric constant ε is a pure number. Because *C* is always greater than C_0 , ε is always greater than unity. Some representative values of ε are given in Table 11. For vacuum, $\varepsilon = 1$ by definition. For air at ordinary temperatures and pressures, ε is about 1.0006; this is so nearly equal to 1 that for most purposes an air capacitor is equivalent to one in vacuum. Note that while water has a very large value of ε , it is usually not a very practical dielectric for use in capacitors.

The reason is that while pure water is a very poor conductor, it is also an excellent ionic solvent. Any ions that are dissolved in the water will cause charge to flow between the capacitor plates, so the capacitor discharges.

Material	Е	Material	З
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.4
Air (100 atm)	1.0548	Glass	5-10
Teflon	2.1	Neoprene	6.7
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3-6	Water	80.4
Mylar	3.1	Strontium titanate	310

Table 11 - Values of dielectric constant ε at 20°C

No real dielectric is a perfect insulator. Hence there is always some *leakage current* between the charged plates of a capacitor with a dielectric. We tacitly ignored this effect when we derived expressions for the equivalent capacitances of capacitors in series, Eq. (356), and in parallel, Eq. (362). But if a leakage current flows for a long enough time to substantially change the charges from the values we used to derive Eqs. (356) and (362), those equations may no longer be accurate.

When a dielectric material is inserted between the plates while the charge is kept constant, the potential difference between the plates decreases by a factor ε . Therefore the electric field between the plates must decrease by the same factor. If E_0 is the vacuum value and E is the value with the dielectric, then

$$E = \frac{E_0}{\varepsilon} \tag{371}$$

Since the electric-field magnitude is smaller when the dielectric is present, the surface charge density (which causes the field) must be smaller as well. The surface charge on the conducting plates does not change, but an *induced* charge of the opposite sign appears on each surface of the dielectric (see fig. 219). The dielectric was originally electrically neutral and is still neutral; the induced surface charges arise as a result of *redistribution* of positive and negative charge

within the dielectric material, a phenomenon called **polarization.** We will assume that the induced surface charge is *directly proportional* to the electric-field magnitude E in the material; this is indeed the case for many common dielectrics. (This direct proportionality is analogous to Hooke's law for a spring.) In that case, ε is a constant for any particular material. When the electric field is very strong or if the dielectric is made of certain crystalline materials, the relationship between induced charge and the electric field can be more complex; we won't consider such cases here.



Figure 219 – Electric field lines with (a) vacuum between the plates and (b) dielectric between the plates

We can derive a relationship between this induced surface charge and the charge on the plates. Let's denote the magnitude of the charge per unit area induced on the surfaces of the dielectric (the induced surface charge density) by.

The magnitude of the surface charge density on the capacitor plates is, as usual. Then the net surface charge on each side of the capacitor has magnitude $(\sigma - \sigma_1)$ as shown in Fig. 219b. As known, the field between the plates is related to the net surface charge density by $E = \sigma_{net}/\varepsilon_0$. Without and with the dielectric, respectively, we have

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{\sigma - \sigma_i}{\varepsilon_0} \tag{372}$$

Using these expressions in Eq. (371) and rearranging the result, we find

$$\sigma_i = \sigma \left(1 - \frac{1}{\varepsilon} \right) \tag{373}$$

This equation shows that when ε is very large, σ_i is nearly as large as ε . In this case, σ_i nearly cancels σ and the field and potential difference are much smaller than their values in vacuum.

The product $\varepsilon \varepsilon_0$ is called the permittivity of the dielectric, denoted by ϵ :

$$\epsilon = \varepsilon \varepsilon_0 \tag{374}$$

In terms ϵ of we can express the electric field within the dielectric as

$$E = \frac{\sigma}{\epsilon} \tag{375}$$

The capacitance when the dielectric is present is given by

$$C = \varepsilon C_0 = \varepsilon \varepsilon_0 \frac{S}{d} = \epsilon \frac{S}{d}$$
(376)

We can repeat the derivation of Eq. (368) for the energy density in an electric field for the case in which a dielectric is present. The result is

$$u = \frac{1}{2}\varepsilon\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2 \tag{377}$$

In empty space, where $\varepsilon = 1$, $\epsilon = \varepsilon_0$ and Eqs. (376) and (377) reduce to Eqs. (349) and (368), respectively, for a parallel-plate capacitor in vacuum. For this reason, ε_0 is sometimes called the "permittivity of free space" or the "permittivity of vacuum." Because ε is a pure number, ϵ and ε_0 have the same units, $C^2/N \cdot m^2$ or /m.

Equation (376) shows that extremely high capacitances can be obtained with plates that have a large surface area S and are separated by a small distance d by a dielectric with a large value of ε . In an *electrolytic double-layer capacitor*, tiny carbon granules adhere to each plate: The value of is the combined surface area of the granules, which can be tremendous. The plates with granules attached are separated by a very thin dielectric sheet. A capacitor of this kind can have a capacitance of 5000 farads yet fit in the palm of your hand.

Several practical devices make use of the way in which a capacitor responds to a change in dielectric constant. One example is an electric stud finder, used by home repair workers to locate metal studs hidden behind a wall's surface. It consists of a metal plate with associated circuitry. The plate acts as one half of a capacitor, with the wall acting as the other half. If the stud finder moves over a metal stud, the effective dielectric constant for the capacitor changes, changing the capacitance and triggering a signal.

We mentioned earlier that when a dielectric is subjected to a sufficiently strong electric field, *dielectric breakdown* takes place and the dielectric becomes a conductor. This occurs when the electric field is so strong that electrons are ripped loose from their molecules and crash into other molecules, liberating even more electrons. This avalanche of moving charge forms a spark or arc discharge. Lightning is a dramatic example of dielectric breakdown in air.

Because of dielectric breakdown, capacitors always have maximum voltage ratings. When a capacitor is subjected to excessive voltage, an arc may form through a layer of dielectric, burning or melting a hole in it. This arc creates a conducting path (a short circuit) between the conductors. If a conducting path remains after the arc is extinguished, the device is rendered permanently useless as a capacitor.

The maximum electric-field magnitude that a material can withstand without the occurrence of breakdown is called its **dielectric strength**. This quantity is affected significantly by temperature, trace impurities, small irregularities in the metal electrodes, and other factors that are difficult to control. For this reason we can give only approximate figures for dielectric strengths. The dielectric strength of dry air is about $3 \times 10^6 V/m$. Table 12 lists the dielectric strengths of a few common insulating materials. Note that the values are all substantially greater than the value for air. For example, a layer of polycarbonate 0.01 mm thick (about the smallest practical thickness) has 10 times the dielectric strength of air and can withstand a maximum voltage of about $(3 \times 10^6 V/m) \cdot (1 \times 10^{-5} m) = 300 V$.

Material	Dielectric constant, ε	Dielectric strength, E_m , (V/m)
Polycarbonade	2.8	3×10^{7}
Polyester	3.3	6×10^{7}
Polypropylene	2.2	7×10^{7}
Polystyrene	2.6	2×10^{7}
Pyrex glass	4.7	1×10^{7}

Table 12 – Dielectric constant and dielectric strength of some insulating materials

3.5 Direct-current circuits

3.5.1 Resistors in series and parallel

Resistors turn up in all kinds of circuits, ranging from hair dryers and space heaters to circuits that limit or divide current or reduce or divide a voltage. Such circuits often contain several resistors, so it's appropriate to consider *combinations* of resistors. A simple example is a string of light bulbs used for holiday decorations; each bulb acts as a resistor, and from a circuit-analysis perspective the string of bulbs is simply a combination of resistors.

Suppose we have three resistors with resistances R_1 , R_2 and R_3 . Figure 220 shows four different ways in which they might be connected between points *a* and *b*. When several circuit elements such as resistors, batteries, and motors are connected in sequence as in Fig. 220a, with only a single current path between the points, we say that they are connected in **series.** We studied *capacitors* in series; we found that, because of conservation of charge, capacitors in series all have the same charge if they are initially uncharged. In circuits we're often more interested in the *current*, which is charge flow per unit time.



Figure 220 – Four different ways of connecting three resistors

The resistors in Fig. 220b are said to be connected in **parallel** between points a and b. Each resistor provides an alternative path between the points. For

circuit elements that are connected in parallel, the *potential difference* is the same across each element. We studied capacitors in parallel.

In Fig. 220c, resistors R_2 and R_3 are in parallel, and this combination is in series with R_1 . In Fig. 220d, R_2 and R_3 are in series, and this combination is in parallel with R_1 .

For any combination of resistors we can always find a *single* resistor that could replace the combination and result in the same total current and potential difference. For example, a string of holiday light bulbs could be replaced by a single, appropriately chosen light bulb that would draw the same current and have the same potential difference between its terminals as the original string of bulbs. The resistance of this single resistor is called the **equivalent resistance** of the combination. If any one of the networks in Fig. 220 were replaced by its equivalent resistance we could write

$$U_{ab} = IR_{eq} \tag{378}$$

or

$$R_{eq} = \frac{U_{ab}}{I} \tag{379}$$

where U_{ab} is the potential difference between terminals *a* and *b* of the network and *I* is the current at point *a* or *b*. To compute an equivalent resistance, we assume a potential difference U_{ab} across the actual network, compute the corresponding current *I*, and take the ratio

We can derive general equations for the equivalent resistance of a series or parallel combination of resistors. If the resistors are in *series*, as in Fig. 220a, the current *I* must be the same in all of them. (As we discussed current is *not* "used up" as it passes through a circuit.) Applying U = IR to each resistor, we have

$$U_{ax} = IR_1, \qquad U_{xy} = IR_2, \qquad U_{yb} = IR_3$$
 (380)

The potential differences across each resistor need not be the same (except for the special case in which all three resistances are equal). The potential difference U_{ab} across the entire combination is the sum of these individual potential differences:

$$U_{av} = U_{ax} + U_{xy} + U_{yb} = I(R_1 + R_2 + R_3)$$
(381)

and so

$$\frac{U_{ab}}{I} = R_1 + R_2 + R_3 \tag{382}$$

The ratio $\frac{U_{ab}}{I}$ is, by definition, the equivalent resistance R_{eq} Therefore

$$R_{eq} = R_1 + R_2 + R_3 \tag{383}$$

It is easy to generalize this to any number of resistors:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \tag{384}$$

The equivalent resistance of *any number* of resistors in series equals the sum of their individual resistances.

The equivalent resistance is *greater than* any individual resistance.

Let's compare this result with Eq. (356) for *capacitors* in series. Resistors in series add directly because the voltage across each is directly proportional to its resistance and to the common current. Capacitors in series add reciprocally because the voltage across each is directly proportional to the common charge but *inversely* proportional to the individual capacitance.

Resistors in Parallel. If the resistors are in *parallel*, as in Fig. 220b, the current through each resistor need not be the same. But the potential difference between the terminals of each resistor must be the same and equal to U_{av} . (Remember that the potential difference between any two points does not depend on the path taken between the points.) Let's call the currents in the three resistors I_1 , I_2 and I_3 . Then from I = U/R,

$$I_1 = \frac{U_{ab}}{R_1} I_2 = \frac{U_{ab}}{R_2} I_3 = \frac{U_{ab}}{R_3}$$
(385)

In general, the current is different through each resistor. Because charge is not accumulating or draining out of point a, the total current I must equal the sum of the three currents in the resistors:

$$I = I_1 + I_2 + I_3 = U_{ab} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$
(386)

or

$$\frac{I}{U_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
(387)

But by the definition of the equivalent resistance R_{eq} , $I/U_{ab} = 1/R_{eq}$, so

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
(388)

Again it is easy to generalize to *any number* of resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$
(389)

For *any number* of resistors in parallel, the *reciprocal* of the equivalent resistance equals the *sum of the reciprocals* of their individual resistances. The equivalent resistance is always *less than* any individual resistance.

Compare this with Eq. (362) for *capacitors* in parallel. Resistors in parallel add reciprocally because the current in each is proportional to the common voltage across them and *inversely* proportional to the resistance of each. Capacitors in parallel add directly because the charge on each is proportional to the common voltage across them and *directly* proportional to the capacitance of each.

For the special case of *two* resistors in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$
(390)

and

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \tag{391}$$

Because $U_{ab} = I_1 R_1 = I_2 R_2$, it follows that

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$
(392)

This shows that the currents carried by two resistors in parallel are *inversely proportional* to their resistances. More current goes through the path of least resistance.

3.5.2 Kirchhoff's Rules

Many practical resistor networks cannot be reduced to simple series-parallel combinations. Figure 221a shows a dc power supply with emf \mathcal{E}_1 charging a battery with a smaller emf \mathcal{E}_1 and feeding current to a light bulb with resistance R. Figure 221b is a "bridge" circuit, used in many different types of measurement and control systems. To compute the currents in these networks, we'll use the techniques developed by the German physicist Gustav Robert Kirchhoff (1824–1887).



Figure 221 – Two networks that cannot be reduced to simple series-parallel combination of resistors

First, here are two terms that we will use often. A **junction** in a circuit is a point where three or more conductors meet. A **loop** is any closed conducting path. In Fig. 221a points a and b are junctions, but points c and d are not; in Fig. 221b the points a, b, c, and d are junctions, but points e and f are not. The blue lines in Figs. 221a and 221b show some possible loops in these circuits.

Kirchhoff's rules are the following two statements:

Kirchhoff's junction rule: *The algebraic sum of the currents into any junction is zero.* That is,

$$\sum I_i = 0 \tag{393}$$

Kirchhoff's loop rule: *The algebraic sum of the potential differences in any loop,* including those associated with emfs and those of resistive elements, *must equal zero.* That is,

$$\sum U_i = 0 \tag{394}$$

The junction rule is based on *conservation of electric charge*. No charge can accumulate at a junction, so the total charge entering the junction per unit time must equal the total charge leaving per unit time (see fig. 222a). Charge per unit time is current, so if we consider the currents entering a junction to be positive and those leaving to be negative, the algebraic sum of currents into a junction must be zero. It's like a T branch in a water pipe (see fig. 222b); if you have a total of 1 liter per minute coming in the two pipes, you can't have 3 liters per minute going out the third pipe. We may as well confess that we used the junction rule (without saying so) in Section 26.1 in the derivation of Eq. (389) for resistors in parallel.



Figure 222 – Kirchhoffs's junction states that as much current flows into a junction as flows out of it

The loop rule is a statement that the electrostatic force is *conservative*. Suppose we go around a loop, measuring potential differences across successive circuit elements as we go. When we return to the starting point, we must find that the *algebraic sum* of these differences is zero; otherwise, we could not say that the potential at this point has a definite value.

In applying the loop rule, we need some sign conventions. Problem-Solving Strategy 26.2 describes in detail how to use these, but here's a quick overview. We first assume a direction for the current in each branch of the circuit and mark it on a diagram of the circuit. Then, starting at any point in the circuit, we imagine traveling around a loop, adding emfs and *IR* terms as we come to them. When we travel through a source in the direction from - to +, the emf is considered to be *positive;* when we travel from + to -, the emf is considered to be *negative* (see fig. 223a). When we travel through a resistor in the *same* direction as the assumed current, the *IR* term is *negative* because the current goes in the direction of decreasing potential. When we travel through a resistor in the direction of the direction of the assumed current, the *IR* term is *positive* because this represents a rise of potential (see fig. 223b).



Figure 223 – Use these sign conservations when you apply Kirchhoffs's loop rule. In each part of the figure "Travel" is the direction that we imaging going around the loop, which is not necessary the direction of the current

Kirchhoff's two rules are all we need to solve a wide variety of network problems. Usually, some of the emfs, currents, and resistances are known, and others are unknown. We must always obtain from Kirchhoff's rules a number of independent equations equal to the number of unknowns so that we can solve the equations simultaneously. Often the hardest part of the solution is not understanding the basic principles but keeping track of algebraicsigns!

3.5.3 Electrical measuring instruments

We've been talking about potential difference, current, and resistance for two chapters, so it's about time we said something about how to measure these quantities. Many common devices, including car instrument panels, battery chargers, and inexpensive electrical instruments, measure potential difference (voltage), current, or resistance using a d'Arsonval galvanometer (see fig. 224). In the following discussion we'll often call it just a meter. A pivoted coil of fine wire is placed in the magnetic field of a permanent magnet (see fig. 225). Attached to the coil is a spring, similar to the hairspring on the balance wheel of a watch. In the equilibrium position, with no current in the coil, the pointer is at zero. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to the current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement.



Figure 224 – This ammeter (top) and voltmeter (bottom) are both d'Arsonval galvanometers. The difference has to do with their internal connections



Figure 225 – A d'Arsonval galvanometer, showing a pivoted coil with attached pointer, a permanent magnet supplying a magnetic field that is uniform in magnitude, and a spring to provide restoring torque, which opposes magnetic-field

Thus the angular deflection of the coil and pointer is directly proportional to the coil current, and the device can be calibrated to measure current. The maximum deflection, typically 90° or so, is called full-scale deflection. The essential electrical characteristics of the meter are the current I_{fs} required for full-scale deflection (typically on the order of 10 μA to 10 mA) and the resistance R_C of the coil (typically on the order of 10 to 1000 Ω).

The meter deflection is proportional to the current in the coil. If the coil obeys Ohm's law, the current is proportional to the potential difference between the terminals of the coil, and the deflection is also proportional to this potential difference. For example, consider a meter whose coil has a resistance $R_c = 200 \Omega$ and that deflects full scale when the current in its coil is $I_{fs} = 1 mA$. The corresponding potential difference for full-scale deflection is

$$U = I_{fs}R_c = (1 \times 10^{-3}A)(200 \,\Omega) = 0.02 \,V$$

Ammeters. A current-measuring instrument is usually called an ammeter (or milliammeter, microammeter, and so forth, depending on the range). An ammeter always measures the current passing through it. An ideal ammeter would have zero resistance, so including it in a branch of a circuit would not affect the current in that branch. Real ammeters always have some finite resistance, but it is always desirable for an ammeter to have as little resistance as possible.

We can adapt any meter to measure currents that are larger than its full-scale reading by connecting a resistor in parallel with it (see fig. 226a) so that some of the current bypasses the meter coil. The parallel resistor is called a shunt resistor or simply a shunt, denoted as R_{sh} .

Suppose we want to make a meter with full-scale current I_{fs} and coil resistance R_c into an ammeter with full-scale reading I_a . To determine the shunt resistance R_{sh} needed, note that at full-scale deflection the total current through the parallel combination is I_a , the current through the coil of the meter is I_{fs} , and the current through the shunt is the difference $I_a - I_{fs}$. The potential difference U_{ab} is the same for both paths, so

$$I_{fs}R_C = (I_a - I_{fs})R_{sh}$$
(395)

Voltmeters. This same basic meter may also be used to measure potential difference or voltage. A voltage-measuring device is called a voltmeter. A voltmeter always measures the potential difference between two points, and its terminals must be connected to these points. An ideal voltmeter would have infinite resistance, so connecting it between two points in a circuit would not alter any of the currents. Real voltmeters always have finite resistance, but a voltmeter should have large enough resistance that connecting it in a circuit does not change the other currents appreciably.

The voltage across the meter coil at full-scale deflection is only $I_{fs}R_c = (1 \times 10^{-3}A)(200\Omega) = 0.02 V$. We can extend this range by connecting a resistor R_s in *series* with the coil (see fig. 226b). Then only a fraction of the total potential difference appears across the coil itself, and the remainder appears across the coil itself, and the remainder appears across the coil R_s . For a voltmeter with full-scale reading U_V , we need a series resistor R_s in Fig. 226b such that

$$U_V = I_{fs}(R_C + R_S) \tag{396}$$



Figure 226 – Using the same meter to measure (a) current and (b) voltage

Ammeters and voltmeters in combination. A voltmeter and an ammeter can be used together to measure *resistance* and *power*. The resistance *R* of a resistor equals the potential difference U_{ab} between its terminals divided by the current *I*; that is, $R = U_{ab}/I$. The power input *P* to any circuit element is the product of the potential difference across it and the current through it: $P = U_{ab}I$. In principle, the most straightforward way to measure *R* or *P* is to measure U_{ab} and *I* simultaneously.

With practical ammeters and voltmeters this isn't quite as simple as it seems. In Fig. 227a, ammeter A reads the current I in the resistor R. Voltmeter V, however, reads the *sum* of the potential difference U_{ab} across the resistor and the potential difference U_{bc} across the ammeter. If we transfer the voltmeter terminal from c to b, as in Fig. 227b, then the voltmeter reads the potential difference U_{ab} correctly, but the ammeter now reads the *sum* of the current I in the resistor and the current I_V in the voltmeter. Either way, we have to correct the reading of one instrument or the other unless the corrections are small enough to be negligible.



Figure 227 – Ammeter-voltmeter method for measuring resistance

Ohmmeters. An alternative method for measuring resistance is to use a d'Arsonval meter in an arrangement called an **ohmmeter**. It consists of a meter, a resistor, and a source (often a flashlight battery) connected in series (see fig. 228). The resistance R to be measured is connected between terminals x and y.



Figure 228 – Ohmmeter circuit. The resistor R_s has a variable resistance, as is indicated by the arrow through the resistor symbol.

The series resistance R_S is variable; it is adjusted so that when terminals x and y are short-circuited (that is, when R = 0), the meter deflects full scale. When nothing is connected to terminals x and y, so that the circuit between x and y is *open* (that is, when $R \rightarrow 0$), there is no current and hence no deflection. For any intermediate value of R the meter deflection depends on the value of R, and the meter scale can be calibrated to read the resistance R directly. Larger currents

correspond to smaller resistances, so this scale reads backward compared to the scale showing the current.

In situations in which high precision is required, instruments containing d'Arsonval meters have been supplanted by electronic instruments with direct digital readouts. Digital voltmeters can be made with extremely high internal resistance, of the order of 1000Ω . Figure 229 shows a digital multimeter, an instrument that can measure voltage, current, or resistance over a wide range.



Figure 229 – This digital multimeter can be as a voltmeter (red arc), ammeter (yellow arc), or ohmmeter (green arc)

The Potentiometer. The *potentiometer* is an instrument that can be used to measure the emf of a source without drawing any current from the source; it also has a number of other useful applications. Essentially, it balances an unknown potential difference against an adjustable, measurable potential difference.

The principle of the potentiometer is shown schematically in Fig. 230a. A resistance wire *ab* of total resistance R_{ab} is permanently connected to the terminals of a source of known emf \mathcal{E}_1 A sliding contact c is connected through the galvanometer G to a second source whose emf \mathcal{E}_2 is to be measured. As contact c is moved along the resistance wire, the resistance R_{cb} between points c and b varies; if the resistance wire is uniform, is proportional to the length of wire between c and b. To determine the value of \mathcal{E}_2 contact c is moved until a position is found at which the galvanometer shows no deflection; this corresponds to zero current passing through \mathcal{E}_2 . With I = 0, Kirchhoff's loop rule gives

$$\mathcal{E}_2 = IR_{cb} \tag{397}$$

With I_2 , the current *I* produced by the emf \mathcal{E}_1 has the same value no matter what the value of the emf \mathcal{E}_2 . We calibrate the device by replacing \mathcal{E}_2 by a source of known emf; then any unknown emf \mathcal{E}_2 can be found by measuring the length of wire *cb* for which $I_2 = 0$. Note that for this to work, U_{ab} must be greater than \mathcal{E}_2 .



Figure 230 – A potentiometer

The term *potentiometer* is also used for any variable resistor, usually having a circular resistance element and a sliding contact controlled by a rotating shaft and knob. The circuit symbol for a potentiometer is shown in Fig. 230b.

3.5.4 R-C circuits

In the circuits we have analyzed up to this point, we have assumed that all the emfs and resistances are constant (time independent) so that all the potentials, currents, and powers are also independent of time. But in the simple act of charging or discharging a capacitor we find a situation in which the currents, voltages, and powers do change with time.

Many devices incorporate circuits in which a capacitor is alternately charged and discharged. These include flashing traffic lights, automobile turn signals, and electronic flash units. Understanding what happens in such circuits is thus of great practical importance.

Figure 231 shows a simple circuit for charging a capacitor. A circuit such as this that has a resistor and a capacitor in series is called an **R-C circuit**. We

idealize the battery (or power supply) to have a constant emf \mathscr{E} and zero internal resistance (r = 0) and we neglect the resistance of all the connecting conductors.

We begin with the capacitor initially uncharged (see fig. 231a); then at some initial time t = 0 we close the switch, completing the circuit and permitting current around the loop to begin charging the capacitor (see fig. 231b). For all practical purposes, the current begins at the same instant in every conducting part of the circuit, and at each instant the current is the same in every part.



Figure 231 – Charging capacitor. (a) Just before the switch is closed, the charge is q zero. (b) When the switch closes (at t = 0), the current jumps from zero to \mathcal{E}/R . As time passes, q approaches Q_f and the current i approaches zero

Because the capacitor in Fig. 231 is initially uncharged, the potential difference u_{bc} across it is zero at t = 0. At this time, from Kirchhoff's loop law, the voltage u_{ab} across the resistor R is equal to the battery emf \mathscr{E} . The initial (t = 0) current through the resistor, which we will call I_0 , is given by Ohm's law: $I_0 = u_{ab}/R = \mathcal{E}/R$.

As the capacitor charges, its voltage u_{bc} increases and the potential difference u_{ab} across the resistor decreases, corresponding to a decrease in current. The sum of these two voltages is constant and equal to \mathscr{E} . After a long time the capacitor becomes fully charged, the current decreases to zero, and the potential difference u_{ab} across the resistor becomes zero. Then the entire battery emf \mathscr{E} appears across the capacitor and $u_{bc} = \mathscr{E}$.

Let q represent the charge on the capacitor and i the current in the circuit at some time t after the switch has been closed. We choose the positive direction for the current to correspond to positive charge flowing onto the left-hand capacitor plate, as in Fig. 231b. The instantaneous potential differences u_{ab} and u_{bc} are

$$u_{ab} = iR \tag{398}$$

$$u_{bc} = \frac{q}{C} \tag{399}$$

Using these in Kirchhoff's loop rule, we find

$$\mathcal{E} - iR - \frac{q}{C} = 0 \tag{400}$$

The potential drops by an amount *iR* as we travel from *a* to *b* and by $\frac{q}{c}$ as we travel from *b* to *c*. Solving Eq. (26.9) for *i*, we find

$$i = \frac{\mathcal{E}}{R} - \frac{q}{RC} \tag{401}$$

At time (t = 0), when the switch is first closed, the capacitor is uncharged, and so q = 0. Substituting q = 0 into Eq. (401), we find that the *initial* current I_0 is given by as $I_0 = \mathcal{E}/R$, we have already noted. If the capacitor were not in the circuit, the last term in Eq. (401) would not be present; then the current would be *constant* and equal to \mathcal{E}/R .

As the charge q increases, the term $\frac{q}{RC}$ becomes larger and the capacitor charge approaches its final value, which we will call Q_f . The current decreases and eventually becomes zero. When i = 0 Eq. (401) gives

$$\frac{\mathcal{E}}{R} = \frac{Q_f}{RC}, Q_f = C\mathcal{E}$$
(402)

Note that the final charge Q_f does not depend on *R*.

Figure 232 shows the current and capacitor charge as functions of time. At the instant the switch is closed (t = 0), the current jumps from zero to its initial

value $I_0 = \mathcal{E}/R$ after that, it gradually approaches zero. The capacitor charge starts at zero and gradually approaches the final value given by Eq. (402), $Q_f = C\mathcal{E}$.







Figure 232 – Current and capacitor charge q as function of time for the circuit of Fig.231. The initial current I_0 and initial capacitor charge is zero. The current asymptotically approaches zero, and the capacitor charge asymptotically approaches a final value of Q_f

We can derive general expressions for the charge q and current i as functions of time. With our choice of the positive direction for current (see fig. 231b), i equals the rate at which positive charge arrives at the left-hand (positive) plate of the capacitor, so Making this substitution in Eq. (401), we have

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\mathcal{E})$$
(403)
We can rearrange this to

$$\frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC} \tag{404}$$

and then integrate both sides. We change the integration variables to q' and t' so that we can use q and t for the upper limits. The lower limits are q' = 0 and t' = 0:

$$\int_{0}^{q} \frac{dq'}{q - C\mathcal{E}} = -\int_{0}^{t} \frac{dt'}{RC}$$

$$\tag{405}$$

When we carry out the integration, we get

$$\ln\left(\frac{q-C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC} \tag{406}$$

Exponentiating both sides (that is, taking the inverse logarithm) and solving for q, we find

$$\frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-\frac{t}{RC}} \tag{407}$$

$$q = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$$
(408)

The instantaneous current i is just the time derivative of Eq. (408):

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$
(409)

The charge and current are both exponential functions of time. Figure 232a is a graph of Eq. (409) and Fig. 232b is a graph of Eq. (408). Time Constant

After a time equal to *RC*, the current in the R-C circuit has decreased to 1/e (about 0.368) of its initial value. At this time, the capacitor charge has reached $\left(1-\frac{1}{e}\right) = 0.632$ of its final value $Q_f = C\mathcal{E}$. The product *RC* is therefore a measure of how quickly the capacitor charges. We call *RC* the **time constant**, or the **relaxation time**, of the circuit, denoted by τ :

$$\tau = RC \tag{410}$$

When τ is small, the capacitor charges quickly; when it is larger, the charging takes more time. If the resistance is small, it's easier for current to flow,

and the capacitor charges more quickly. If R is in ohms and C in farads, τ is in seconds.

In Fig. 232a the horizontal axis is an *asymptote* for the curve. Strictly speaking, *i* never becomes exactly zero. But the longer we wait, the closer it gets. After a time equal to *10RC*, the current has decreased to 0.000045 of its initial value. Similarly, the curve in Fig. 232b approaches the horizontal dashed line labelled Q_f as an asymptote. The charge *q* never attains exactly this value, but after a time equal to *10RC*, the difference between *q* and is only 0.000045 of Q_f . We invite you to verify that the product *RC* has units of time.



Figure 233 – Discharging a capacitor. (a) Before the switch is closed at time t = 0, the capacitor charge is Q_0 and the current is zero. (b) At time t after the switch is closed, the capacitor charge is q and the current is i. The actual current direction is opposite to the direction shown; i is negative. After a long time, q and i both approach zero

Now suppose that after the capacitor in Fig. 232b has acquired a charge Q_0 , we remove the battery from our *R*-*C* circuit and connect points *a* and *c* to an open switch (see fig. 233a). We then close the switch and at the same instant reset our stopwatch to t = 0; at that time, $q = Q_0$. The capacitor then *discharges* through the resistor, and its charge eventually decreases to zero.

Again let *i* and *q* represent the time-varying current and charge at some instant after the connection is made. In Fig. 233b we make the same choice of the positive direction for current as in Fig. 231b. Then Kirchhoff's loop rule gives Eq. (401) but with $\mathcal{E} = 0$ that is,

$$i = \frac{dq}{dq} = -\frac{q}{RC} \tag{411}$$

The current *i* is now negative; this is because positive charge *q* is leaving the lefthand capacitor plate in Fig. 233b, so the current is in the direction opposite to that shown in the figure. At time t = 0, when $q = Q_0$, the initial current is $I_0 = -Q_0/RC$.

To find q as a function of time, we rearrange Eq. (411), again change the names of the variables to q' and t', and integrate. This time the limits for q' are Q_0 to q. We get

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC} \int_{0}^{t} dt'$$

$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

$$q = Q_0 e^{-\frac{t}{RC}}$$
(412)
(412)
(413)

The instantaneous current i is the derivative of this with respect to time:

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$
(414)

We graph the current and the charge in Fig. 234; both quantities approach zero exponentially with time. Comparing these results with Eqs. (408) and (409), we note that the expressions for the current are identical, apart from the sign of I_0 . The capacitor charge approaches zero asymptotically in Eq. (412), while the *difference* between q and Q approaches zero asymptotically in Eq. (408).

Energy considerations give us additional insight into the behavior of an *R*-*C* circuit. While the capacitor is charging, the instantaneous rate at which the battery delivers energy to the circuit is $P = \mathcal{E}i$. The instantaneous rate at which electrical

energy is dissipated in the resistor is $i^2 R$, and the rate at which energy is stored in the capacitor is $iu_{bc} = iq/C$. Multiplying Eq. (26.9) by *i*, we find

$$\mathcal{E}i = i^2 R + \frac{iq}{C} \tag{414}$$





Figure 234 Current *i* and capacitor charge *q* as function of time for the circuit of Fig. 233. The initial current I_0 and the initial capacitor charge is Q_0 . Both *i* and *q* asymptotically approach zero

This means that of the power $\mathcal{E}i$ supplied by the battery, part (i^2R) is dissipated in the resistor and part $\left(\frac{iq}{c}\right)$ is stored in the capacitor.

The *total* energy supplied by the battery during charging of the capacitor equals the battery emf \mathcal{E} multiplied by the total charge Q_f , or $\mathcal{E}Q_f$. The total energy stored in the capacitor, from Eq. (366), is $Q_f \mathcal{E}/2$. Thus, of the energy supplied by the battery, *exactly half* is stored in the capacitor, and the other half is dissipated in the resistor. This half-and-half division of energy doesn't depend on C, R, or \mathcal{E} . You can verify this result by taking the integral over time of each of the power quantities in Eq. (414).

Questions yourself

- 1. What does mechanics study? Kinematics? Dynamics? Statics? What is a mechanical motion?
- 2. *Give definitions of concepts:* a mass point, a distance, a displacement, a velocity, an acceleration, a trajectory.
- 3. What is a force? Give examples of forces.
- 4. Formulate Newton's laws. What is the principle of superposition?
- 5. *Give definitions of concepts:* a mass, a momentum, a solid body, an angular momentum, a work, an energy.
- 6. Formulate mechanical energy and momentum conservation's laws. What is a closed system?
- 7. Formulate the law of gravity. What is the free fall acceleration? What does it equal on the Earth's surface?
- 8. Einsteins's postulates.. What is the value of speed of light in vacuum?
- 9. *Give definitions of concepts:* a simple pendulum, the amplitude, wavelength, period, frequency, angular frequency
- 10. What is a deformation? Formulate Hooke's law.
- 11. What is the ideal fluid? What is called an incompressible fluid?
- 12. What does a hydrodynamics study? Hydrostatic? Archimedes' law
- 13.Bernoulli equation for an ideal fluid. Stokes' formula, Poiseuille' formula and Torricelli's formula
- 14. What does molecular physics study? Thermodynamics?
- 15.*Give definitions of concepts:* relative molecular mass, relative molar mass, mole, the amount of substance.
- 16. What are the thermodynamic parameters? What thermodynamic parameters do you know?
- 17. What is a thermodynamic process? Equilibrium process?
- 18. What is an ideal gas? Isoprocess? Laws of an Ideal gas.
- 19.Mendeleev Clapeyron's equation. Clapeyron's equation. The basic equation of molecular-kinetic theory of gases.
- 20.Formulate thermodynamics' laws.
- 21.Record the first law of thermodynamics for isoprocesses.
- 22. What is internal energy? Formulas for work at different iso. What is SI unit of the work?
- 23. Give definitions of concepts: an engine, heater, refrigerator, efficiency.
- 24.Record Fourier's, Newton's, and Fick's equations.
- 25.Write down van der Waals's equation for real gases. What is the critical point? What are the critical parameters?

- 26. What is a phase change? What types of phase transformations are there?
- 27.*Give definitions of concepts:* boiling, melting, crystallization, evaporation, condensation, sublimation
- 28. What is a triple point? Record Clausius-Clapeyron' equation.
- 29. What does electrostatics study? Electrodynamics?
- 30. What is an electric field? What is an electric charge?
- 31. The electric charge's law conservation and the superposition of electrostatic fields.
- 32. Which device is called a capacitor? What is a capacitance? What is a SI unit of a capacitance?
- 33.*Give definitions of concepts:* voltage, current, resistance, electromotive force, potential, conductivity, current density
- 34. What is a direct current? Alternative current? What is a SI unit of a current?
- 35.Write down Ohm's law for the branch circuit, for the complete circuit in integral form. Ohm's law in differential form.
- 36.Formulate Joule's law in differential form and integral forms. Capacitors and conductors in parallel and in series.
- 37.Formulate Kirchhoff's rules.
- 38. What is a magnetic field?
- 39. Formulate the Biot Savart- Laplace's law. Ampere's law
- 40. What are magnets? What is the Curie's point?
- 41.Formulate law of electromagnetic induction. Lenz's rule.
- 42. What are the electromagnetic waves? What is the electrodynamics constant?
- 43. What is electromagnetic oscillation? Thomson's formula.
- 44.Ohm's law for AC circuits. What is the impedance of the circuit?

Test yourself

1.1. What is a SI unit of length? A) m (metre) B) s (second) C) mole D) K (kelvin) E) kg (kilogram) ******** 2.1. What is a SI unit of mass? A) g (gram) B) kg (kilogram) C) h (hour) D) mole E) °C (Celsius) ******* 3.1. What is a SI unit of time? A) h (hour) B) J (jear) C) s (second) D) d (day) E) w (week) ****** 4.1. What is a SI unit of electric current? A) N (Newton) B) cd (candela) C) K (Kelvin) D) A (ampere) E) C (coulomb) ****** 5.1. What is a SI unit of temperature? A) cd (candela) B) J (joule) C) °C (Celsius) D) N (Newton) E) K (Kelvin) ***** 6.1. What is a SI unit of amount of substance? A) mole B) beetle C) hour D) hertz

E) candela ******* 7.1.What is a SI unit of luminous intensity? A) K (Kelvin) B) cd (candela) C) J (joule) D) C (coulomb) E) F (farad) ****** 8.1.What is a SI unit of force? A) J (joule) B) K (Kelvin) C) N (Newton) D) kg (kilogram) E) E (Einstein) ****** 9.1.What is a SI unit of electric charge? A) N (Newton) B) K (Kelvin) C) J (joule) D) C (coulomb) E) Hz (hertz) ****** 10.1.What is a SI unit of velocity? A) s B) m/s^2 C) m D) N E) m/s******* 11.1.What is a SI unit of acceleration? A) m/s^2 B) m/sC) kg/s D) s/kgE) m·s² ******* 12.1.What is a SI unit of work? A) K (Kelvin) B) m/s^2

C) J (joule) D) C (coulomb) E) Hz (hertz) *********

13.1.What is a SI unit of frequency?
A) C (coulomb)
B) N (Newton)
C) m/s²
D) Hz (hertz)

E) K (Kelvin) *********

14.1.The Concorde is a type of aeroplane that flies very fast. The top speed of the Concorde is 844 km/hr. Convert the Concorde's top speed to m/s.

- A) 844 m/s
- B) 3038 m/s
- C) 0.844 m/s
- D) 300 m/s
- E) 234 m/s

15.1.A measurement of a location,

- with reference to an origin is a ...
- A) position
- B) displacement
- C) force
- D) velocity
- E) acceleration

16.1. What is the symbol used to indicate position?

A) μ B) x C) £ D) ∞ E) ¥ **********

17.1. What is the position for the object A?



18.1. What is the position for the object B?



19.1. What is the position for the object C?



B) -1. m C) 1. m D) 4 m E) unknown ******

20.1. What is the position for the object D?



- A) 1. m B) 3 m C) 0 m
- D) -1. m
- E) -3 m
- __/ _ ___ **********

21.1. What is the position for the object E?



- A) 4 m
- B) 3 m
- C) 2 m
- D) 1. m
- E) -1. m
- *********

22.1.What is the position for the reference point?



- A) force
- B) momentum
- C) velocity
- D) torque
- E) Displacement

- 24.1. Choose a scalar quantity:
- A) velocity
- B) mass
- C) force
- D) acceleration
- E) displacement
- *****
- 25.1. Choose a scalar quantity:
- A) torque
- B) time
- C) force
- D) acceleration
- E) velocity
- *****
- 26.1.Choose a scalar quantity:
- A) velocity
- B) momentum
- C) displacement
- D) position
- E) force
- ********
- 27.1. Choose a scalar quantity:
- A) distance
- B) displacement
- C) acceleration
- D) torque
- E) momentum
- ******
- 28.1. Choose a vector quantity:
- A) time
- B) distance
- C) mass
- D) displacement
- E) position
- *****
- 29.1. Choose a vector quantity:
- A) distance
- B) mass
- C) time
- D) velocity
- E) position
- / I **********

30.1. Choose a vector quantity:

32.1. The first derivative of a displacement with respect to time is

- •••
- A) force
- B) position
- C) acceleration
- D) velocity
- E) impulse
- *****

33.1.The rate of change of position is

•••

- A) distance
- B) momentum
- C) velocity
- D) torque
- E) mass

34.1. The first derivative of a displacement with respect to time is

•••

- A) velocity
- B) momentum
- C) acceleration
- D) force
- E) position
- *******

35.1. Choose the modulus of vector velocity:

A)
$$\upsilon = \omega R$$
.
B) $\upsilon = \left| \frac{d\vec{r}}{dt} \right|$

C) $|\vec{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$ $|\vec{v}| = \frac{dS}{dt}$ E) $v = v_0 - at$.

36.1. Given kinematics equation of a motion of a point: $x=2+4t+2t^3$. Find a position of a point at the moment t=2 s:

- A) 16 m
- B) 26 m
- C) 28 m
- D) 2 m
- E) 8 m
- *********

37.1. Given kinematics equation of a motion of a point: $x=2+4t+2t^3$. Find a velocity of a point at the moment t=2 s:

- A) 16 m/s
- B) 28 m/s
- C) 2 m/s
- D) 8 m/s
- E) 26 m/s
- *********

38.1. Given kinematics equation of a motion of a point: $x=2+4t+2t^3$. Find an acceleration of a point at the moment t=2 s: A) 16 m/s² B) 24 m/s² C) 2 m/s² D) 8 m/s² E) 26 m/s² ******

39.1. A position of a point changes with a law: $x=2+4t+2t^3$. Find a position of a point at the moment t=1. s: A) 1. m

- B) 2 m
- C) 4 m
- D) 6 m

E) 8 m *********

40.1. A position of a point changes with a law: $x=2+4t+2t^3$. Find a velocity of a point at the moment t=1. s:

A) 2 m

- \dot{B} 4 m
- C) 6 m
- D) 8 m
- E) 10m

__/ _ - - ---***********

41.1. A position of a point changes with a law: $x=2+4t+2t^3$. Find an acceleration of a point at the moment t=1. s: A) 2 m/s²

B) 4 m/s^2 C) 6 m/s^2 D) 8 m/s^2 E) 12 m/s^2

42.1. The second derivative of a displacement with respect to time is

- A) acceleration
- B) velocity
- C) force

. . .

- D) torque
- E) momentum

43.1. The rate of change of velocity is

- •••
- A) momentum
- B) velocity
- C) acceleration
- D) force
- E) position

44.1. How can you define an acceleration?

A)
$$\overline{a} = \frac{d\vec{\upsilon}}{dt}$$
.

B)
$$a = \frac{d^2 s}{dt^2}$$

C) $a = \frac{dv}{dt}$
D) $a = \frac{v^2}{R}$
E) $a=g$

45.1. How can you define an

acceleration?
A)
$$|\vec{a}| = \sqrt{\left(\frac{dv_x}{dt}\right)^2 + \left(\frac{dv_y}{dt}\right)^2 + \left(\frac{dv_z}{dt}\right)^2}$$

B) $a = \frac{d^2s}{dt^2}$
C) $a = \frac{dv}{dt}$
D) $a = \frac{v^2}{R}$
E) $a=g$

47.1. An aeroplane accelerates uniformly from an initial velocity of 200 m/s to a final velocity of 100 m/s in 10 seconds. Calculate his acceleration

A)
$$-10 \text{ m/s}^2$$

B) 30 m/s^2
C) -30 m/s^2
D) 10 m/s^2
E) 3000 m/s^2

48.1. A bus accelerates uniformly from an initial velocity of 15 m/s to a

49.1. Choose a normal component of acceleration:

A)
$$\overrightarrow{a_n} = \frac{d\overrightarrow{v}}{dt}$$

B) $\overrightarrow{a_n} = \omega^2 r$
C) $\overrightarrow{a_n} = \frac{d^2 r}{dt^2} \overrightarrow{n}$
D) $\overrightarrow{a_n} = \frac{v^2}{R} \overrightarrow{\tau}$
E) $\overrightarrow{a_n} = \frac{v^2}{R} \overrightarrow{n}$

50.1. Choose a centripetal component of acceleration:

A)
$$\overrightarrow{a_n} = \frac{d\overrightarrow{v}}{dt}$$

B) $\overrightarrow{a_n} = \omega^2 r$
C) $\overrightarrow{a_n} = \frac{d^2 r}{dt^2} \overrightarrow{n}$
D) $\overrightarrow{a_n} = \frac{v^2}{R} \overrightarrow{\tau}$
E) $\overrightarrow{a_n} = \frac{v^2}{R} \overrightarrow{n}$

51.1. Choose an tangential component of acceleration:

A)
$$\vec{a}_{\tau} = \frac{d\vec{v}}{dt}\tau$$

B) $\vec{a}_{\tau} = \omega R$
C) $\vec{a}_{\tau} = \frac{d^2\vec{v}}{dt^2}n$
D) $\vec{a}_{\tau} = \frac{\vec{v}}{R}\tau$
E) $\vec{a}_{\tau} = \frac{\vec{v}^2}{R}n$

52.1. Choose a equation of motion:

A) $\vec{F} = \mu \cdot \vec{N}$ B) $x = -k \cdot \Delta x$ C) $z = m\vec{a} + \frac{v}{2}$ D) $S = v_0 t + \frac{at^2}{2}$ E) $\vec{v} = \vec{v}_0 + \vec{a}t$ *********

54.1. A train starts from rest, and accelerates at 1. m/s^2 for 10 s. How far does it move?

A) 10 m B) 25 m C) 50 m D) 100 m E) 75m **********

55.1. A bus is going 30 m/s and stops in 5 s. What is its stopping distance for this speed?

A) 35 m B) 25 m C) 50 m D) 75 m E) 225 m *********

56.1. A racing car going at 20 m/s stops in a distance of 20 m. What is its acceleration? A) -10 m/s^2

B) -1. m/s² C) -20 m/s² D) -40 m/s² E) -5 m/s² ********* 57.1. If the car travelling at 120 km/h, how long will it take the a car to travel 100 m?

- A) 1. s
- B) 2 s
- C) 5 s
- D) 3 s
- E) 10 s
- ******

58.1. An object will remain in a state of rest or continue travelling at constant velocity, unless acted upon by an unbalanced (net) force. It is ...

- A) Newton's first law
- B) Newton's second law
- C) Newton's third law
- D) gravity law

E) straight line motion law *******

59.1. Choose the second law of Newton:

A)
$$\vec{F} = m\vec{a}$$

B) $\vec{P} = m\vec{g}$
C) $\vec{p} = m\vec{v}$
D) $F = G \frac{m_1 m_2}{r^2}$
E) $\vec{F} = \vec{F_1} + \vec{F_2} + \dots + \vec{F_i}$

60.1. Choose the second law of Newton:

A) $\vec{P} = m\vec{g}$ B) $F = Mr \sin \alpha$ C) $\vec{F} = \frac{d\vec{p}}{dt}$ D) $F = G \frac{m_1 m_2}{r^2}$ E) $\vec{p} = m\vec{v}$ *********

61.1.Choose the second law of Newton:

A)
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_i$$

B) $F = G \frac{m_1 m_2}{r^2}$
C) $M = Fr \sin \alpha$
D) $\vec{P} = m\vec{g}$

E) $\vec{F} = m \frac{d\vec{v}}{dt}$ *********

- 62.1. $\vec{F} = m\vec{a}$. It is ...
- A) Newton's first law
- B) Newton's second law
- C) Newton's third law

D) gravity law

E) straight line motion law ********

63.1.
$$\vec{F} = m \frac{d\vec{v}}{dt}$$
. It is ...

- A) Newton's first law
- B) Newton's second law
- C) Newton's third law
- D) gravity law

E) straight line motion law *******

64.1.
$$\vec{F} = \frac{d\vec{p}}{dt}$$
. It is ...
A) Newton's first law
B) Newton's second lay

- B) Newton's second law
- C) Newton's third law
- D) gravity law
- E) straight line motion law ******

65.1. A car of mass 850 kg accelerates at 2 m/s². Calculate the magnitude of the resultant force that is causing the acceleration.

A) 850 N B) 1700 N C) 425 N D) 3400 N E) 2016 N *******

66.1. Find the force needed to accelerate a 3 kg object at 4 m/s². A) 1. N B) 7 N C) 12 N D) 0.75 N E) 1.25 N ***** 67.1. Calculate the acceleration of an object of mass 1000 kg accelerated by a force of 100 N. A) $0.1. \text{ m/s}^2$

B) 1100 m/s²
C) 900 m/s²
D) 10 m/s²
E) 100000 m/s²

68.1. An object of mass 7 kg is accelerating at 2,5 m/s^2 . What resultant force acts on it?

A) 9.5 N

- B) 4.5 N C) -5.5 N
- D) 17.5 N
- E) 2.8 N

69.1. Find the mass of an object if a force of 40 N gives it an acceleration of 2 m/s².
A) 20 kg
B) 80 kg

- C) 38 kg
- D) 42 kg
- E) 0.25 kg

70.1. Find the acceleration of a body of mass 1. 000 kg that has a 150 N force acting on it.

A) 1150 m/s² B) 6.67 m/s² C) 850 m/s² D) 13 m/s² E) 0.15 m/s² *********

71.1. Find the mass of an object which is accelerated at 2 m/s^2 by a force of 800 N.

A) 400 kg

- B) 1600 kg
- C) 100 kg
- D) 2400 kg
- E) 800 kg

72.1. Determine the acceleration of a mass of 24 kg when a force of 6 N acts on it. What is the acceleration if the force were doubled and the mass was halved? A) 24.7 m/s² B) 8 m/s²

C) 18 m/s^2 D) 30 m/s^2

E) 0.25 m/s²

73.1. A mass of 8 kg is accelerating at 5 m/s². Determine the resultant force that is causing the acceleration.

A) 13 N B) 40 N C) 1.6 N D) 0.625 N E) 169 N *****

74.1. If body A exerts a force on body B, then body B exerts a force of equal magnitude on body A, but in the opposite direction.

- A) Newton's first law
- B) Newton's second law
- C) Newton's third law
- D) law of gravity
- E) straight line motion law ******

75.1. Every point mass attracts every other point mass by a force directed along the line connecting the two.
This force is proportional to the product of the masses and inversely proportional to the square of the distance between them. It is ...
A) Newton's first law
B) Newton's second law
C) Newton's third law
D) Law of universal gravitation

E) Galiley's law

76.1. Choose a Newton's law universal gravitation:

A)
$$F = G \frac{\overline{m_1 m_2}}{r^2}$$

B) $\vec{F} = \vec{F_1} + \vec{F_2} + \dots + \vec{F_i}$
C) $\vec{F} = m \frac{d\vec{v}}{dt}$
D) $\vec{F} = \mu \cdot \vec{N}$
E) $M = Fr \sin \alpha$

77.1. What Is a weight of a body? A) $\vec{F} = m\vec{a}$ B) $\vec{p} = m\vec{v}$ C) $M = Fr \sin \alpha$ D) $\vec{P} = m\vec{g}$ E) $F = G \frac{m_1 m_2}{r^2}$

78.1... is calculated from the product of the mass and velocity of an object.

A) torque

B) momentum

C) force

D) acceleration

E) weight

79.1. How can you calculate a momentum?

A)
$$\vec{F} = m\vec{a}$$

B) $F = -k \cdot \Delta x$
C) $\vec{P} = m\vec{g}$
D) $\vec{F} = \vec{F_1} + \vec{F_2} + \dots + \vec{F_i}$
E) $\vec{p} = m\vec{v}$

80.1. What is a SI unit of the momentum? A) kg B) Newton C) kg·m/s D) m/s E) N/kg ********

81.1. A soccer ball of mass 420 g is kicked at 20 m/s towards the goal

post. Calculate the momentum of the ball. A) 8,4 kg·m/s B) 8400 kg·m/s C) 21. kg·m/s D) 440 kg·m/s E) 400 kg·m/s ******

83.1. Product if the net force and the time interval for which the force acts is ...

84.1. How can you calculate impulse?

A) $m\vec{g}$ B) $m\vec{v}$ C) $m\vec{g} - m\vec{v}$ D) $\vec{F} \cdot dt$ E) $\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_i$ *********

86.1. What is a SI unit of the impulse? A) kg B) m/s² C) N·s D) kg·m/s² E) N/s *********

87.1. Choose a formula for a torque: A) $M = Fr \sin \alpha$ B) $\vec{P} = m\vec{g}$ C) $F = G \frac{m_1 m_2}{r^2}$ D) $F = -k \cdot \Delta x$ E) $\vec{F} = \mu \cdot \vec{N}$ **********

88.1. A driving force of 750 N acts on a car of mass 600 kg. Calculate the car's acceleration. A) 0.75 m/s^2 B) 1.25 m/s^2

B) 1.25 m/s⁻ C) 1. m/s² D) 1.3 m/s² E) 1.5 m/s² *********

89.1. A driving force of 900 N acts on a car of mass 600 kg. Calculate the car's speed after 20 s.

A) 10 m/s

- B) 20 m/s
- C) 30 m/s
- D) 40 m/s
- E) 50 m/s

90.1. What is a resultant of forces? A) $F = -k \cdot \Delta x$ B) $\vec{F} = \mu \cdot \vec{N}$ C) $\vec{p} = m\vec{v}$ D) $\vec{F} = m\vec{a}$ E) $\vec{F} = \vec{F_1} + \vec{F_2} + \dots + \vec{F_i}$ **********

91.1. Two forces act for a point. Modulus of forces $f_1=3$ N and $f_2=4$ N. The angle between the forces equal 90°.What is a resultant of forces? A) 1. N B) 0 N C) 25 N D) 5 N E) 3 N *****

95.1. What is a magnitude of free fall acceleration?

A) 8.9 B) 3 C) 8.31 D) 6.02 E) 9.8 *****

96.1. Motion in the Earth's gravitation field when no other forces act on the object is ...A) lineal

97.1. An apple falls from a tree. It takes 1. s to reach the ground. What is the velocity of the apple when it reaches the ground? A) 4.9 m/s

B) 1. m/s

- C) 9.8 m/s
- D) 11.2 m/s
- E) 19.6 m/s
- *****

98.1. A stone is dropped from the top a 5 m. How long does it takes the stone to reach the ground?

A) 0.1. s B) 0.5 s C) 50 s D) 1.7 s E) 1. s **********

B)
$$\vec{F} = m\vec{v}$$

C) $\vec{F} = \mu \cdot \vec{N}$
D) $\vec{F} = \frac{d\vec{p}}{dt}$
E) $F = Fr \sin \alpha$

101.1. Which type of mechanical energy do exist?A) kinetic and potentialB) potential and intern

C) intern and nuclear D) nuclear and kinetic E) electric and magnetic ******* 102.1. is the energy an object has due to its position state: A) kinetic B) nuclear C) potential D) magnetic E) intern ***** 103.1. What is a SI unit of potential energy? A) W (Watt) B) N (Newton) C) g (m/s²) D) G (without unit) E) J (Joule) ********

104.1. Chose a formula for the gravitational potential energy above Earth: mv^2

A)
$$\frac{mv^2}{2}$$

B) mgh
C) $\frac{J\omega^2}{2}$
D) ρgV
E) $6\pi\eta rv$

105.1. A brick with a mass 2 kg is lifted to the top of a 4 m high roof. Calculate the potential energy of the brick at the top of the roof.

A) 88 J B) 180 J C) 58,8 J D) 78,4 J E) 66 J *****

106.1. The energy an object has due to its motion is ...A) internB) nuclear

C) kinetic D)potential E)electric *********

107.1. Choose a formula for the kinetic energy:

A) $\frac{\rho v l}{\eta}$ B) $\frac{mv^2}{2}$ C) $\sqrt{2gh}$ D) mgh E) $-G \frac{Mm}{r}$ *********

108.1. Choose a formula for the kinetic energy:

A) $-k \cdot \Delta x$ B) $\frac{p^2}{2m}$ C) $G \frac{m_1 m_2}{r^2}$ D) $m \frac{d\vec{v}}{dt}$ E) $\frac{\rho v^2}{2t}$

109.1. Choose a formula for the kinetic energy of translational motion: A) $Fr \sin \alpha$

B) $m\vec{v}$ C) $\frac{d\vec{p}}{dt}$ D) $\frac{1}{2}mv^2$ E) $\frac{J\omega^2}{2}$

110.1. Choose a formula for the kinetic energy of translational motion: A) $J\omega^2$ B) $m\vec{v}h$ C) $\frac{L^2}{2J}$ D) $\frac{1}{2m}p^2$ E) $\frac{d\vec{p}}{dt}$ ****** 111.1. Choose a formula for the kinetic energy of rotational motion: A) $2L^2$ B) $m\vec{v}/2$ C) $\frac{J\omega^2}{2}$ D) $J\omega^2$ E) $m\vec{v}p$ ********

112.1. Choose a formula for the kinetic energy of rotational motion:

A)
$$\frac{p^2}{2m}$$

B) $J\omega$
C) $\frac{L^2}{2J}$
D) $\rho \vec{g} h$
E) $-k \cdot \Delta x/2$

113.1. Choose a formula for the elastic potential energy:

A)
$$\frac{p^2}{2m}$$

B) $J\omega$
C) $\frac{kx^2}{2}$
D) $\rho \vec{g} h$
E) $-k \cdot \Delta x/2$

114.1. Choose a formula for the angular momentum:

A) $L = Fp \sin \alpha$ B) $L = pr \sin \alpha$ C) $L = Fr \sin \alpha$ D) $L = prv \cos \alpha$ E) $L = Fr \cos \alpha$ *********

116.1. A car with a mass of 700 kg is travelling at a constant velocity 0f 100 km/hr. Calculate the kinetic energy of the car.

- A)7000 J
- B) 3.5 kJ
- C) 1.08 MJ
- D) 270 kJ
- E) 540123 J
- ******

117.1. What is a mechanical energy?

- A) magnetic
- B) potential
- C) electric
- D) intern
- E) nuclear
- ********

118.1. What is a mechanical energy?

- A) nuclear
- B) intern
- C) electric
- D) kinetic
- E) magnetic
- *****

119.1. Choose a mechanics energy conservation law:

A)
$$E_p + E_k = const$$

B) $E_p + E_k = 0$
C) $E_p - E_k = 0$
D) $E_p = E_k$
E) $\frac{E_p}{E_k} = const$

120.1. Choose a mechanics energy conservation law:

A)
$$E_{p1} + E_{k1} = E_{p2} + E_{k2}$$

B) $E_p + E_{k2} = 0$
C) $E_{p1} - E_{k2} = E_{p1} + E_{k2}$
D) $E_{p12} = E_{k12}$
E) $\frac{E_{p1}}{E_{k1}} = \frac{E_{p2}}{E_{k2}}$

121.1. Choose a mechanics energy conservation law:

A)
$$\frac{dE}{dt} = const$$

B) $dE + dt = const$
C) $\frac{dE}{dt} = 0$
D) $dE + dt = 0$
E) $dE_p - dE_k = const$

122.1. Choose a momentum conservation law:

A) $\sum \vec{p}_i = 0$ B) $\sum \vec{p}_i = const$ C) $\sum \vec{p}_i = E_p 1 + E_{k2}$ D) $\sum \vec{p}_i = -1$ E) $\sum \vec{p}_i = 1$ ***********

123.1. Choose a momentum conservation law:

A)
$$\frac{\vec{p}_1}{\vec{p}_2} = \frac{\vec{p}'_1}{\vec{p}'_2}$$

B) $\sum \vec{p}_i = -1$
C) $\vec{p}_1 + \vec{p}_2 = \vec{p}'_1 / \vec{p}'_2$
D) $\frac{d\vec{p}}{dt} = 0$
E) $\sum \vec{p}_i = 0$

124.1. Choose a momentum conservation law:

A)
$$\frac{\vec{p}_{1}}{\vec{p}_{2}} = \frac{\vec{p}_{1}}{\vec{p}_{2}}$$

B) $\frac{d\vec{p}}{dt} = const$
C) $\sum \vec{p}_{i} = 1$
D) $\frac{d\vec{p}}{dt} = 1$
E) $\vec{p}_{1} + \vec{p}_{2} = \vec{p}_{1}' + \vec{p}_{2}'$

125.1. Choose an angular momentum conservation law:

A)
$$\sum \vec{L}_i = 0$$

B) $\sum \vec{L}_i = const$
C) $\sum \vec{L}_i = E_p 1 + E_{k2}$
D) $\sum \vec{L}_i = -1$
E) $\sum \vec{L}_i = 1$

126.1. Choose an angular momentum conservation law:

A)
$$\frac{\vec{L}_{1}}{\vec{L}_{2}} = \frac{\vec{L}_{1}}{\vec{L}_{2}}$$

B) $\sum \vec{L}_{i} = -1$
C) $\vec{L}_{1} + \vec{L}_{2} = \vec{L}_{1}^{'}/\vec{L}_{2}$
D) $\frac{d\vec{L}}{dt} = 0$
E) $\sum \vec{L}_{i} = 0$

127.1. Choose an angular momentum conservation law:

A)
$$\frac{\vec{L}_1}{\vec{L}_2} = \frac{L'_1}{L'_2}$$

B) $\frac{d\vec{L}}{dt} = const$
C) $\sum \vec{L}_i = 1$
D) $\frac{d\vec{L}}{dt} = 1$
E) $\vec{L}_1 + \vec{L}_2 = \vec{L}_1 + \vec{L}_2$

128.1. Choose an angular momentum conservation law:

A)
$$\frac{\vec{L}_1}{\vec{L}_2} = \frac{\vec{L}_1}{\vec{L}_2}$$

B) $\frac{d\vec{L}}{dt} = const$
C) $\sum \vec{L}_i = 1$
D) $\frac{d\vec{L}}{dt} = 1$
E) $J_1 \vec{\omega}_1 = J_2 \vec{\omega}_2$

129.1. Mechanical energy

conservation law:

A) energy cannot be crated or

destroyed

B) energy cannot be created, but can be destroyed

C) energy can be created, but cannot be destroyed

D) energy can be created and can be destroyed

E) energy in the universe is equal zero ********

130.1. What is a work: A) $A = F + S \cos \alpha$ B) $A = FS \cos \alpha$ C) $A = FS \sin \alpha$ D) $A = Fg \cos \alpha$ E) $A = Fv \sin \alpha$ **********

131.1. If you push a box 20 m forward by applying a force of 15 N in the forward direction, what is the work you have done on the box?

A) 35 J B) 5 J C) 200 J D) 300 J E) 150 J *****

132.1. A ball of mass 1. kg is dropped from a height of 10 m. Calculate the work done on the ball at the point it hits the ground assuming that there is no air resistance?

A) 35 J B) 5 J C) 10 J D) 9 J E) 100 J *****

133.1. Calculate the work done on a box, if it is pulled 5 m along the ground by applying a force of F=10 N at an angle of 60° to the horizontal

134.1. Choose a harmonic motion equation:

A) $\ddot{x} + \omega_0^2 \dot{x} = 0$ B) $x + \omega_0^2 / x = 0$ C) $\dot{x} \omega_0^2 - x = 0$ D) $\ddot{x} - \omega_0^2 x = 0$ E) $\ddot{x} + \omega_0^2 x = 0$ **********

135.1. What is a solution of a harmonic motion equation? A) $x = a \cos(\omega_0 t + \alpha)$ B) $x = \cos(\omega_0 t + \alpha a)$ C) $x = a \omega_0 \cos(t + \alpha)$ D) $x = \cos(\omega_0 + \alpha)$ E) $x = a \cos(\alpha t + \omega_0)$ **********

136.1. What is *a* in equation $x = a \cos(\omega_0 t + \alpha)$? A) case B) amplitude C) displacement D) angular frequency E) particular case **********

137.1. What does it mean x in equation $x = a \cos(\omega_0 t + \alpha)$? A) amplitude B) case C) particular case D) displacement E) angular frequency ****** 138.1. What is ω_0 in equation $x = a\cos(\omega_0 t + \alpha)?$ A) displacement B) particular case C) amplitude D) case E) angular frequency **********139.1. What does it mean($\omega_0 t + \alpha$) in equation x = $a\cos(\omega_0 t + \alpha)?$

A) displacement

B) angular frequency

C) case

D) particular case

E) amplitude ****************140.1. What is α in equation $x = a \cos(\omega_0 t + \alpha)$? A) particular case B) displacement C) angular frequency D) amplitude E) case ***** 141.1. Relationship between frequency and period: A) v/T = 1B) $\frac{1}{v} = 1/T$ C) $\nu = \frac{1}{T}$ D) $\nu = T$ E) $\nu = 2\pi T$ ******* 142.1. Relationship between frequency and period: A) $T = \frac{1}{2}$ B) $\nu / T = 1$ C) $\nu = T$ D) $\frac{1}{T} = \frac{1}{v}$ E) $T = \pi v$ *******

143.1. Relationship between angular frequency and period:

A)
$$\omega = \frac{1}{2\pi T}$$

B) $\omega = 2\pi T$
C) $\omega = \frac{T}{2\pi}$
D) $\omega = \frac{\pi}{T}$
E) $\omega = \frac{2\pi}{T}$

144.1. Relationship between angular frequency and period:

A)
$$T = \frac{\pi}{2\omega}$$

B) $T = 2\pi\omega$
C) $T = \frac{2\pi}{\omega}$
D) $T = \frac{\omega}{2\pi}$

E)
$$T = \frac{2}{\pi\omega}$$

145.1. Relationship between angular frequency and frequency:

146.1. Relationship between angular frequency and frequency:

147.1. What SI unit of frequency? A) rad/s

- B) Hertz
- C) Pertz
- D) dimensionless

E) rad/s²

148.1. A point mass suspended by a massless, unstrechable string is called:

- A) simple pendulum
- B) physical pendulum
- C) spring pendulum
- D) autumn pendulum
- E) elasticity pendulum

149.1. A period of the simple pendulum:

A)
$$T = 2\pi \sqrt{\frac{m}{k}}$$

B) $T = 2\pi \sqrt{\frac{J}{mgl}}$
C) $T = 2\pi \sqrt{\frac{g}{l}}$

D)
$$T = 2\pi \sqrt{\frac{l}{g}}$$

E) $T = 2\pi \sqrt{gl}$

150.1. What is SI unit of a period?

A) m B) h C) s D) s⁻¹ E) rad/s *********

151.1. An angular frequency of a simple pendulum:

A)
$$\omega = \sqrt{\frac{m}{g}}$$

B) $\omega = \sqrt{\frac{g}{l}}$
C) $\omega = \sqrt{\frac{mgl}{J}}$
D) $\omega = \sqrt{\frac{m}{k}}$
E) $\omega = 2\pi \sqrt{\frac{l}{g}}$

152.1. What does it mean l in the

formula =
$$2\pi \sqrt{\frac{l}{g}}$$
?

A) frequencyB) free fall accelerationC) mass of a string

D) gravity constant

E) length of string ******

153.1. What does it mean g in the

formula =
$$2\pi \sqrt{\frac{l}{g}}$$
?

- A) length of stringB) free fall acceleration
- B) free fail acceleration C also that is the set of C is interval.
- C) elasticity coefficient
- D) mass of a body

E) gravity constant *******

154.1. Find the period of a simple pendulum 1. m long at a location where g=9.8A) 20 s B) 3.13 s C) 2 s D) 0.6 s E) 0.32 s *********

155.1. Find the frequency of a simple pendulum 1. m long at a location where g=9.8A) 3.13 Hz B) 2 Hz C) 1.5 Hz D) 0.5 Hz E) 4.93 Hz ********

156.1. Find the angular frequency of a simple pendulum 1. m long at a location where g=9.8 A) 2 rad/s

- B) 3.13 rad/s
- C) 0.5 rad/s
- D) 0.6 rad/s
- \vec{E}) 20 rad/s
- *********

157.1. The period of a spring pendulum:

A)
$$T = 2\pi \sqrt{\frac{m}{k}}$$

B) $T = 2\pi \sqrt{\frac{l}{g}}$
C) $T = 2\pi \sqrt{\frac{mgl}{J}}$
D) $T = 2\pi \sqrt{\frac{mg}{k}}$
E) $T = 2\pi \sqrt{\frac{Jm}{kl}}$

158.1. The angular frequency of a physical pendulum:

A)
$$\omega = 2\pi \sqrt{\frac{Jm}{kl}}$$

B)
$$\omega = \sqrt{\frac{m}{g}}$$

C) $\omega = \pi \sqrt{\frac{m}{k}}$
D) $\omega = \sqrt{\frac{mgl}{J}}$
E) $\omega = \sqrt{\frac{Jm}{k}}$

159.1. The period of a physical pendulum:

A)
$$T = 2\pi \sqrt{\frac{l}{g}}$$

B) $T = 2\pi \sqrt{\frac{Jm}{k}}$
C) $T = 2\pi \sqrt{\frac{m}{k}}$
D) $T = 2\pi \sqrt{\frac{J}{mgl}}$
E) $T = 2\pi \sqrt{\frac{Jm}{kgl}}$

160.1. The angular frequency of a spring pendulum:

A)
$$\omega = \sqrt{\frac{k}{mgl}}$$

B) $\omega = \sqrt{\frac{k}{m}}$
C) $\omega = \frac{m}{k}$
D) $\omega = \sqrt{\frac{m}{k}}$
E) $\omega = \sqrt{mk}$

161.1. What does it mean m in the

formula = $2\pi \sqrt{\frac{m}{k}}$? A) mass of a body B) elasticity coefficient C) spring constant D) mass of the spring E) mass of the cat *********

B) Hooke's constantC) free fall accelerationD) mass of the spring

E) moment of inertia

164.1. Find a period of a spring pendulum of a mass 2 kg and spring constant of 32 N/m:

A) 1.5 s B) 1.8 s C) 1.57 s D) 4.02 s E) 3.6 s **********

165.1. Find a frequency of a spring pendulum of a mass 8 kg and spring constant of 50 N/m:

A) 0.8 Hz B) 2.5 Hz C) 1. Hz D) 0.4 Hz E) 7.6 Hz **********

166.1. A 2 kg metal ball is suspended from a rope. If it is released from point A and swings down to the point B (the bottom of its arc):



. Calculate the

velocity of the ball at point B

A) 1. m/s B) 1.5 m/s C) 2 m/s D) 3.13 m/s E) 9.86 m/s ******

167.1. A pendulum bob of mass 1.5 kg, swings from a height A to the bottom of its arc at B. The velocity of the bob at B is 4 m/s. Calculate the height A from which the bob was released. Ignore the effects of air friction.

A) 0.1. m B) 1. m C) 0.3 m D) 0.6 m E) 0.8 m *********

168.1. Find a angular frequency of a spring pendulum of a mass 20 kg and spring constant of 500 N/m:

169.1. Choose a Bernoulli' equation:

A)
$$\rho gh + \frac{\rho v^2}{2} + p = const$$

B) $v = \sqrt{2gh}$
C) $F = 6\pi\eta r v$
D) $Q = \frac{\pi R^4 (p_1 - p_2)}{8\eta l}$

E)
$$R_e = \frac{\rho v l}{\eta}$$

170.1. Choose a Bernoulli' equation:
A)
$$v = \sqrt{2gh}$$

B) $\rho gh_1 + \frac{\rho v_1^2}{2} + p_1 = \rho gh_2 + \frac{\rho v_2^2}{2} + p_2$
C) $F = 6\pi\eta r v$
D) $Q = \frac{\pi R^4 (p_1 - p_2)}{8\eta l}$
E) $R_e = \frac{\rho v l}{\eta}$

171.1. Choose a Bernoulli' equation for horizontal tube:

172.1. Choose a Torricelli's equation:
A)
$$\rho gh + \frac{\rho v^2}{2} + p = const$$

B) $v = \sqrt{2gh}$
C) $F = 6\pi\eta rv$
D) $Q = \frac{\pi R^4 (p_1 - p_2)}{8\eta l}$
E) $R_e = \frac{\rho v l}{\eta}$

173.1. Choose a Reynolds number:
A)
$$\rho gh + \frac{\rho v^2}{2} + p = const$$

B) $v = \sqrt{2gh}$
C) $F = 6\pi\eta rv$
D) $Q = \frac{\pi R^4(p_1 - p_2)}{8\eta l}$
E) $R_e = \frac{\rho v l}{\eta}$

174.1. Choose a Bernoulli's law for the horizontal pipe:

A)
$$\frac{\rho v^2}{2} + p = const$$

B) $v = \sqrt{2gh}$
C) $F = 6\pi\eta r v$
D) $Q = \frac{\pi R^4 (p_1 - p_2)}{8\eta l}$
E) $R_e = \frac{\rho v l}{\eta}$

175.1. Choose a Bernoulli's law for the horizontal pipe:

η ***************

178.1. Choose a continuity equation: A) Sv = const

B)
$$v = \sqrt{2gh}$$

C) $F = 6\pi\eta r v$
D) $Q = \frac{\pi R^4 (p_1 - p_2)}{8\eta l}$
E) $R_e = \frac{\rho v l}{\eta}$

A) volume of fluid

B) viscosity of the liquid

C) density of the fluid

D) radius of the sphere

E) velocity of the sphere

181.1. What does Avogadro's number equal?

A) $6.02 \cdot 10^{23} 1/mol$ B) $1.38 \cdot 10^{-23} 1/mol$ C) $8.85 \cdot 10^{-12} 1/mol$ D) $8.31 \cdot 10^{25} 1/mol$ E) $6.67 \cdot 10^{11} 1/mol$ *********

182.1. What does Boltzmann's constant equal?

A) $6.02 \cdot 10^{23} J/K$ B) $1.38 \cdot 10^{-23} J/K$ C) $8.85 \cdot 10^{-12} J/K$ D) $8.31 \cdot 10^{25} J/K$ E) $6.67 \cdot 10^{11} J/K$ *********

183.1. What does ideal-gas constant equal?

184.1. What is an amount of substance? A) $\nu = \frac{mN}{M}$

B)
$$\nu = \frac{m}{MN_A}$$

C) $\nu = \frac{M}{MN}$
D) $\nu = \frac{mN_A}{M}$
E) $\nu = \frac{m}{M}$

185.1. What is a relationship between constants k, R, N_A :

A)
$$kRN_A = 1$$

B) $R = kN_A$
C) $k = RN_A$
D) $R = \frac{k}{N_A}$
E) $k+N_A = R$

186.1. What is relationship between a molar mass and molecular mass?

A)
$$M = M_r \cdot 10^{-3} g/mol$$

B) $M = \frac{M_r}{10^{-3}} kg/mol$
C) $M = \frac{M_r}{10^3} g/mol$
D) $M = M_r \cdot 10^{-3} kg/mol$
E) $M = M_r \cdot 10^3 kg/mol$

187.1. How can you calculate a number of gas molecules?

A)
$$N = mMN_A$$

B) $N = \nu N_A$
C) $N = \frac{m}{M}kN_A$
D) $N = \frac{\nu}{N_A}$
E) $N = MN_A$

188.1. How can you calculate a number of gas molecules?

A)
$$N = mMN_A$$

B) $N = \frac{m}{M}N_A$
C) $N = \frac{m}{M}kN_A$
D) $N = \frac{v}{N_A}$
E) $N = MN_A$

189.1. Calculate a molar mass of water H_2O :

190.1. Calculate a molar mass of water NH₃:
A) 31. *kg/mole*B) 19000 *kg/mol*

C) 0.017 kg/mol
D) 0.019 kg/mol
E) 0.022 kg/mol

191.1. A balloon contains 2 g of a nitrogen. Calculate the amount of substance.

192.1. A tank consists 176 g of carbon dioxide. Find a number of molecules.

A) $24 \cdot 10^{23}$ B) $38 \cdot 10^{23}$ C) $33 \cdot 10^{23}$ D) $528 \cdot 10^{23}$ E) $15 \cdot 10^{23}$ ************

193.1. What is a standard temperature and pressure?

A) 0 K, 100 Pa B) 0° C, 100 Pa C) -273° C, 100 kPa D) 273 K, 100 kPa E) 0° C, 100 MPa *********

194.1. What is a major equation of molecular-kinetic theory?

A)
$$\frac{pv}{T} = const$$

B) $\left(p + \frac{av^2}{v^2}\right)(V - vb) = vRT$

C) $\delta Q = dU + pdV$ D) $p = \frac{1}{3}nm_0 \langle v \rangle^2$ E) $pV = \frac{m}{M}RT$ **********

195.1. What is a major equation of molecular-kinetic theory?

196.1. Choose a formula of ideal gas law (Mendeleev-Clapeyron's equation):

A) p = NkTB) pV = vRTC) pV = NRTD) vp = VkTE) $p/V = N_AkT$ *********

197.1. Choose an ideal gas equation:
A)
$$\delta Q = dU + pdV$$

B) $\langle \varepsilon \rangle = \frac{i}{2}kT$
C) $pV = vRT$
D) $v = \sqrt{\frac{2RT}{N}}$

E) $\Delta S = \int \frac{\delta Q}{T}$

198.1. Choose a Mendeleev-Clapeyron's equation: A) p = NkTB) $pV = \frac{m}{M}RT$ C) pV = NRTD) vp = VkTE) $p/V = N_AkT$ **********

199.1. Choose a Clapeyron's equation:

A)
$$pV = vRT$$

B) $N = \frac{m}{M}kN_A$
C) $n = n_0 e^{-Mgh/RT}$
D) $\delta Q = dU + pdV$
E) $\frac{pV}{T} = const$

200.1. Choose a Clapeyron's equation: A) nV = vRT

B)
$$N = \frac{m}{M} k N_A$$

C) $n = n_0 e^{-Mgh/RT}$
D) $\delta Q = dU + pdV$
E) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

201.1. What equation is called Dalton's law?

A)
$$\langle \varepsilon \rangle = \frac{5}{2}RT$$

B) $\delta Q = dU + pdV$
C) $v = \sqrt{\frac{2RT}{M}}$
D) $p = p_1 + p_2 + \dots + p_i$
E) $p = NkT$

202.1. What is a concentration of molecules?

A) n = N/vB) n = N/VC) $n = N_A/V$ D) n = vN/VE) n = N/vV**********

203.1. What does equal average translational kinetic energy of an ideal gas molecule?

A)
$$\langle \varepsilon \rangle = \frac{3}{2}kT$$

B) $\langle \varepsilon \rangle = \frac{2}{3}kT$
C) $\langle \varepsilon \rangle = \frac{3}{2}RT$
D) $\langle \varepsilon \rangle = \frac{5}{2}RT$
E) $\langle \varepsilon \rangle = \frac{2}{5}kT$

204.1. What does equal average kinetic energy of an ideal gas ideal gas molecule?

A)
$$\langle \varepsilon \rangle = \frac{ik}{2}T$$

B) $\langle \varepsilon \rangle = \frac{i}{2T}k$
C) $\langle \varepsilon \rangle = \frac{i}{2}kT$
D) $\langle \varepsilon \rangle = \frac{i}{2}RT$
E) $\langle \varepsilon \rangle = \frac{i}{2R}T$

205.1. What is relationship between a pressure, a temperature and a concentration? A) n = nk/T

A)
$$p = nk/T$$

B) $k = npT$
C) $T = nkp$
D) $n = pkT$
E) $p = nkT$

206.1. How can you determine the root-mean-square speed of a gas molecule?

A)
$$\langle v \rangle = \sqrt{\frac{2RT}{M}}$$

B) $\langle v \rangle = \sqrt{\frac{3RT}{M}}$
C) $\langle v \rangle = \sqrt{\frac{3k}{MT}}$
D) $\langle v \rangle = \sqrt{\frac{3T}{RM}}$
E) $\langle v \rangle = \sqrt{\frac{M}{3RT}}$

207.1. Barometric formula:
A)
$$p = p_0 e^{-Mgh/RT}$$

B) $\Delta S = \int \frac{\delta Q}{T}$
C) $T = p + \frac{a}{V_m^2}$
D) $A = C_V T - \frac{av}{V}$
E) $(V + vb) \left(V - \frac{av^2}{V^2}\right) = vRT$

208.1. Boltzmann's distribution for gas molecules:

A)
$$v = \sqrt{\frac{2RT}{M}}$$

B) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$
C) $\left(p + \frac{av^2}{V^2}\right)(V - vb) = vRT$
D) $n = n_0 e^{-Mgh/RT}$
E) $\delta Q = dU + pdV$

209.1. Boltzmann's distribution for gas molecules:

212.1. Charles's law: A) $\frac{pV}{T} = const$ B) pV = constC) $\frac{V}{T} = const$

D) $\frac{p}{r} = const$ E) vRT = const****** 213.1. Find the Boyle's law: A) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ B) v R p = constC) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ D) $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ E) $p_1V_1 = p_2V_2$ ********** 214.1. Find the Gay-Lussac's law: A) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ B) $\nu RT = const$ C) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ D) $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ E) $p_1V_1 = p_2V_2$ ********** 215.1. Find the Charle's law: A) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ B) vRT = constC) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ D) $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ E) $p_1V_1 = p_2V_2$ ********* 216.1. If the temperature of a gas is a constant that process is called: A) adiabatic B) isothermal C) isochoric D) isobaric

- E) isokinetic
- *********

217.1. If the pressure of a gas is a constant that process is called:

- A) adiabatic
- B) isothermal
- C) isochoric
- D) isobaric
- E) isokinetic

218.1. If the volume of a gas is a constant that process is called: A) adiabatic B) isothermal C) isochoric D) isobaric E) isokinetic ****** 219.1. A process is called isothermal if: A) p=const B) T=const C) M=const D) m=const E) V=const ******** 220.1. A process is called isochoric if: A) p=const B) T=const C) M=const D) m=const E) V=const ****** 221.1. A process is called isobaric if: A) p=const B) T=const

- C) M=const
- D) m=const
- E) V=const
- ******

222.1. A tank contains 1320 g of carbon dioxide. How many moles of carbon dioxide are in a tank?

- A) 10 mol
- B) 41. mol
- C) 30 mol
- D) 13 mol
- E) 47 mol
- *******

223.1. A cylindrical tank has 512 mg of an oxygen. How many molecules are in a cylinder? A) 1.5 · 10²³

B) 91.2 · 10²² C) $2.4 \cdot 10^{21}$ D) 9.6 · 10²¹ E) $9.6 \cdot 10^{24}$ *****

224.1. A 20-L tank consists 3200 g of oxygen at 16°C. What is the pressure in the tank? A) 12 MPa B) 12 kPa C) 0,66 MPa D) 664 MPa E) 2000 Pa *******

225.1. Four moles of an ideal gas are in a rigid cubical box with sides of length 0.2 m. What is a temperature of the gas if the pressure 20 atm? A) 240°C B) 200°C C) 208°C D) 481°C E) 108°C *******

226.1. A balloon has 300 g of hydrogen at 36°C and under pressure of 5 atm. What is a volume of the balloon? A) $0,11. \text{ m}^3$ B) 0.33 m^3 C) $0.55m^3$ D) 0.66 m^3 E) 0.77 m^3 *******

227.1. Calculate the density of the atmosphere of Jupiter where the pressure is 1. atm and the temperature is -107 °C with a hydrogen atmosphere? A) 0.14 kg/m^3 B) 1.2 kg/m^3 C) 0.4 kg/m^3 D) 10 kg/m^3 E) 0.045 kg/m^3

228.1. A canister of volume 50-L has $3 \cdot 10^{25}$ molecules. Find a concentration of molecules. A) $1.5 \cdot 10^{27}m^{-3}$ B) $6 \cdot 10^{26}m^{-3}$ C) $6 \cdot 10^{23}m^{-3}$ D) $6 \cdot 10^{25}m^{-3}$ E) $1.5 \cdot 10^{24}m^{-3}$ ***********

229.1. Calculate the gas temperature in a balloon if the pressure is 1.5 MPa and the concentration of molecules is $30 \cdot 10^{25}$? A) 177 °C B) 45 °C C) 454 °C D) 89 °C E) 276 °C ******

230.1. What is the pressure in cylinder if it has the temperature 162° C and concentration of a molecules $30 \cdot 10^{25}$? A) 5977 Pa B) 6700 kPa C) 1. MPa D) 6 MPa E) 1800 kPa ****

232.1. A 10-L balloon has 16 g of helium under the pressure 1,5 MPa. What is a temperature of a helium in this balloon? A) 32°C B) 208°C C) 504°C D) 481°C E) 43°C **********

234.1. A 6-liter container has 10 g of a gas under pressure of 2 atm and a temperature of 131. °C. What kind of gas is in a container?

A) O₂ B) He C) N₂ D) H₂ E) Ar ***********

energy of an oxygen molecule if a temperature is normal:

A) $90 \cdot 10^{-23} J$ B) $17 \cdot 10^{-19} J$ C) $9,4 \cdot 10^{-21} J$ D) $7 \cdot 10^{-21} J$ E) $3.2 \cdot 10^{-20} J$ ******** 237.1. Calculate an average kinetic energy of an argon (monatomic gas) molecule if a temperature is 65°C:

A)
$$9,4 \cdot 10^{-21} J$$

B) $17 \cdot 10^{-19} J$
C) $7 \cdot 10^{-21} J$
D) $3.2 \cdot 10^{-20} J$
E) $0.15 \cdot 10^{-23} J$

238.1. A balloon with ammonia (NH₃) has a temperature 500°C. What is an average kinetic energy of NH₃molecule? A) $1.5 \cdot 10^{-23} J$ B) $3.2 \cdot 10^{-20} J$

B) $3.2 \cdot 10^{-20} J$ C) $17 \cdot 10^{-19} J$ D) $9.4 \cdot 10^{-21} J$ E) $7 \cdot 10^{-21} J$ *****

240.1. The root-mean-square speed of gas molecules is 540 m/s under temperature of 54,5°C. What is gas this?

- A) carbon dioxide
- B) helium
- C) nitrogen
- D) oxygen
- E) hydrogen

241.1. What is a temperature of carbon dioxide if root-mean-square speed of molecules is 124 m/s? A) 300°C B) 30°C

D) 27°C E) 54,5°C ******

242.1. Equation of state for the real gas is:

A)
$$\left(p + \frac{av^2}{v^2}\right)(V - vb) = vRT$$

B) $p = (dU + pdV)e^{-Mgh/RT}$
C) $\Delta S = \int \frac{\delta Q}{T} + \frac{mdQ}{dT}$
D) $\frac{i}{2}vRT = vRT \ln \frac{V2}{V1} \left(p + \frac{av^2}{V^2}\right)$
E) $\sqrt{\frac{2RT}{M}} - p(V_2 - V_1) = 1$

243.1. Choose the Van der Waals' equation:

A)
$$(p + vRT)(vb - pV) = \frac{av^2}{v^2}$$

B) $\left(vb + \frac{av^2}{v^2}\right)(V - p) = VRT$
C) $\left(p + \frac{av^2}{v^2}\right)(V - vb) = vRT$
D) $(V + vb)\left(V - \frac{av^2}{v^2}\right) = vRT$
E) $\left(p + \frac{av^2}{v^2}\right)(vRT - vb) = V$

244.1. Choose the Van der Waals' equation for the one mole of real gas:

A)
$$\left(p + \frac{a}{v_m^2}\right) (V_m - b) = 1$$

B) $(p + RT)(1 - b) = \frac{a}{v_m^2}$
C) $(p + 1)(V_m - b) = RT$
D) $\left(1 + \frac{a}{v_m^2}\right) (1 - b) = pRT$
E) $\left(p + \frac{a}{v_m^2}\right) (V_m - b) = RT$

245.1. How does determine the intern energy of a real gas?

A)
$$U = v \frac{av}{v} (C_V T - RT)$$

B) $U = v C_V T \left(1 - \frac{av}{v}\right)$
C) $U = \frac{av}{v} (C_V T - v)$
D) $U = v \left(C_V T - \frac{av}{v}\right)$

E)
$$U = C_V T \left(\nu - \frac{a\nu}{v} \right)$$

246.1. What is an intern pressure of a real gas?

A)
$$\int \frac{\delta Q}{T} + \frac{m dQ}{dT}$$

B) $p + \frac{a}{V_m^2}$
C)
$$\sqrt{\frac{2RT}{M}} - p$$

D) $V_m - b$
E) $-Mgh/RT$

247.1. What is a SI unit of a Van der Waals' a-constant ? A) m^3/mol B) $\frac{mol \cdot J}{K^2}$ C) $mol \cdot K \cdot J$ D) $\frac{kg \cdot m^4}{s^2}$ E) $\frac{N \cdot m^4}{mol^2}$ ******

248.1. What is a SI unit of a Van der Waals' b-constant ?

A) $\frac{mol \cdot J}{K^2}$ B) m^3/mol C) $mol \cdot K$ D) $\frac{N \cdot m^4}{mol^2}$ E) mol/m^3 *********

249.1. Formula for the work of a gas: A) $A = \int_{V_1}^{V_2} p dV$ B) $A = p + \frac{a}{V_m^2}$ C) $A = \sqrt{\frac{2RT}{M}} - p(V_2 - V_1)$ D) A = dU + p dVE) $A = \int \frac{\delta Q}{T}$ *********

250.1. What is the work of a gas in isobaric process? A) A = 1

B)
$$A = 0$$

C) $A = p(V_2 - V_1)$
D) $A = vRT \ln \frac{V^2}{V_1}$
E) $A = -1$

251.1. What is the work of a gas in isochoric process?

A)
$$A = p + \frac{a}{V_m^2}$$

B) $A = 0$
C) $A = p(V_2 - V_1)$
D) $A = vRT \ln \frac{V^2}{V_1}$
E) $A = \infty$

252.1. What is the work of a gas in isothermal process?

A) A = 1B) A = 0C) $A = p(V_2 - V_1)$ D) $A = vRT \ln \frac{V^2}{V_1}$ E) $A = C_V T - \frac{av}{V}$ *********

253.1. What is the work of a gas if the temperature is a constant?

A)
$$A = 1$$

B) $A = vRT \ln \frac{V2}{V1}$
C) $A = p(V_2 - V_1)$
D) $A = 0$
E) $A = C_V T - \frac{av}{V}$

254.1. What is the work of a gas if the pressure is a constant?

A)
$$A = p(V_2 - V_1)$$

B) $A = 0$
C) $A = -1$
D) $A = vRT \ln \frac{V^2}{V_1}$
E) $A = -1$

255.1. What is the work of a gas if the volume is a constant?

A)
$$A = p + \frac{a}{V_m^2}$$

B)
$$A = -\infty$$

C) $A = p(V_2 - V_1)$
D) $A = vRT \ln \frac{v_2}{v_1}$
E) $A = 0$

257.1. The sum of kinetic energies of all particles of a system, plus the sum of all the potential energies of interaction among these particles is: A) work

B) heat

C) entropy

D) heat capacity

E) internal energy

258.1. The molar heat capacity at constant volume for monatomic gas:

259.1. The molar heat capacity at constant volume for diatomic gas:

A)
$$C_V = \frac{3}{2}R$$

B) $C_V = \frac{5}{2}R$
C) $C_V = \frac{1}{2}R$
D) $C_V = \frac{7}{2}R$
E) $C_V = \frac{i+2}{2}R$

260.1. The molar heat capacity at constant volume:

A)
$$C_V = \frac{3}{2}R$$

B) $C_V = \frac{5}{2}R$
C) $C_V = \frac{i}{2}R$
D) $C_V = \frac{7}{2}R$
E) $C_V = \frac{i+2}{2}R$

261.1. The molar heat capacity at constant pressure for monatomic gas:

A)
$$C_P = \frac{3}{2}R$$

B) $C_P = \frac{5}{2}R$
C) $C_P = \frac{1}{2}R$
D) $C_P = \frac{7}{2}R$
E) $C_p = \frac{i}{2}R$

262.1. The molar heat capacity at constant pressure for diatomic gas:

263.1. The molar heat capacity at constant pressure:

A)
$$C_P = \frac{3}{2}R$$

B) $C_P = \frac{5}{2}R$
C) $C_P = \frac{1}{2}R$
D) $C_P = \frac{7}{2}R$
E) $C_p = \frac{i+2}{2}R$

264.1. Robert Mayer's equation: A) $C_P = R - C_V$ B) $C_P + C_V = R$

266.1. Find formula for the adiabatic index:

A)
$$\gamma = \frac{C_P}{C_V} R$$

B) $\gamma = \frac{C_P}{C_V}$
C) $\gamma = \frac{C_V}{C_P}$
D) $\gamma = \frac{C_P}{RC_V}$
E) $\gamma = \frac{1}{C_P C_V}$

268.1. Specific heat capacity:
A)
$$c = \frac{mdQ}{dT}$$

B) $c = \frac{dT}{dQ}$
C) $c = \frac{dQ}{mdT}$
D) $c = \frac{dQ}{vdT}$
E) $c = \frac{dT}{dQ}$

269.1. Molar heat capacity:

A)
$$C_m = \frac{dT}{dQ}$$

B) $C_m = \frac{dQ}{mdT}$
C) $C_m = \frac{vdQ}{dT}$
D) $C_m = \frac{dQ}{vdT}$
E) $C_m = \frac{vdQ}{vdT}$

270.1. The first law of thermodynamics: A) $\delta Q = dU + \delta A'$ B) $\Delta S = \int \frac{\delta Q}{T} > 0$ C) pV = vRTD) $n = n_0 e^{-Mgh/RT}$ E) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ *********

271.1. The first law of thermodynamics: A) $p = p_0 e^{-Mgh/RT}$ B) $\Delta S = \int \frac{\delta Q}{T} > 0$ C) $pV = \nu RT$ D) $\delta Q = U_2 - U_1 + \delta A'$ E) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} + C$ *********

272.1. The first law of thermodynamics for isothermal process: A) $\delta Q = dU + \delta A'$ B) $\delta Q = \delta A'$ C) $\delta Q = dU$ D) $\delta A' = -dU$ E) $\delta Q = dU = \delta A'$ ******** 273.1. The first law of thermodynamics for isobaric process: A) $\delta Q = dU + \delta A'$ B) $\delta Q = \delta A'$ C) $\delta Q = dU$ D) $\delta A' = -dU$ E) $\delta Q = dU = \delta A'$ *******
274.1. The first law of thermodynamics for isochoric process:

A)
$$\delta Q = dU + \delta A'$$

B) $\delta Q = \delta A'$
C) $\delta Q = dU$
D) $\delta A' = -dU$
E) $\delta Q = dU = \delta A'$

275.1. The first law of thermodynamics for adiabatic process: A) $\delta O = dU + \delta A'$

B)
$$\delta Q = dU + bA$$

B) $\delta Q = \delta A'$
C) $\delta Q = dU$
D) $\delta A' = -dU$
E) $\delta Q = dU = \delta A'$

276.1. Choose the formula for the internal energy of ideal gas:

A)
$$U = \frac{l}{2}vRT$$

B) $U = nkT$
C) $U = \int \frac{\delta Q}{T}$
D) $U = \frac{pV}{T}$
E) $U = \sqrt{\frac{2RT}{M}}$

277.1. Choose the formula for the internal energy of ideal gas:

A)
$$U = \frac{i}{2R}T$$

B)
$$U = p + \frac{av^2}{v^2}$$

C)
$$U = N \cdot \frac{i}{2}kT$$

D)
$$U = vN/V$$

E)
$$U = dU + pdV$$

278.1. Choose the formula for the internal energy of ideal gas:

A)
$$U = \frac{i}{2R}T$$

B) $U = p + \frac{av^2}{v^2}$
C) $U = N\langle \varepsilon \rangle$

D) U = vN/VE) U = dU + pdV*********

279.1. Choose the formula for the internal energy of ideal gas:

A)
$$U = \frac{3k}{MT}$$

B) $U = V - \nu b$
C) $U = NkT$
D) $U = \frac{\delta Q}{T}$
E) $U = \frac{i}{2} \frac{m}{M} RT$

280.1. Choose the formula for the internal energy of ideal gas:

A)
$$U = vC_V T$$

B) $U = e^{-Mgh/RT}$
C) $U = \frac{ik}{2}T$
D) $U = \frac{m}{M}kN_A$
E) $U = \frac{2RT}{M}$

281.1. Choose the formula for the internal energy of ideal gas:

A)
$$U = \frac{m}{M}kN_A$$

B) $U = \frac{m}{M}C_VT$
C) $U = \frac{i}{2}kT$
D) $U = \frac{m}{M}kN_A$
E) $U = Mgh/RT$

282.1. In the isobaric process volume of the gas changes from 30 to 10 L.
How much work does the gas do if the pressure is 1.2 atm?
A) 12kJ
B) 0.036 MJ
C) 24 kJ
D) 48 kJ
E) 40000 J

283.1. What is a change of a volume if the gas do a work 50 kJ under pressure 2.9 atm?

A) 0.27 m³ B) 0.17 m³ C) 10 m³ D) 71. m³ E) 145 m³ *********

284.1. In the isobaric process volume of the gas changes from 50 to 100 L. How much is the pressure if a work is 75 kJ? A) 1500 Pa

B) 500 kPa

- C) 11250 Pa
- D) 1.5 MPa

E) 225000 Pa *********

285.1. A tank consists 320 g of oxygen at 16°C. What is the internal energy of the gas?

A) 5120 J
B) 4335 J
C) 24 kJ
D) 60 kJ

E) 0.2 kJ

286.1. A 50-L balloon has ideal gas under pressure of 5 atm. What is an internal energy of a gas?

A) 25 kJ

B) 250 J

C) 0.25 J

D) 25 MJ

E) 1. kJ *********

287.1. A container of volume 258-L consist $3 \cdot 10^{25}$ molecules of diatomic gas with concentration $3 \cdot 10^{25} m^{-3}$ under the temperature 300 K. Find an internal energy of a gas.

- A) 2700 J
- B) 48 kJ
- C) 20 J
- D) 567 kJ
- E) 80 kJ

288.1. A container of volume 258-L consist $3 \cdot 10^{25}$ molecules of monatomic gas with concentration $3 \cdot 10^{25}m^{-3}$ under the temperature 300 K. Find an internal energy of a gas? A) 2700 J B) 12600 J C) 48000 J D) 0.2 MJ E) 0.08 MJ ******

289.1. Calculate the molar heat capacity at constant volume for helium:

A) 20.78 J/(mol*K) B) 12,5 J/(mol*K) C) 20.8 J/(mol*K) D) 29.085 J/(mol*K) E) 24.9 J/(mol*K) ********

290.1. Calculate the molar heat capacity at constant volume for nitrogen:

A) 29.085 J/(mol*K) B) 33.24 J/(mol*K) C) 20,8 J/(mol*K) D) 24,9 J/(mol*K) E) 12,5 J/(mol*K) ********

291.1. Calculate the molar heat capacity at constant pressure for nitrogen:

A) 12.5 J/(mol*K) B) 32.15 J/(mol*K) C) 33.24 J/(mol*K) D) 24.9 J/(mol*K) E) 29.085 J/(mol*K) ********

292.1. Calculate the molar heat capacity at constant pressure for argon (monatomic gas): A) 33.24 J/(mol*K) 293.1. Calculate the molar heat capacity at constant pressure for oxygen: A) 29.085 J/(mol*K) B) 20.78 J/(mol*K) C) 33.24 J/(mol*K)

D) 24,9 J/(mol*K) E) 12,5 J/(mol*K) ********

294.1. Calculate the molar heat capacity at constant pressure for NH₃: A) 12.5 J/(mol*K) B) 33.24 J/(mol*K) C) 20,8 J/(mol*K) D) 24.9 J/(mol*K) E) 29.085 J/(mol*K) *****

295.1. Calculate adiabatic index for the neon (monatomic gas):

- A) 1.67
- B) 0.33
- C) 1.4
- D) 0.6

E) 1.33 *********

296.1. Calculate adiabatic index for the hydrogen:

- A) 1.67
 B) 0.33
 C) 1.4
 D) 0.6
- E) 1.33

297.1. Calculate adiabatic index for the SO₂: A) 1.67 B) 0.33 C) 1.4 D) 0.6 E) 1.33 *****

internal energy of the gas?

300.1. A container consists 3 · 10²⁵ molecules of carbon dioxide under temperature of 300 K. What is an internal energy of the gas?
A) 373 kJ
B) 187 kJ
C) 1024 kJ
D) 311. kJ
E) 900 kJ

L) 700 KJ

302.1. A container consists 4 moles of oxygen under temperature of 40°C. What is an internal energy of the gas?

303.1. A container consists 6 moles of SO₃ under temperature of 50°C. What is an internal energy of the gas? A) 5790 J

- B) 11330 J
- C) 16280 J
- D) 26010 J
- E) 48314 J
- ******

304.1. A tank has 5 g of helium (monatomic gas) under temperature of 60°C. What is an internal energy of the gas?

A) 3.5 kJ
B) 5 kJ
C) 1.2 kJ
D) 2.8 kJ
E) 7.8 kJ

305.1. A tank has 7,5 g of nitrogen under temperature of 70°C. What is an internal energy of the gas?

A) 1861J

- B) 1945 J
- C) 1272 J
- D) 1908 J
- E) 2016 J
- *****

306.1. A tank has 10 g of NO_2 under temperature of 80°C. What is an internal energy of the gas? A) 4.6 kJ

- B) 1,9 kJ
- $C > 2.1 \ 1_{-1}$
- C) 2.1. kJ
- D) 3.3 kJ E) 1.2 kJ

L) 1.2 KJ *********

307.1. What is an entropy?

A)
$$S = \int_{V1}^{V2} p dV$$

B) $S = \sqrt{\frac{2RT}{M}}$
C) $S = \int \frac{\delta Q}{T}$
D) $S = dU + p dV$
E) $S = vRT \ln \frac{V2}{V1}$

formula for entropy: A) $S = k \ln W$ B) S = -Mgh/RTC) $S = vRT \ln \frac{V^2}{V_1}$ D) $S = vb + \frac{av^2}{V^2}$

E)
$$S = \frac{i}{2}RT$$

310.1. The thermal efficiency of a heat engine:

A)
$$\eta = 1 + \frac{Q_C}{Q_H}$$

B) $\eta = \frac{A}{Q_H}$
C) $\eta = \frac{Q_H}{A}$
D) $\eta = \frac{Q_H + Q_C}{Q_H}$
E) $\eta = \frac{Q_H}{Q_C} - 1$

311.1. The thermal efficiency of a heat engine:

A)
$$\eta = \frac{Q_H + Q_C}{Q_H}$$

B) $\eta = \frac{Q_H}{Q_C} - 1$
C) $\eta = \frac{Q_H}{A}$

D) $\eta = \frac{Q_H - Q_C}{Q_H}$ E) $\eta = 1 + \frac{Q_C}{Q_H}$ **********

312.1. The thermal efficiency of a heat engine:

A)
$$\eta = \frac{Q_H}{Q_C} - 1$$

B) $\eta = \frac{Q_H}{A}$
C) $\eta = 1 + \frac{Q_C}{Q_H}$
D) $\eta = \frac{Q_H + Q_C}{Q_H}$
E) $\eta = 1 - \frac{Q_C}{Q_H}$

313.1. The thermal efficiency of a heat engine:

A)
$$\eta = 1 + \frac{Q_2}{Q_1}$$

B) $\eta = \frac{A}{Q_1}$
C) $\eta = \frac{Q_1}{A}$
D) $\eta = \frac{1+Q_2}{Q_1}$
E) $\eta = \frac{Q_1}{Q_2} - 1$

314.1. The thermal efficiency of a heat engine:

A)
$$\eta = \frac{Q_1 + Q_2}{Q_1}$$

B) $\eta = \frac{Q_1}{Q_2} - 1$
C) $\eta = \frac{Q_1}{A}$
D) $\eta = \frac{Q_1 - Q_2}{Q_1}$
E) $\eta = 1 + \frac{Q_2}{Q_1}$

315.1. The thermal efficiency of a heat engine:

A)
$$\eta = \frac{Q_1}{Q_2} - 1$$

B) $\eta = \frac{Q_1}{A}$
C) $\eta = 1 + \frac{Q_2}{Q_1}$

D)
$$\eta = \frac{Q_1 + Q_2}{Q_1}$$

E) $\eta = 1 - \frac{Q_2}{Q_1}$

316.1. The thermal efficiency of the Carnot cycle:

A)
$$\eta = \frac{T_1 + T_2}{T_1}$$

B) $\eta = \frac{T_1}{T_2} - 1$
C) $\eta = \frac{T_1}{A}$
D) $\eta = \frac{T_1 - T_2}{T_1}$
E) $\eta = 1 + \frac{T_2}{T_1}$

317.1. The thermal efficiency of the Carnot cycle:

A)
$$\eta = \frac{T_1}{T_2} - 1$$

B) $\eta = \frac{T_1}{A}$
C) $\eta = 1 + \frac{T_2}{T_1}$
D) $\eta = \frac{T_1 + T_2}{T_1}$
E) $\eta = 1 - \frac{T_2}{T_1}$

318.1. A diesel engine performs 2200J of mechanical work and discards4300 of heat each cycle. What is the thermal efficiency of the engine?

319.1. An aircraft engine takes in
9000 J of heat and discards 6400 J
each cycle. What is the thermal
efficiency of the engine?
A) 71%
B) 171%
C) 58%
D) 41%
E) 29%

320.1. A Carnot cycle is operated between heat reservoirs at temperatures of 520 K an 300 K.What is the thermal efficiency of the engine?A) 37%

- B) 29%C) 63%
- D) 42%
- E) 58%

321.1. How many types of electric charges do exist?

- A) 1
 B) 2
 C) 3
 D) 4
- E) 5

322.1. What is an elementary charge? A) 0 B) 1.6[.]10⁻¹⁹ C C) 8.85[.]10⁻¹² C D) 6.02[.]10⁻²³ C E) 1.38[.]10⁻¹⁶ C *********

325.1. What is a charge of proton? A) 0 B) -1.38·10⁻¹⁶ C C) $-1.6 \cdot 10^{-19}$ C D) $6.02 \cdot 10^{-23}$ C E) $+1.6 \cdot 10^{-19}$ C **********

326.1. Choose an electric constant:

A) $1.38 \cdot 10^{-16}$ F/m B) $1.6 \cdot 10^{-19}$ F/m C) $6.02 \cdot 10^{-23}$ F/m D) $8.85 \cdot 10^{-12}$ F/m E) $1.38 \cdot 10^{-16}$ F/m *********

327.1Choose a major law of electrostatic:

A)
$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

B)
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

C)
$$\oint \vec{E} d\vec{S} = \frac{Q}{\varepsilon_0}$$

D)
$$A = \frac{q_1 q_2}{4\pi\varepsilon_0 r} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

E)
$$W = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

328.1. The magnitude of the electric force between two point charges is directly proportional to the product of the charge and inversely proportional to the square of the distance between them. It is:

A) Ohm's law

- B) Gauss's law
- C) Coulomb's law

D) Kirchhoff's junction rule

E) Kirchhoff's junction loop rule

329.1. Choose a Coulomb's law:

A)
$$W = q(\varphi_1 - \varphi_2)$$

B) $\oint \vec{E} d\vec{S} = \frac{Q}{\varepsilon_0}$
C) $I = \frac{\varepsilon}{R+r}$
D) $F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$
E) $A = \frac{q_1q_2}{4\pi\varepsilon_0 r} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

330.1. What is characteristic of electric fields? A) $E = \frac{\sigma}{\varepsilon_0}$ B) $A = -q\Delta\varphi$ C) $\Phi_E = ES \cos \alpha$ D) $C = \frac{q}{\varphi_1 - \varphi_2}$ E) $\vec{E} = \frac{\vec{F}}{q}$ *********** 331.1. The electric force using on a charged body: A) $\vec{F} = ES \cos \alpha$ B) $\vec{F} = q\vec{E}$ C) $\vec{F} = \frac{\sigma}{\varepsilon_0}$ D) $\vec{F} = q(\varphi_1 - \varphi_2)$ E) $\vec{F} = \frac{q}{\varphi_1 - \varphi_2}$ 332.2 The electric dipole moment: A) $\vec{p} = q\vec{l}$ B) $\vec{F} = q\vec{E}$ C) $\vec{E} = \frac{\vec{F}}{q}$ D) $\Phi_E = ES \cos \alpha$ E) $A = -q\Delta\varphi$ ******* 333.3 The magnitude of electric dipole moment:

A)
$$p = q(\varphi_1 - \varphi_2)$$

B) $p = \frac{\varepsilon}{R+r}$
C) $\vec{p} = \gamma \vec{E}$
D) $p = ql$
E) $p = \frac{U}{R}$

334.1. What is electric field of a point charge:

A)
$$E = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

B) $E = \frac{q_1 q_2}{4\pi\varepsilon_0 r} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$
C) $E = \frac{q}{4\pi\varepsilon_0 r^2}$
D) $E = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

E) $\oint \vec{E} d\vec{S} = \frac{Q}{\varepsilon_0}$ *********

335.1. What is linear charge density?

A)
$$\tau = \frac{dq}{dl}$$

B) $I = \frac{dq}{dt}$
C) $\sigma = \frac{dq}{ds}$
D) $j = \frac{dI}{ds}$
E) $\rho = \frac{dq}{dV}$

336.1. What is surface charge density? A) $\tau = \frac{dq}{dl}$

B)
$$j = \frac{dI}{dS}$$

C) $\sigma = \frac{dq}{dS}$
D) $I = \frac{dq}{dt}$
E) $\rho = \frac{dq}{dV}$

337.1. What is volume charge density?

A)
$$\tau = \frac{dq}{dl}$$

B) $j = \frac{dI}{dS}$
C) $I = \frac{dq}{dt}$
D) $\sigma = \frac{dq}{dS}$
E) $\rho = \frac{dq}{dV}$

338.1. Superposition of electric field:

A)
$$\sum \frac{1}{\vec{E}_i} = \sum \frac{1}{\varphi_i}$$

B)
$$\sum \frac{1}{\vec{E}_i} = q_1 + q_2 + q_3 + \cdots q_i$$

C)
$$q = q_1 + q_2 + q_3 + \cdots q_i$$

D)
$$\vec{E} = \sum \vec{E}_i$$

E)
$$\frac{1}{\vec{E}} = \sum \frac{1}{\vec{E}_i}$$

339.1. Superposition of electric field: A) $R = R_1 + R_2 + R_3 + \dots + R_i$

B)
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_i$$

C) $q = q_1 + q_2 + q_3 + \dots + q_i$
D) $\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \dots + \frac{1}{c_i}$
E) $A = A_1 + A_2 + A_3 + \dots + A_i$

340.1. Electric flux of a nonuniform field:

A)
$$\Phi_E = \int_{\sigma} \vec{E} d\vec{S}$$

B) $\Phi_E = \frac{\sigma}{\varepsilon_0}$
C) $\Phi_E = q(\varphi_1 - \varphi_2)$
D) $\Phi_E = \frac{\varepsilon}{R+r}$
E) $\Phi_E = \frac{\vec{F}}{q}$

341.1. Electric flux of a uniform field:

A)
$$\omega = \frac{\varepsilon_0 E^2}{2}$$

B) $C = \frac{q}{\varphi_1 - \varphi_2}$
C) $A = q(\varphi_1 - \varphi_2)$
D) $\Phi_E = ES \cos \alpha$
E) $E = \frac{\sigma}{\varepsilon_0}$

342.1. Electric flux of a nonuniform field:

344.1. Choose a Gauss's law:

A)
$$I = q(\varphi_1 - \varphi_2)$$

B) $I = \frac{\varepsilon}{R+r}$
C) $\oint \vec{E} d\vec{S} = \frac{Q}{\varepsilon_0}$
D) $\vec{I} = \gamma \vec{E}$
E) $U = \int \vec{E} d\vec{l}$

345.1. The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided be electric constant. It is:

A) Kirchhoff's junction rule

B) Ohm's law

C) Coulomb's law

D) Kirchhoff's junction loop rule

E) Gauss's law *********

346.1. What can we say about electric field inside conductor?

A) E=0 B) E=1 C) E= ∞ D) E=-1 E) $E = \frac{\sigma}{\varepsilon_0}$ **********

347.1. What can we say about electric field outside charged conductor (with surface charge density σ)?

A)
$$E = \frac{q_1 q_2}{4\pi\sigma} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

B)
$$E = \frac{\sigma}{\varepsilon_0}$$

C)
$$E = \frac{q}{\varphi_1 - \varphi_2}$$

D)
$$E = q(\varphi_1 - \varphi_2)$$

E)
$$E = \frac{U}{R}$$

348.1. Electric charge conservation's law:

A)
$$q = E = \frac{q}{4\pi\varepsilon_0 r^2}$$

B) $q = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_i = 0$
C) $q = \frac{\vec{F}}{q}$

D)
$$q = \sum_{\sigma} q_i = const$$

E) $q = \frac{\sigma}{\varepsilon_0}$

349.1. Electric charge conservation's law: A) $q_1 + q_2 + q_3 + \cdots q_i = 0$ B) $q_1 + q_2 + q_3 + \cdots q_i = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_i$ C) $q_1 + q_2 + q_3 + \cdots q_i = const$ D) $q_1 + q_2 + q_3 + \cdots q_i = \oint \vec{E} d\vec{S}$ E) $q_1 + q_2 + q_3 + \cdots q_i = \frac{\sigma}{\varepsilon_0}$ *********

350.1. Electric charge conservation's law:

A)
$$\sum q_i = 0$$

B) $\sum q_i = 1$
C) $\sum q_i = \frac{\varepsilon_0 E^2}{2}$
D) $\sum q_i = \infty$
E) $\sum q_i = const$

351.1. The electric potential energy for two point charges depends by an equation:

A)
$$W = \frac{\varepsilon_0 E^2}{2}$$

B) $W = \frac{q_1 q_2}{4\pi \varepsilon_0 r}$
C) $W = \frac{CU^2}{2}$
D) $W = \frac{q}{\varphi_1 - \varphi_2}$
E) $W = \frac{\varepsilon}{R+r}$

352.1. The formula for the electric potential of a point charge:

A)
$$\varphi = \frac{q}{4\pi\varepsilon_0 r}$$

B) $\varphi = \frac{q_1q_2}{4\pi\varepsilon_0 r}$
C) $\varphi = \frac{CU^2}{2}$
D) $\varphi = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$
E) $\varphi = ES \cos \alpha$

353.1. Work done in an electric field:
A)
$$A = \frac{q}{\varphi_1 - \varphi_2}$$

B) $A = \frac{\varepsilon}{R+r}$
C) $A = \frac{\varepsilon_0 E^2}{2}$
D) $A = q(\varphi_1 - \varphi_2)$
E) $A = \frac{q}{4\pi\varepsilon_0 r^2}$

354.1. Work done in an electric field:

A)
$$A = \frac{\varepsilon_0 E^2}{2}$$

B) $A = -q\Delta\varphi$
C) $A = ES \cos\alpha$
D) $A = \frac{\sigma}{\varepsilon_0}$
E) $A = \frac{q}{\varphi_1 - \varphi_2}$

355.1. Work done in an electric field:

A)
$$A = \frac{q}{\varphi_1 - \varphi_2}$$

B) $\vec{A} = \frac{\vec{F}}{q}$
C) $A = qU$
D) $A = ES \cos \alpha$
E) $A = \frac{CU^2}{2q}$

356.1. Work done in an electric field:

A)
$$A = \frac{\varepsilon}{R+r}$$

B) $A = \frac{q}{\varphi_1 - \varphi_2}$
C) $A = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$
D) $\oint \vec{A} d\vec{S} = \frac{Q}{\varepsilon_0}$
E) $A = \frac{q_1 q_2}{4\pi\varepsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

357.1. Electric potential with several point charges:

A)
$$\frac{1}{\varphi} = \sum \frac{1}{\varphi_i}$$

B) $\varphi = \sum \frac{1}{\varphi_i}$
C) $\varphi = \frac{1}{\varphi_1 + \varphi_2 + \varphi_3 + \dots + \varphi_i}$
D) $\varphi = \sum \varphi_i$

E)
$$\frac{1}{\varphi} = \frac{1}{\varphi_1} + \frac{1}{\varphi_2} + \frac{1}{\varphi_3} + \dots + \frac{1}{\varphi_4}$$

358.1. Electric potential with several point charges:

A)
$$\varphi = \frac{1}{\varphi_1 + \varphi_2 + \varphi_3 + \dots + \varphi_i}$$

B)
$$\varphi = \frac{1}{\varphi_1} + \frac{1}{\varphi_2} + \frac{1}{\varphi_3} + \dots + \frac{1}{\varphi_i}$$

C)
$$\varphi = \varphi_1 + \varphi_2 + \varphi_3 + \dots + \varphi_i$$

D)
$$\frac{1}{\varphi} = \sum \frac{1}{\varphi_i}$$

E)
$$\varphi = \sum \frac{1}{\varphi_i}$$

359.1. What is a potential difference? A) $U = \frac{q}{\varphi_1 - \varphi_2}$ B) $U = \varphi_1 - \varphi_2$ C) $\vec{U} = \gamma \vec{E}$ D) $U = \frac{\varepsilon}{R+r}$ E) $\vec{U} = \frac{\vec{F}}{q}$ *********

360.1. What is a potential difference?
A)
$$U = q(\varphi_1 - \varphi_2)$$

B) $U = \frac{q}{\varphi_1 - \varphi_2}$
C) $U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$
D) $\oint \vec{U}d\vec{S} = \frac{Q}{\varepsilon_0}$
E) $U = \int \vec{E}d\vec{l}$

361.1. What is a relationship between electron volt and joule? A) 1. eV=6.02·10⁻²³J

B) 1. eV=8.85 · 10⁻¹² J C) 1. eV=1.6 · 10⁻¹⁹ J D) 1. eV=8.31. J E) 1. eV=1.38 · 10⁻¹⁶ J ********

362.1Any two conductors separated by an insulator (or a vacuum) form: A) resistor

- B) capacitor
- C) source
- D) conductor

E) isolator *********

B)
$$C = -q\Delta\varphi$$

B) $C = -q\Delta\varphi$
C) $C = \frac{\varepsilon}{R+r}$
D) $C = \frac{-q}{\varphi_1 - \varphi_2}$
E) $C = \frac{U}{R}$

366.1. How can you calculate a capacitance of parallel-plate capacitor with dielectric?

A)
$$C = \frac{\varepsilon \varepsilon_0 S}{d}$$

B) $C = \frac{S}{\varepsilon \varepsilon_0 d}$
C) $C = \frac{\varepsilon_0 S}{\varepsilon d}$
D) $C = \frac{d}{\varepsilon \varepsilon_0 S}$
E) $C = \frac{\varepsilon_0 d}{\varepsilon S}$

367.1. How can you calculate a capacitance of parallel-plate capacitor in vacuum?

A)
$$C = \frac{D}{\varepsilon_0 S}$$

B)
$$C = \frac{S}{\varepsilon_0 d}$$

C) $C = \frac{\varepsilon_0 D}{S}$
D) $C = \frac{\varepsilon_0 S}{d}$
E) $C = d\varepsilon_0 S$

368.1. Capacitance of capacitors of the series combination:

A)
$$C = \sum C_i$$

B) $\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \dots + \frac{1}{c_i}$
C) $C = C_1 + C_2 + C_3 + \dots + C_i$
D) $C = \sum \frac{1}{c_i}$
E) $C = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \dots + \frac{1}{c_i}$

369.1. Capacitance of capacitors of the series combination:

A)
$$C = C_1 + C_2 + C_3 + \dots + C_i$$

B) $C = \frac{1}{C_1 + C_2 + C_3 + \dots + C_i}$
C) $C = \sum C_i$
D) $C = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_i}$
E) $\frac{1}{C} = \sum \frac{1}{C_i}$

370.1. Capacitance of capacitors of the parallel combination:

A)
$$C = C_1 + C_2 + C_3 + \dots + C_i$$

B) $\frac{1}{c} = \sum \frac{1}{c_i}$
C) $\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \dots + \frac{1}{c_i}$
D) $C = \frac{1}{c_1 + c_2 + c_3 + \dots + c_i}$
E) $C = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \dots + \frac{1}{c_i}$

371.2 Capacitance of capacitors of the series combination:

A)
$$\frac{1}{c} = \sum \frac{1}{c_i}$$

B) $C = \sum \frac{1}{c_i}$
C) $C = \frac{1}{c_1 + c_2 + c_3 + \dots + c_i}$
D) $C = \sum C_i$

E)
$$\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \dots + \frac{1}{c_i}$$

372.3 Potential energy stored in a capacitor:

A)
$$W = \frac{q}{4\pi\varepsilon_0 r}$$

B) $W = q(\varphi_1 - \varphi_2)$
C) $W = \frac{q^2}{2c}$
D) $W = \frac{q_1q_2}{4\pi\varepsilon_0} \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$
E) $\sum W_i = 0$

373.1. Potential energy stored in a capacitor:

A)
$$W = \int \vec{E} d\vec{l}$$

B) $W = \frac{CU^2}{2}$
C) $W = \frac{q}{4\pi\varepsilon_0 r}$
D) $W = \frac{\varepsilon}{R+r}$
E) $\vec{W} = \gamma \vec{E}$

374.1. Potential energy stored in a capacitor:

A)
$$W = \frac{q_1 q_2}{4\pi\varepsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

B)
$$W = \frac{q}{4\pi\varepsilon_0 r}$$

C)
$$W = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

D)
$$\sum W_i = 0$$

E)
$$W = \frac{qU}{2}$$

375.1. What is an electric energy density in a vacuum?

A)
$$\sum I_i = 0$$

B) $I = \frac{\varepsilon}{R+r}$
C) $I = \frac{\varepsilon_0 E^2}{2}$
D) $\vec{I} = \gamma \vec{E}$
E) $I = \frac{U}{R}$

376.1. What is a current? A) $I = \frac{q}{\varphi_1 - \varphi_2}$

from temperature? A) $\rho = 1 + (\rho_0 + \alpha \Delta T)$ B) $\rho = \rho_0 (1 - \alpha \Delta T)$ C) $\rho = (\rho_0 + \Delta T)$ D) $\rho = \rho_0 (1 + \alpha \Delta T)$ E) $\rho = \alpha (1 - \rho_0 \Delta T)$ **********

384.1How can you calculate a resistance of a cylindrical conductor?

A)
$$R = \rho \frac{s}{l}$$

B) $R = \rho \frac{l}{s}$
C) $R = \frac{l}{\rho s}$
D) $R = \rho l S$
E) $R = \frac{\rho}{sl}$
385.1. What is relationship between a conductivity and a resistivity:

A)
$$\rho = \frac{1}{1+\gamma}$$

B) $\rho = \frac{1}{1-\gamma}$
C) $\rho = \frac{1}{\gamma}$
D) $\rho = \gamma + 1$
E) $\rho = \frac{1}{\gamma-1}$

386.1. Choose the Ohm's law for a complete circuit:

E) $\vec{P} = \gamma \vec{E}$ **********

390.1. If the resistors connects in series that equivalent resistance is: A) $R = R_1 + R_2 + R_3 + \dots + R_i$ B) $R = \frac{1}{R_1 + R_2 + R_3 + \dots + R_i}$ C) $\frac{1}{R} = \sum \frac{1}{R_i}$ D) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R} + \dots + \frac{1}{R_i}$

E)
$$R = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R} + \dots + \frac{1}{R_i}$$

391.2 If the resistors connects in parallel that equivalent resistance is:

A)
$$R = \frac{1}{R_1 + R_2 + R_3 + \dots + R_i}$$

B) $R = R_1 + R_2 + R_3 + \dots + R_i$
C) $\frac{1}{R} = \frac{1}{\Sigma R_i}$
D) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R} + \dots + \frac{1}{R_i}$
E) $R = \frac{1}{\Sigma R_i}$

392.1. If the resistors connects in series that equivalent resistance is: A) $R = \sum R_i$ B) $\frac{1}{R} = \sum \frac{1}{R_i}$ C) $R = \frac{1}{R_1 + R_2 + R_3 + \dots + R_i}$ D) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R} + \dots + \frac{1}{R_i}$ E) $\frac{1}{R} = R_1 + R_2 + R_3 + \dots + R_i$ ********

393.1. How many Kirchhoff's rules does exist?

A) 1 B) 2 C) 3 D) 4 E) 5 *********

394.1. The algebraic sum of the currents into any junction is zero. It is:

- A) Gauss's law
- B) Kirchhoff's junction loop rule
- C) Kirchhoff's junction rule
- D) Ohm's law
- E) Coulombs' law

395.1. The algebraic sum of the potential differences in any loop must equal zero. It is:A) Kirchhoff's junction rule

398.1. Kirchhoff's junction loop rule:

A)
$$U = \int E dl$$

B) $W = \frac{CU^2}{2}$
C) $\sum I_i R_i = 0$
D) $I = \frac{\varepsilon}{R+r}$
E) $\vec{j} = \gamma \vec{E}$

401.1. You have a pure gold ring with mass 17.7 g. Gold has atomic mass of 197 g/mol and atomic number of 79. How many protons are in the ring? A) $2.6 \cdot 102^5$ B) $6.56 \cdot 10^{21}$ C) $4.26 \cdot 10^{24}$ D) $4.44 \cdot 10^{26}$ E) $9.8 \cdot 10^{27}$ ********

402.1. You have a pure gold ring with mass 17.7 g. Gold has atomic mass of 197 g/mol and atomic number of 79. What is their total positive charge? A) 22 MC B) 34 kC C) 720 kC D) 0.68 MC E) 1.4 MC *********

403.1. You have a pure gold ring with mass 17.7 g. Gold has atomic mass of 197 g/mol and atomic number of 79. If the ring carries no net charge, how many electrons are in it? A) $2.6 \cdot 10^{25}$

A) 2.0 10 B) $4.26 \cdot 10^{24}$ C) $4.4 \cdot 10^{16}$ D) $9.8 \cdot 10^{27}$ E) $6.46 \cdot 10^{24}$ ********

404.1. Two small spheres spaced 20 cm apart have equal charge. How many excess electrons must be present on each sphere if the magnitude of the force of repulsion between them is $5.57 \cdot 10^{-11}$ N? A) $1.5 \cdot 10^{9}$ B) $7.8 \cdot 10^{3}$ C) $9.3 \cdot 10^{5}$ D) $8.9 \cdot 10^{7}$ E) $9.8 \cdot 10^{7}$

405.1. In experiment in space, one proton is held fixed and another proton is released from rest a distance of 2.5 mm away. What is the initial acceleration of the proton after it is released? (mass of a proton $1.67 \cdot 10^{-27}$ kg)

A) 22 km/s² B) 3300 m/s² C) 4.4 m/s² D) 5.5 km/s² E) 66 km/s² **********

407.1. A proton is placed in a uniform electric field or $2.75 \cdot 10^3$ N/C.

Calculate the proton's acceleration.

A) $2.6 \cdot 10^{5}$ m B) $4.4 \cdot 10^{-16}$ N C) $2.6 \cdot 10^{11}$ m/s² D) $9.8 \cdot 10^{7}$ E) $4.26 \cdot 10^{24}$ *********

408.1. A proton is placed in a uniform electric field or $2.75 \cdot 10^3$ N/C. Calculate the proton's speed after 1. μ s in the field, assuming it starts from rest.

A) $2.6 \cdot 10^5$ m/s B) $2.6 \cdot 10^{11.}$ m/s C) $4.4 \cdot 10^{-16}$ m/s D) $9.8 \cdot 10^7$ m/s E) $2.6 \cdot 10^5$ m/s ********* 409.1. An electron is released from rest in an uniform electric field. The electron accelerates vertically upward, travelling 4.5 m in the first 3 µs after it is released. What is the magnitude of the electric field?

A) 11.2 V/m B) 5.7 V/m C) 0.55 V/m D) 3.5 V/m E) 7.3 V/m **********

410.1. Two point charges are separated be 25 cm. Find the net electric field these charges produce at point A.



411.1. Two point charges are separated be 25 cm. Find the net electric field these charges produce at point B.



412.1. Find the magnitude of electric flux through the sphere of a radius 20 cm with a charge of 26.55 nC in it.

A) 531. V·m B) 130 V·m C) 10400 V·m D) 3000 V·m E) 7000 V·m **********

413.1. Find the magnitude of electric flux through the cube of a length 20 cm with a fifty five electrons in it. A) $1\cdot10^{-7}$ V·m B) $8\cdot10^{-7}$ V·m C) $3.2\cdot10^{-7}$ V·m

D) 5·10⁻⁷ V·m E) 7·10⁻⁷ V·m

414.1. A 6.2-nC charge is at the centre of a cube with sides of length 0.5 m. What is the electric flux through one of the six faces of the cube?

A) 31. V·m B) 334 V·m C) 117 V·m D) 25 V·m E) 155 V·m **********

415.1. Two point charges $q_1=2.4$ nC and $q_2=-6.5$ nC. Find the potential at point A (see a figure).



416.1. Two point charges $q_1=2.4$ nC and $q_2=-6.5$ nC. Find the potential at

point B (see a figure).



417.1. At a certain distance from a point charge, the potential and electric-field magnitude due to that charge 4.98 V and 12 V/m, respectively. What is the magnitude of the charge? A) 0.59 nC B) 0.23 nC C) 0.41. nC D) 0.48 nC E) 0.29 nC ******

419.1. An electron is to be acceleration from $3 \cdot 10^6$ m/s to $8 \cdot 10^6$ m/s. Through what potential difference must the electron pass to accomplish this? A) 212 V 420.1. The plates of a parallel capacitor are 2.5 mm apart and each carries a charge of magnitude 80 nC. The plates are in vacuum. The electric field between the plates has magnitude of $4 \cdot 10^6$ V/m. What is a potential difference between the plates?

A) 10 kV B) 5 kV C) 2 kV D) 128 kV E) 32 kV

421.1. The plates of a parallel capacitor are 2.5 mm apart and each carries a charge of magnitude 80 nC. The plates are in vacuum. The electric field between the plates has magnitude of $4 \cdot 10^6$ V/m. What is area of each plate?

A) 66. 2 cm² B) 22.6 cm² C) 36.6 cm² D) 75.5 cm²

E) 55.5 cm^2

422.1. The plates of a parallel capacitor are 2.5 mm apart and each carries a charge of magnitude 80 nC. The plates are in vacuum. The electric field between the plates has magnitude of $4 \cdot 10^6$ V/m. What is the capacitance? A) 40 pF B) 5 pF C) 24 pF D) 8 pF E) 1.6 pF *********

423.1. A 10-μF parallel-plate capacitor with circular plates is connected to a 12-V battery. What is the charge on each plate?
A) 0.12 mC
B) 0.83 mC
C) 0.95 mC
D) 1.44 mC
E) 0.07 mC

424.1. For the system of capacitors shown in figure, find the equivalent capacitance between b and c.



425.1. For the system of capacitors shown in figure, find the equivalent

capacitance between a and c.



426.1. In figure let $C_1=3\mu F$, $C_2=5\mu F$ and U=52 V. Calculate the charge on each capacitor.



427.1. In figure let C1= 3μ F, C2= 5μ F and U=52 V. Calculate the potential

difference between a and c.



428.1. In figure let $C_1=3\mu F$, $C_2=5\mu F$ and U=52 V. Calculate the potential difference between b and c.



429.1. Figure shows a system of four capacitors, where the potential difference across ab is 50 V. Find the equivalent capacitance of this system

between a and b.



D) 0.28 µF

E) 4.5 µF

430.1. Figure shows a system of four capacitors, where the potential difference across ab is 50 V. How much charge is stored in each of the 10- μ F and 9- μ F?



A) 0.44 mC B) 0.64 mC C) 1.56 mC D) 2.27 mC E) 0.18 mC ***********

431.1. During lighting strikes from a cloud to the ground, current as high as 25000 A can occur and last for about 40 μ s. Yow much charge is transferred from the cloud to the earth during such a strike?

- A) 1000 C
- B) 1.6 kC
- C) 1. C

D) 625 MC

E) 87.5 C

434.1. A silver wire 2.6 mm in diameter transfers a charge of 420 C in 80 min. Silver contains $5.8 \cdot 10^{28}$ free electrons per cubic meter. What is the magnitude of the drift velocity of the electrons in the wire?

A) 1.07·10⁻³ mm/s B) 2.1·10⁻³ mm/s C) 1.1·10⁻³ mm/s D) 1.8·10⁻³ mm/s E) 0.75·10⁻³ mm/s *********

435.1. A 5-A current runs through a 12-gauge copper wire (diameter 2.05 mm) and through a light bulb. Copper has $8.6 \cdot 10^{28}$ free electrons per cubic meter. How many electrons pass through the light bulb each second? A) $1 \cdot 10^{19}$ B) $2 \cdot 10^{19}$ C) $3 \cdot 10^{19}$ D) $4 \cdot 10^{19}$ E) $5 \cdot 10^{19}$ ***********

436.1. A 5-A current runs through a 12-gauge copper wire (diameter 2.05 mm) and through a light bulb. Copper has $8.6 \cdot 10^{28}$ free electrons per cubic meter. What is the current density in the wire?

A) 7.5 MA/m² B) 1.5 MA/m² C) 2.3 MA/m² D) 6.4 MA/m² E) 3.1. MA/m² **********

437.1. A 5-A current runs through a 12-gauge copper wire (diameter 2.05 mm) and through a light bulb. Copper has $8.6 \cdot 10^{28}$ free electrons per cubic meter. At what speed does a typical electron pass by any given point in the wire?

A) $3.1 \cdot 10^{-4}$ m/s B) $1.1 \cdot 10^{-4}$ m/s C) $2.2 \cdot 10^{-4}$ m/s D) $6.4 \cdot 10^{-4}$ m/s E) $7.5 \cdot 10^{-4}$ m/s *********

438.1. A copper wire has a square cross section 2.3 mm on a side. The wire is 4 m long and carries a current 3.6 A. The density of free electrons is $8.5 \cdot 10^{28}$ m⁻³. Find the magnitude of the current density in the wire.

439.1. A copper wire has a square cross section 2.3 mm on a side. The wire is 4 m long and carries a current 3.6 A. The density of free electrons is $8.5 \cdot 10^{28}$ m⁻³. Find the magnitude of the drift velocity of electrons in the wire.

440.1. A copper wire has a square cross section 2.3 mm on a side. The wire is 4 m long and carries a current 3.6 A. The density of free electrons is $8.5 \cdot 10^{28}$ m⁻³. How much time is required for an electron to travel the length of the wire?

A) 1500 s B) 5 ms C) 0.6 Ms D) 700 s E) 80 ks ******

441.1. Consider the circuit shown in figure. The terminal voltage of the 24-V battery is 21.2 V. What is the internal resistance of the battery?



A) 5.3 Ohm B) 6.7 Ohm C) 0.7 Ohm D) 11.3 Ohm E) 6 Ohm **********

442.1. Consider the circuit shown in figure. The terminal voltage of the 24-V battery is 21.2 V. What is the

resistance of the circuit resistor?



- A) 11.3 Ohm B) 6 Ohm
- C) 0.7 Ohm
- D) 5.3 Ohm
- E) 6.7 Ohm

444.1. A copper transmission cable 100 km long and 10 cm in diameter carries a current of 125 A. How much electrical energy is dissipated as thermal energy every hour? (resistivity of a copper is 17 p Ω ·m) A) 1060 Kj B) 10.6 MJ C) 15.5 kJ D) 2.12 MJ E) 12 MJ *****

445.1. A copper transmission cable 100 km long and 10 cm in diameter carries a current of 125 A. How much electrical power is dissipated as thermal energy? (resistivity of a copper is 17 p Ω ·m) A) 2125 W 446.1. An ideal voltmeter A is connected to a 2- Ω resistor and a battery with emf (electromotor force) 5 V and internal resistance 0.5 Ω . What is the current in the resistor?

A) 5 A B) 3.33 A C) 2.5 A D) 2 A E) 1.75 A **********

447.1. An ideal voltmeter A is connected to a 2-Ω resistor and a battery with emf (electromotor force)
5 V and internal resistance 0.5 Ω.
What is the terminal voltage of the battery?
A) 4 V
B) 3.08 V
C) 7.08 V

D) 1.08 V E) 5.08 V

************ 448.1. An ideal vo

449.1. When switch S in figure is open, the voltmeter V of the battery reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and ammeter A reads 1.65 A.

Find the emf.



450.1. When switch S in figure is open, the voltmeter V of the battery reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and ammeter A reads 1.65 A. Find the internal resistance of the



451.1. When switch S in figure is open, the voltmeter V of the battery reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and ammeter A reads 1.65 A. Find the circuit resistance R.



452.1. A battery-powered global positioning system receiver operating on 9 V draws a current of 0.13 A. How much electrical energy does it consume during 1.5 h?

A) 1,755 Kw B) 1,755 W C) 6.3 kW D) 547 W E) 374 W **********

453.1. A32-Ω resistor and a 20-Ω resistor are connected in parallel, and the combination is connected across a 240-V dc line. What is the resistance of the parallel combination? A) 55 Ω

B) 12.3 Ω C) 0,08 Ω D) 12 Ω E) 52 Ω ******

454.1. A32- Ω resistor and a 20- Ω resistor are connected in parallel, and the combination is connected across a 240-V dc line. What is the total current through the parallel combination? A) 20 A 455.1. A32-Ω resistor and a 20-Ω resistor are connected in parallel, and the combination is connected across a 240-V dc line. What is the current through each resistor? A) 9.5 A; 10 A B) 4 A; 15.5 A C) 7.5 A; 12 A D) 1.5 A; 18 A

E) 13.5 A; 6 A

456.1. A triangular array of resistors is shown in figure. What current will this array draw from a 35-V battery having negligible internal resistance if we connect it across ab?



457.1. A triangular array of resistors is shown in figure. What current will this array draw from a 35-V battery having negligible internal resistance if



458.1. A triangular array of resistors is shown in figure. What current will this array draw from a 35-V battery having negligible internal resistance if we connect it across ac?



459.1. A triangular array of resistors is shown in figure. What current will this array draw from a 35-V battery having internal resistance of $3-\Omega$ if we connect it across bc?



C) 3.5 A D) 3.15 A E) 4.5 A *********

460.1. In the circuit shown in figure, the voltage across $2-\Omega$ resistor is 12 V. What is current through the $6-\Omega$



resistor? A) 1. A

- B) 2 A
- C) 3 A
- D) 4 A
- E) 5 A

461.1. In the circuit shown in figure, the voltage across $2-\Omega$ resistor is 12 V. What is the emf of the battery?



E) 24 V *********

462.1. Two light bulbs have resistance of 400 Ω and 800 Ω . If the two light bulbs are connected in series across a 120-V line, find the current through each bulbs.

A) 0.3 A B) 0.1. A

C) 1. A

D) 0.15 A E) 1.44 A **********

463.1. Two light bulbs have resistance of 400 Ω and 800 Ω . If the two light bulbs are connected in series across a 120-V line, find the power dissipated in each bulbs.

A) 12 W; 4 W B) 16 W; 8 W C) 8 W; 24 W D) 12 W; 4 W E) 4 W; 8 W **********

464.1. Two light bulbs have resistance of 400 Ω and 800 Ω . If the two light bulbs are connected in series across a 120-V line, find the total power dissipated in both bulbs.

A) 12 W B) 18 W C) 24 W D) 36 W E) 6 W **********

465.1. Two light bulbs have resistance of 400 Ω and 800 Ω . If the two light bulbs are connected in parallel across a 120-V line, find the current through each bulbs.

A) 0.6 A; 0.15 A B) 1.875 A; 0.75 A C) 0.75 A; 1.5 A D) 0.3 A; 0.15 A E) 0.75 A; 0.25 A ********

466.1. Two light bulbs have resistance of 400 Ω and 800 Ω . If the two light bulbs are connected in parallel across a 120-V line, find the power dissipated in each bulbs. A) 12 W; 36 W B) 36 w; 54 W C) 54 W; 36 W D) 36 W; 18 W E) 18 W; 12 W *********

467.1. Two light bulbs have resistance of 400 Ω and 800 Ω . If the two light bulbs are connected in parallel across a 120-V line, find the total power dissipated in both bulbs.

A) 288 W

- B) 144 W
- C) 96 W
- D) 48 W
- E) 54 W

468.1. In the series circuit of suppose R=300 Ω , L=60 mH, C=0.5 μ F, U=50 V and ω =10 krad/s. Find the voltage amplitude across capacitor.

A) 60 V

- B) 30 V
- C) 40 V
- D) 20 V
- E) 50 V

469.1. A friend returns to the United States from Europe with a 960-W coffeemaker, designed to operate from a 240-V line. What current will the coffeemaker draw from the 120-V line?

A) 1. A

- B) 4 A
- C) 8 A
- D) 10 A
- E) 12 A

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470.1. How many types of magnetic charges do exist?

A) 1

- B) 2
- C) 3
- D) 4

E) don't exist ********* 472.1. The magnitude of the magnetic force is given by:

A) $F = qvB \sin \alpha$ B) $F = qE \cos \alpha$ C) $F = IBl \sin \alpha$ D) $F = p_m B \sin \alpha$ E) $F = BS \cdot \cos \alpha$ ********

473.1. What is a magnetic force on a moving charge?

A)
$$\vec{F} = \vec{E} + \vec{v}\vec{B}$$

B) $\vec{F} = I\vec{l} \times \vec{B}$
C) $\vec{F} = IS\vec{n}$
D) $\vec{F} = q\vec{v} \times \vec{B}$
E) $\vec{F} = \vec{p}_m \times \vec{B}$

475.1. When an electric charge moves through electric and magnetic field that total force:

A)
$$\vec{F} = q\vec{E}$$

B) $\vec{F} = q\vec{v}\vec{B}$
C) $\vec{F} = q(\vec{E} + \vec{v}\vec{B})$
D) $\vec{F} = IS\vec{n} + \vec{p}_m \times \vec{B}$
E) $\vec{F} = I\vec{l} \times \vec{B} + \vec{E}$

476.1. What is a magnetic field lines of a straight current-carrying wire? A) ellipsis B) infinity C) squares D) circles E) don't exist ***** 477.1. Magnetic flux: A) $d\Phi = BdS \cdot \cos \alpha$ B) $d\Phi = IBdl \sin \alpha$ C) $d\Phi = qdE \cos \alpha$ D) $d\Phi = IdB \cos \omega t$ E) $d\Phi = p_m dB \sin \alpha$ ******** 478.1. Magnetic flux: A) $\Phi = p_m B \sin \alpha$ B) $\Phi = I \cos \omega t$ C) $\Phi = qE \cos \alpha$ D) $\Phi = BS \cdot \cos \alpha$ E) $\Phi = IBl \sin \alpha$ ******

479.1. What is a SI unit of a magnetic flux? A) Newton

B) Tesla

- C) Weber
- D) Siemens
- E) Henry
- *******

480.1. Choose a Gauss's law for magnetism:

A)
$$\int \vec{B} d\vec{S} = 1$$

B) $\int \vec{B} d\vec{S} = \frac{Q}{\varepsilon_0}$
C) $\int \vec{B} d\vec{S} = \frac{1}{\mu_0} \sum I$
D) $\int \vec{B} d\vec{S} = \mu_0 I_{enclosed}$
E) $\int \vec{B} d\vec{S} = 0$

481.1. The total magnetic flux through a closed surface is:A) 0B) 1

C) -1 D) unknown E) ∞ ***********

482.1. The total magnetic flux through a closed surface is:

A)
$$-\frac{Q}{\varepsilon_0}$$

B) $\mu_0 I$
C) $\frac{Q}{\varepsilon_0}$
D) 0
E) $\frac{1}{\mu_0} \sum I$

483.1. The total magnetic flux through a closed surface is always zero. It is:A) Magnetic forceB) Gauss's law for magnetism

C) Lentz's rule

D) Faraday's law of induction

E) Electrostatic theorem ********

484.1. What does the cyclotron frequency equal?

A)
$$\omega = \frac{qm}{B}$$

B) $\omega = \frac{mB}{q}$
C) $\omega = \frac{q}{Bm}$
D) $\omega = \frac{1}{mqB}$
E) $\omega = \frac{qB}{m}$

485.1. Magnetic force on a straight wire segment:

A)
$$\vec{F} = q\vec{v} \times \vec{B}$$

B) $\vec{F} = \vec{p}_m \times \vec{B}$
C) $\vec{F} = I\vec{l} \times \vec{B}$
D) $\vec{F} = \vec{S} \times \vec{B}$
E) $\vec{F} = IS\vec{n} \times \vec{B}$

486.1. Magnetic force on an infinitesimal wire section:

A)
$$d\vec{F} = \vec{p}_m \times d\vec{B}$$

B) $d\vec{F} = d\vec{S} \times \vec{B}$
C) $d\vec{F} = ISd\vec{n} \times \vec{B}$
D) $d\vec{F} = qd\vec{v} \times \vec{B}$
E) $d\vec{F} = I\vec{dl} \times \vec{B}$

487.1. Magnitude of torque on a current loop:

A) $M = IdB \cos \omega t$ B) $M = IBS \sin \alpha$ C) $M = \frac{1}{2}LI^2$ D) $M = p_m I \sin \alpha$ E) M = qdEtgI*********

488.1. What is a magnetic dipole of the current loop?

A)
$$p_m = BS$$

B) $p_m = \frac{1}{2}LI^2$
C) $p_m = \frac{B^2}{2\mu_0}$
D) $p_m = \frac{\mu_0 I}{2\pi r}$
E) $p_m = IS$

489.1. What is a magnetic dipole of the current loop?

A)
$$\vec{p}_m = IS\vec{n}$$

B) $\vec{p}_m = II\vec{B}$
C) $\vec{p}_m = IS\vec{B}$
D) $\vec{p}_m = Ip\vec{B}$
E) $\vec{p}_m = MS\vec{B}$

490.1. Magnitude of torque on a current loop:

A)
$$M = I \cos \omega t$$

B) $M = p_m B \sin \alpha$
C) $M = \frac{d\Phi_M}{dt}$
D) $M = I_{rms} U_{rms} \cos \varphi$
E) $M = qdEtgI$

491.1. Vector torque on a current loop:

A) $\vec{M} = I \vec{dl} \times \vec{B}$ B) $\vec{M} = S \vec{n} \times \vec{B}$ C) $\vec{M} = q d \vec{v} \times \vec{B}$ D) $\vec{M} = \vec{p}_m \times \vec{B}$ E) $\vec{M} = I_{rms} U_{rms} \cos \varphi$ *********

492.1. How can you calculate a magnetic field of a straight current-carrying wire:

A)
$$B = \frac{\mu_0 I}{2\pi r}$$

B) $B = \frac{M^2}{2\mu_0}$
C) $B = L \frac{rdI}{dt}$
D) $B = I_{rms} U_{rms} \cos \varphi$
E) $B = r \omega CI$

493.1. How can you calculate a magnetic field of a circle wire in its centre:

A)
$$B = \frac{\mu_0 I}{2R}$$

B) $B = \frac{1}{2} L I^2$
C) $B = \sqrt{LC}$
D) $B = \mu_0 I_{enclosed}$
E) $B = \frac{1}{\mu_0 R} \sum I$

494.1. Ampere's law (general statement):

A)
$$\int BdS = \mu_0 I_{enclosed}$$

B) $\int \vec{B}d\vec{S} = \frac{Q}{\varepsilon_0}$
C) $\int \vec{B}d\vec{S} = 1$
D) $\int \vec{B}d\vec{S} = \frac{1}{\mu_0} \sum I$
E) $\int \vec{B}d\vec{S} = 0$

495.1. The induced emf in a closed loop equals the negative if the time rate if a change of magnetic flux through the loop. It is:

- A) Electrostatic theorem
- B) Lentz's rule

C) Magnetic force
D) Faraday's law
E) Gauss's law for magnetism

496.1. Equation $\varepsilon = -\frac{d\Phi_M}{dt}$ means:

- A) Magnetic force
- B) Lentz's rule
- C) Faraday's law of induction
- D) Electrostatic theorem
- E) Gauss's law for magnetism

497.1. The direction of any magnetic induction effect is such as to opposite the cause of the effect. It is:

- A) Lenz's law
- B) Farady's law
- C) Ampere's law
- D) Bio-Savar-Laplas' law
- E) Weber's law
- *****

498.1. Choose Faraday's law of induction:

A)
$$\varepsilon = \frac{Q}{\varepsilon_0} - U_{rms} \cos \varphi$$

B) $\varepsilon = -\frac{d\Phi_M}{dt}$
C) $\varepsilon = -I \cos \omega t$
D) $\varepsilon = \int \vec{B} d\vec{S}$
E) $\varepsilon = \frac{1}{\mu_0 R} \sum I + 1$

499.1. Self-inductance electromotive force:

A)
$$\varepsilon = -L \frac{rdI}{dt}$$

B) $\varepsilon = \frac{\mu_0 I}{2\pi r}$
C) $\varepsilon = -L \mu_0 I_{enclosed}$
D) $\varepsilon = \frac{B^2}{2\mu_0}$
E) $\varepsilon = -L \frac{dI}{dt}$

500.1. Formula for the magnetic energy given by equation: A) $W = \frac{\mu_0 I}{2\pi r}$ B) $W = \int \vec{B} d\vec{S} + \mu_0 I_{enclosed}$ C) $W = \frac{1}{2}LI^2$ D) $W = L\frac{dI}{dt}$ E) W = qdE*********

501.1. Magnetic energy density in vacuum:

A)
$$u = \frac{B^2}{2\mu_0}$$

B)
$$u = \frac{d\Phi_M}{dt}$$

C)
$$u = 2\pi\sqrt{LC}$$

D)
$$u = \frac{1}{\mu_0}\sum I$$

E)
$$u = qdB\cos\alpha$$

502.1. Angular frequency of oscillation in an L-C circuit:

A)
$$\omega = 2\pi \sqrt{\frac{L}{c}}$$

B) $\omega = \sqrt{LC}$
C) $\omega = \sqrt{\frac{c}{L}}$
D) $\omega = 2\pi \sqrt{LC}$
E) $\omega = \sqrt{\frac{1}{LC}}$

503.1. Period of oscillation in an L-C circuit:

A)
$$T = \sqrt{2\pi LC}$$

B) $T = 2\pi\sqrt{LC}$
C) $T = 2\pi LC$
D) $T = 2\pi \sqrt{\frac{L}{c}}$
E) $T = \sqrt{\frac{C}{L}}$

504.1. Alternating current given by equation:

A) $i = \omega t \cos I$ B) $i = I \cos \omega t$ C) $i = U \cos \omega t$ D) $i = I \omega \cos U t$ E) $i = IU \cos \omega t$

505.1. What is inductive resistance? A) $X_L = 1/\omega L$ B) $X_L = \omega/L$ C) $X_L = \omega C$ D) $X_L = \omega L$ E) $X_L = \omega LC$ ***********

506.1. What is capacitive resistance?

A)
$$X_C = \omega C$$

B) $X_C = \frac{\omega}{c}$
C) $X_C = \frac{\omega}{\omega}$
D) $X_C = \frac{1}{\omega c}$
E) $X_C = \frac{L}{\omega c}$

507.1. What is the impedance? A) $Z = \sqrt{R^2 + (X_L - X_C)^2}$ B) $Z = \sqrt{R^2 - (X_L - X_C)^2}$ C) $Z = \sqrt{X_L - (R^2 + X_C)^2}$ D) $Z = \sqrt{R^2 + (X_C - X_L)^2}$ E) $Z = \sqrt{X_C - (X_L - R^2)^2}$

508.1. What is the impedance?

A)
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

B)
$$Z = \sqrt{R^2 + \omega L - \frac{1}{\omega C}}$$

C)
$$Z = \sqrt{C^2 + RL + (\omega C)^2}$$

D)
$$Z = \sqrt{R^2 - \left(\omega C - \frac{1}{\omega L}\right)^2}$$

E)
$$Z = \sqrt{\omega L - \left(R^2 + \frac{1}{\omega C}\right)^2}$$

509.1. Ohm's law for the alternative current (AC) circuit:

A)
$$I = \frac{U_m}{RZ}$$

B) $I = \frac{U_m}{Z+R}$
C) $I = \frac{U_m}{Z}$

D)
$$I = \frac{RU_m}{RZ}$$

E) $I = \frac{U_m}{R-Z}$

510.1. What is f root-mean-square value of a sinusoidal voltage?

A)
$$U_{rms} = \frac{\sqrt{2}}{U}$$

B) $U_{rms} = \frac{\sqrt{2}}{\sqrt{2}}$
C) $U_{rms} = \sqrt{2}U$
D) $U_{rms} = \sqrt{2}U$
E) $U_{rms} = \frac{\sqrt{U}}{2}$

511.1. What is f root-mean-square value of a sinusoidal current?

A)
$$I_{rms} = \frac{\sqrt{I}}{2}$$

B) $I_{rms} = \sqrt{2I}$
C) $I_{rms} = \sqrt{2I}$
D) $I_{rms} = \frac{\sqrt{2}}{I}$
E) $I_{rms} = \frac{1}{\sqrt{2}}$

512.1. Power in a general AC circuit given by:

A) $P = I_{rms}U_{rms}\cos\varphi$ B) $P = IU\cos\varphi$ C) P = IUD) $P = U_{rms}I_{rms}\sin\varphi$ E) $P = U_{rms}/I_{rms}$ *********

513.1. If the primary winding of transformer has less turns than the secondary that you have:

A) step-down transformer

B) step-left transformer

C) step-up transformer

D) step-right transformer

E) step-zero transformer

514.1. If the primary winding of transformer has more turns than the secondary that you have:

A) step-up transformer

B) step-zero transformer

C) step-right transformer

D) step-down transformer

E) step-left transformer

515.1. If the secondary winding of transformer has more turns than the primary that you have:

A) step-up inductor

B) step-left conductor

C) step-up transformer

D) step-down source

E) step-down transformer

516.1. If the secondary winding of transformer has less turns than primary the that you have:

A) step-left source

B) triple-point transformer

- C) step-up transformer
- D) step-down transformer
- E) step-up transfmitter

517.1. For the step-up transformer:

A) $N_1 > N_2$ B) $N_1 = N_2$ C) $N_1 < N_2$ D) $N_2 < N_1$ E) $N_1N_2 = 1$

519.1. For the step-down transformer: A) $N_1 > N_2$ B) $N_1 = N_2$ C) $N_1 N_2 = 1$ D) $N_1 < N_2$ E) $\frac{N_1}{N_2} = 1$ ********

520.1. For the step-down transformer: A) $U_1 = U_2$ B) $U_2 > U_1$ C) $U_1 < U_2$

D) $U_1 U_2 = 1$

E) $U_1 > U_2$ **********

521.1. Speed of electromagnetic wave in vacuum:

A) $8 \cdot 10^8 m/s$ B) $3 \cdot 10^8 m/s$ C) $8 \cdot 10^3 m/s$ D) $6 \cdot 10^{23} m/s$ E) $9 \cdot 10^9 m/s$ *********

522.1. Speed of electromagnetic wave in vacuum:

A)
$$c = \frac{1}{\sqrt{\varepsilon_0 - \mu_0}}$$

B) $c = \frac{\varepsilon_0}{\sqrt{\mu_0}}$
C) $c = \frac{\mu_0}{\sqrt{\varepsilon_0}}$
D) $c = \sqrt{\varepsilon_0 \mu_0}$
E) $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

523.1. Speed of electromagnetic wave in a dielectric:

A)
$$v = \frac{1}{\sqrt{\epsilon \varepsilon_0}} \mu \mu_0$$

B) $v = \frac{\epsilon \varepsilon_0}{\sqrt{\mu \mu_0}}$
C) $v = \frac{1}{\sqrt{\epsilon \varepsilon_0 \mu \mu_0}}$
D) $v = \frac{1}{\sqrt{\epsilon \varepsilon_0 \mu_0}} \epsilon \mu$
E) $v = \frac{1}{\sqrt{\epsilon \mu}} \varepsilon_0 \mu_0$

524.1. Speed of electromagnetic wave in a dielectric:

A)
$$v = \frac{\varepsilon \mu}{\sqrt{c}}$$

B) $v = \frac{c\varepsilon}{\sqrt{\mu}}$
C) $v = \frac{c\mu}{\sqrt{\varepsilon}}$

D)
$$v = \frac{c}{\sqrt{\varepsilon\mu}}$$

E) $v = \frac{1}{\sqrt{\varepsilon\mu}}$

525.1. What is relationship between electric field and magnetic field in the electromagnetic wave?

A)
$$E = cB$$

B) $E = B$
C) $E = B/c$
D) $E = c/B$
E) $E = c - B$

526.1. What is relationship between electric field and magnetic field in the electromagnetic wave (in vacuum)?

A) B = cEB) $B = \varepsilon_0 \mu_0 / E$ C) $B = \varepsilon_0 \mu_0 cE$ D) $B = \varepsilon \mu cE$ E) $B = \varepsilon_0 \mu_0 / cE$ *********

527.1. What is relationship between electric field and magnetic field in the electromagnetic wave?

A)
$$B = E - c$$

B) $B = E + c$
C) $B = c/E$
D) $B = Ec$
E) $B = E/c$

528.1. What is relationship between electric field and magnetic field in the electromagnetic wave (in vacuum)??

A) $E = Bc/\varepsilon_0\mu_0$ B) $E = B/\varepsilon_0\mu_0c$ C) $E = \mu_0c/B$ D) $E = B\varepsilon_0/\mu_0c$ E) $E = B\varepsilon_0\mu_0c$ **********

529.1. Poyting's vector: A) $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ B) $\vec{S} = \frac{1}{\mu_0} \sum I$ C) $\vec{S} = \int \vec{B} d\vec{S} + \mu_0 I_{enclosed}$ D) $\vec{S} = IEBS\vec{n}$ E) $\vec{S} = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$ ********

530.1. Calculate magnetic force on a proton moving in 2 mT magnetic field at $4 \cdot 10^5$ m/s velocity. The angle between magnetic field and velocity is 30° .

A) $9.6 \cdot 10^{-14} N$ B) $1.38 \cdot 10^{-23} N$ C) $4.6 \cdot 10^{-19} N$ D) $6.4 \cdot 10^{-17} N$ E) $1.64 \cdot 10^{-27} N$ *********

531.1. A beam of electrons moves at $3 \cdot 10^5$ m/s through a uniform 2-T magnetic field along the magnetic fields lines. Find the force on an electron.

A) $9.2 \cdot 10^{-7} N$ B) $9.6 \cdot 10^{-14} N$ C) $1.38 \cdot 10^{-23} N$ D) $6.4 \cdot 10^{-17} N$ E) $3.64 \cdot 10^{-25} N$ *********

532.1. Find the magnetic flux trough a flat surface with area 3 cm² in a uniform magnetic field 6-T. The angle between a normal and magnetic field is 0°

A) 1.8 mWb B) 2 Wb C) 0 mWb D) 9 mWb E) 27 mWb *****

533.1. Find the magnetic flux trough a flat surface with area 6 cm^2 in a uniform magnetic field 60-T. The angle between a normal and magnetic field is 60° A) 9 mWb

534.1. Find the magnetic flux trough a flat surface with area 3 cm² in a uniform magnetic field 6-T. The angle between a normal and magnetic field is 90°

A) 18 mWb B) 9 mWb C) 27 mWb D) 1. Wb E) 0 mWb

535.1. A metallic conductor at 0.5 m is in 2-T uniform magnetic field.
Calculate the force on it if the current 2 A and an angle between a normal and magnetic field lines is 90°
A) 0.555 N
B) 3 N

- C) 4 N
- D) 1. N
- \dot{E} 2 N
- *****

536.1. A magnetron in a microwave oven emits electromagnetic waves with frequency 2236 MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?

A) 0.83 T B) 0.08 T C) 52 T D) 0.04 T E) 4.68 T **********

537.1. A magnetron in a microwave oven emits electromagnetic waves with frequency 7000 Mrad/s. What magnetic field strength is required for electrons to move in circular paths with this frequency? A) 0.08 T B) 1.43 T C) 3.1. T D) 0.54 T E) 0.04 T *******

538.1. A circular coil 0.0500 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a counter clockwise current of 5.00 A. Find the magnitudes of the magnetic moment.

A) $0.375 \text{ A} \cdot \text{m}^2$ B) $0.29 \text{ A} \cdot \text{m}^2$ C) $1.18 \text{ A} \cdot \text{m}^2$ D) $8.11. \text{ A} \cdot \text{m}^2$ E) $4.68 \text{ A} \cdot \text{m}^2$ *********

539.1. A square coil 0.0500 m long, with 30 turns of wire, lies in a horizontal plane. It carries a counter clockwise current of 5.00 A. Find the magnitudes of the magnetic moment. A) $0.375 \text{ A} \cdot \text{m}^2$

B) $3.57 \text{ A} \cdot \text{m}^2$ C) $4.68 \text{ A} \cdot \text{m}^2$ D) $1.18 \text{ A} \cdot \text{m}^2$ E) $0.545 \text{ A} \cdot \text{m}^2$ ************

540.1. A circular coil 0.0500 m in diameter, with 30 turns of wire, lies in a horizontal plane. It carries a counter clockwise current of 5.00 A. Find the magnitudes of the magnetic moment. A) 0.275 A m^2

A) $0.375 \text{ A} \cdot \text{m}^2$ B) $1.43 \text{ A} \cdot \text{m}^2$ C) $0.04 \text{ A} \cdot \text{m}^2$ D) $0.29 \text{ A} \cdot \text{m}^2$ E) $1.18 \text{ A} \cdot \text{m}^2$ *********

541.1. A circular coil 0.0500 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a counter

clockwise (as viewed from above) current of 5.00 A. The coil is in a uniform 1.20-T magnetic field directed toward the right. Find the torque on the coil.



A) 0.54 N[.]m B) 90 N[.]m C) 4.68 N[.]m

- D) 3.1. N[.]m
- E) 1.41. N[.]m

542.1. An electron experiences a magnetic force of magnitude $4.6 \cdot 10^{-15}$ *N*when moving at an angle 60° of with respect to a magnetic field of magnitude 3.5 m T. Find the speed of the electron. A) 1.44 km/s

B) 9.5 Mm/s

- C) 0.83 m/s
- D) 12 km/h

E) 0.5 Mm/s *********

543.1. A deuteron (the nucleus of an isotope of hydrogen) has a mass of $3.34 \cdot 10^{-27} kg$ and a charge of +e. The deuteron travels in a circular path with a radius of 6.96 mm in a magnetic field with magnitude 2.50 T. Find the speed of the deuteron. A) 0.5 Mm/s

- B) 1.1. mm/s C) 0.83 Mm/s D) 1.44 km/s
- E) 0.93 m/s

544.1. A deuteron (the nucleus of an isotope of hydrogen) has a mass of $3.34 \cdot 10^{-27} kg$ and a charge of +e. The deuteron travels in a circular path with a radius of 6.96 mm in a magnetic field with magnitude 2.50 T. Find the time required for it to make half a revolution.

- A) 33 Ms B) 52 ns
- C) 111. µs
- D) 42 s
- E) 16 ms
- *******

545.1. An electron in the beam of a TV picture tube is accelerated by a potential difference of 2.00 kV. Then it passes through a region of transverse magnetic field, where it moves in a circular arc with radius 0.180 m. What is the magnitude of the field?

A) 1.1. mT B) 0.54 mT C) 14.4 mT D) 0.84 mT E) 10 mT **********

546.1. A long, straight conductor carries a 1.0-A current. At what distance from the axis of the conductor does the resulting magnetic field have magnitude? A) 4 mm B) 3.1. mm C) 90 mm D) 14.4 mm E) 0.5 mm

E) 0.5 mm

547.1. Lightning bolts can carry currents up to approximately 20 kA. We can model such a current as the equivalent of a very long, straight wire. If you were unfortunate enough to be 5.0 m away from such a lightning bolt, how large a magnetic field would you experience? A) 1000 mT B) 4.68 mT C) 14.4 mT D) 3.1. mT E) 0.8 mT

548.1. A very long, straight horizontal wire carries a current such that $3.5 \cdot 10^{18}$ electrons per second pass any given point going from west to east. What are the magnitude and direction of the magnetic field this wire produces at a point 4.00 cm directly above it?

549.1. If the current in the toroidal solenoid of 40 μ H (microhenry) increases uniformly from 0 to 6.0 A in 30 μ s, find the magnitude of the self-induced emf.

A) 110 V B) 42 V

- C) 33 V
- D) 80 V
- E) 70 V

550.1. Charged 25- μ F capacitor connected across a 10-mH inductor. Find the frequency of oscillation of the circuit.

A) 220 HzB) 320 HzC) 120 Hz

- D) 240 Hz
- E) 125 Hz

- 551.1. Charged 25-μF capacitor connected across a 10-mH inductor. Find the angular frequency of oscillation of the circuit.
 A) 10000 rad/s
 B) 100 rad/s
 C) 6660 rad/s
- D) 2000 rad/s
- E) 500 rad/s ********

552.1. Charged 25- μ F capacitor connected across a 10-mH inductor. Find the period of oscillation of the circuit.

A) 13.1. ms B) 6.15 ms C) 11.2 ms D) 12 ms E) 3.1. ms *********

553.1. The inductor has inductance 0.260 H, di/dt=-0.018 A/s. Find the self-induced emf. A) 12 mV B) 468 mV C) 684 mV D) 4.68 mV E) 22.2 mV ********

554.1. At the instant when the current in an inductor is increasing at a rate of 0.064 A/m the magnitude of the self-induced emf is 0.0160 V. What is the inductance of the inductor?

A) 3.1. mH B) 0.25 H C) 1.43 mH D) 0.54 H E) 90 H **********

555.1. When the current in a toroidal solenoid is changing at a rate of 0.026 A/s, the magnitude of the induced emf is 12.6 mV. When the current equals

1.40 A, the average flux through each turn of the solenoid is 0.00285 Wb. How many turns does the solenoid have?

A) 123

- B) 783
- C) 953
- D) 238
- E) 333
- __/ - -***********

556.1. In a proton accelerator used in elementary particle physics experiments, the trajectories of protons are controlled by bending magnets that produce a magnetic field of 4.80 T. What is the magnetic-field energy in a volume 10 cm⁻³ of space where B=4.8 T?

A) 12 J B) 0.5 kJ C) 92 J D) 75 J

E) 125 mJ

557.1. A 7.50-nF capacitor is charged up to 12.0 V, then disconnected from the power supply and connected in series through a coil. The period of oscillation of the circuit is then measured to be $8.6 \cdot 10^{-5} s$. Calculate the inductance of the coil. A) 14 µH

- B) 0.5 mH C) 1.43 mH
- D) 25 mH
- E) 0.54 µH
- *****

558.1. A 7.50-nF capacitor is charged up to 12.0 V, then disconnected from the power supply and connected in series through a coil. The period of oscillation of the circuit is then measured to be $8.6 \cdot 10^{-5} s$. Calculate the maximum charge on the capacitor.

A) 90 nC B) 80 nC C) 70 nC D) 50 nC E) 30 nC ******

559.1. A 7.50-nF capacitor is charged up to 12.0 V, then disconnected from the power supply and connected in series through a coil. The period of oscillation of the circuit is then measured to be $8.6 \cdot 10^{-5} s$. Calculate the total energy of the circuit.

A) 33.5 μJ B) 2.12 μJ C) 1.41. μJ D) 0.83 μJ E) 0.54 μJ *****

560.1. The current amplitude in a pure inductor in a radio receiver is to be 250 μ A when the voltage amplitude is 3.60 V at a frequency of 1.60 MHz (at the upper end of the AM broadcast band). What inductive reactance is needed?

561.1. The current amplitude in a pure inductor in a radio receiver is to be 250 μ A when the voltage amplitude is 3.60 V at a frequency of 1.60 MHz (at the upper end of the AM broadcast band). What inductance is needed? A) 4.27 mH B) 5.14 mH C) 8.33 mH D) 1.43 mH E) 7.62 mH *********

562.1. In the series circuit of suppose R=300 Ω , L=50 mH, C=0.5 μ F, U=50 V and ω =10 rad/s. Find the inductive resistance.

A) 100 ΩB) 200 Ω

- C) 600 Ω
- C) 000 32D) 400 Ω
- D $= 00 \Delta$

E) 500 Ω *********

563.1. In the series circuit of suppose R=300 Ω , L=60 mH, C=0.5 μ F, U=50 V and ω =10 krad/s. Find the capacitive resistance. A) 100 Ω

- D) 200 0
- B) 200 Ω
- C) 600 Ω
- D) 400 Ω
- E) 500 Ω
- ******

564.1. In the series circuit of suppose R=300 Ω , L=60 mH, C=0.5 μ F, U=50 V and ω =10 krad/s. Find the impedance. A) 100 Ω

- B) 200 Ω
- C) 600 Ω
- C) 000 32
 D) 400 Ω
- E) 500 Ω

565.1. In the series circuit of suppose R=300 Ω , L=60 mH, C=0.5 μ F, U=50 V and ω =10 krad/s. Find current amplitude.

- A) 1. A
- B) 4 A
- C) 8 A
- D) 10 A
- E) 12 A

566.1. In the series circuit of suppose R=300 Ω , L=60 mH, C=0.5 μ F, U=50 V and ω =10 krad/s. Find the voltage amplitude across resistor.

A) 60 V B) 30 V C) 40 V D) 20 V E) 50 V ******

567.1. In the series circuit of suppose R=300 Ω , L=60 mH, C=0.5 μ F, U=50 V and ω =10 krad/s. Find the voltage amplitude across inductor.

- A) 60 V B) 30 V C) 40 V D) 20 V E) 50 V
- ********

568.1. In the series circuit of suppose R=300 Ω , L=60 mH, C=0.5 μ F, U=50 V and ω =10 krad/s. Find the voltage amplitude across capacitor.

A) 60 V B) 30 V C) 40 V D) 20 V E) 50 V **********

569.1. A friend returns to the United States from Europe with a 960-W coffeemaker, designed to operate from a 240-V line. What current will the coffeemaker draw from the 120-V line?

A) 1 A B) 4 A C) 8 A D) 10 A E) 12 A **********

570.1. What is the speed of light in vacuum?
A) $8 \cdot 10^8 m/s$ B) $3 \cdot 10^8 m/s$ C) $6 \cdot 10^{23} m/s$ D) 9 · 10⁹*m/s* E) 8 · 10³*m/s* *****

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