Ministry of education and science of the Republic of Kazakhstan

A. Baitursynov Kostanai State university

Department of electricity and physics

M. Dunsky

THEORETICAL MECHANICS

Manual

Kostanay, 2010

UDC 53
LBC 22.3
D 83

Author:
Dunsky Mikhail, master of physics, senior lecturer of the department of Electricity and Physics

Readers:
Telegina Oksana, head of the department of physical and technical disciplines, candidate of technical science
Koshkin Igor, head of the department of electricity and physics, candidate of technical science
Poezjalov Vladimir, docent, candidate of physics and mathematics science

Dunsky M.
D83 Theoretical mechanics: Manual in theoretical mechanics for specialty 5B060400-Physics. – Kostanay: A. Baitursynov Kostanai State university, 2018. – 151 p.

ISBN 978-601-7640-57-6

This manual contains questions, solution problems, problems for independent work and tests.

UDC 53
LBC 22.3
D 83

Approved and recommended to the Learning-Methodology Council of A. Baitursynov Kostanay State university, ______ year, Protocol № ____

Content

Introduction

Theoretical mechanics is the first discipline of the theoretical physics course, which is read by students of the specialty "Physics" and it causes considerable difficulties for students. This is primarily due to the use of theoretical structures and high-level mathematical apparatus, which is an integral part of theoretical physics and is difficult for students to perceive due to lack of strong enough mathematical training.

The main textbook recommended for the study of this discipline is the first volume of theoretical physics course in 10 volumes by L.D. Landau, E.M. Lifshits, which in itself is rather difficult due to the high scientific level of presentation and, as a consequence, the omission of the vast majority of mathematical calculations in obtaining one or another equation. Laconic transformations at their detailed decision at times occupy some pages of the text. Despite the popularity of the course (has withstood 5 editions, translated into many languages), it is designed for well-educated readers with strong mathematical training. The omission of many calculations is accompanied by the expressions "from where it is obvious ...", "it is easy to show that ...", "having performed elementary transformations, we find ...", and a detailed explanation of the physical meaning is often left "off-screen". Nevertheless, a typical theoretical mechanics curriculum for Physics was fully consistent with the content of the first volume of this course.

It should be noted that the often criticized style of presentation of the theoretical physics course in general and the first volume of "Mechanics" in particular (omission of many nontrivial computations replaced by the words "obvious", "how easy it is to show", etc., almost complete absence of references to specific works, and mentioning only the names of the authors, sometimes excessive mathematization) is the object of discussion from the first editions of the course, but it is not an original invention of its authors. Exactly the same claims were made to the five-volume "Heavenly Mechanics" by Laplace (1799-1825). Thus, Nathaniel Boudich from Boston, who translated four volumes of Laplace's work into English, once said: "Whenever I met Laplace's statement, it's easy to see...", I was sure that I would need hours of hard work until I filled in the blank, guess and show how easy it is to see.

This tutorial is designed to help students to master the course of theoretical mechanics and is a kind of replenishment of missed mathematical calculations of the textbook. The manual is very detailed from a mathematical point of view, providing an explanation of how to obtain a formula or expression with references to elementary formulas, which should help students to master the methods of theoretical physics in general and theoretical mechanics in particular, as well as help in mastering other disciplines of the theoretical physics course, such as electrodynamics, quantum mechanics, atomic and nuclear physics, and others.

The manual also includes questions for self-testing before the tasks on a particular Chapter are solved, as well as tasks for self-review. After each chapter, there are test assignments for theoretical material, which also fully correspond to

Includes questions and tasks on all main sections of the theoretical mechanics course for physical specialties of universities.

Chapter 1 The equations of motion

1.1 Verification questions

1. What does classical mechanics study?
2. What are the space and the time?
3. How can you define the position of a material point in space?
4. How can you define the position of a system N material point in space?
5. What is called the number of degrees of freedom?
6. How many number of degrees of freedom have got a material point?
7. How many number of degrees of freedom have got a rigid body?
8. What is a path (trajectory)?
9. If you know an equation of motion how can you get a path?
10. What is called generalized coordinates?
11. What is called generalized velocities?
12. What is called generalized accelerations?
13. How the position of point in Cartesian coordinate system is specified?
14. How the position of point in cylindrical coordinate system is specified?
15. How the position of point in spherical coordinate system is specified?
16. What are relations between Cartesian, cylindrical and spherical coordinate systems?
17. What is called Lame's coefficients?
18. How can you Lame's coefficients in Cartesian coordinate system define?
19. How can you Lame's coefficients in polar coordinate system define?
20. How can you Lame's coefficients in cylindrical coordinate system define?
21. How can you Lame's coefficients in spherical coordinate system define?
22. How can you arc length differential in Cartesian coordinate system define?
23. How can you arc length differential in polar coordinate system define?
24. How can you define arc length differential in cylindrical coordinate system?
25. How can define you arc length differential in spherical coordinate system?
26. Which quantities should you know for completely determination of the state of mechanical system?
27. What is called an equation of motion?
28. Formulate and write down the second Newton's law
29. On which does depend a force in classical mechanics?
30. What does equal the speed of propagation of interaction between bodies in Newton's mechanics? Why? Is it correct?

31. Which is called inertial system?
32. What is an action?
33. Formulate principle of least action
34. Tell about properties of Lagrangian
35. Write down Lagrange's equations
36. If you know Lagrangian what do you get using Lagrange's equations?
37. Tell about properties of the space and the time
38. Formulate law of inertia
39. Formulate Galileo's relativity principle
40. Get a Galilean transformation
41. Write down Lagrangian for a free particle
42. Write down Lagrangian for a system
43. Can a mass be negative? Why?
44. What is a closed system?
45. Get the second Newton's law using Lagrangian for a particle
46. What field is called uniform?
47. What does equal a potential energy of a point in a uniform field?

1.2 Problems Solution

Problem 1.

Find Lame's coefficients for the polar coordinate system.

Solution

Relation between polar and Cartesian coordinate systems is specified with expression: . Using general formula

rewrite it for case in two dimensions and take , , , . Here you should not confuse angle as the second coordinate in polar system and as general notation of coordinate system's equations . Then for the finding coefficients it is necessary calculate expressions: for the coordinate and for the coordinate . We find partial derivatives of functions with respect two coordinates:

, т.к. , а

, т.к.

, т.к. , а

,

т.к. , а .

We substitute the obtained expressions into formulas for и :

using basic trigonometrically identity, similar

.

Then write down finally Lame's coefficients for the polar system:
.

In applying we often should find a distance between two points in this or other coordinate system. The general expression for the arc length differential in the curvilinear coordinates is defined with formula:

.

We substitute in this expression , , , , and get the arc length differential in the polar system in this form:

Determine amplitudes of velocities and acceleration in polar system using two methods: the first using formulas of vectors' amplitude and the second using Lame's coefficients.

The first method. The amplitude of velocity's vector is defined equations: , and acceleration's vector . In polar system case we have two dimensions: , . So having relation between Cartesian and polar systems we need to find the first and the second derivatives with respect to time:

Expression we find using the rule of indirect differentiation , coordinate depend implicitely on the time.

We substitute in of

We combine similar terms and put the common factors in brackets:

So, amplitude of velocity vector in the polar system has the form .

For finding amplitude acceleration vector we should find the second derivatives from coordinates of, that the same, the first derivatives from velocity projections, which we found:

+

+

+

+

Substitute the second derivatives in the formula for the amplitude acceleration vector :

So, amplitude acceleration vector in the polar system has the form:

.

The second method. The expression we can get from the formula for proections of velocity vector via generalized coordinates:

We jet know Lame's coefficients for the polar system , and substituting , , we have next expressions for the projections of velocity vector:

Projection is called radial velocity, and projection is called tranverse projection.

For the amplitude of the vector we get:

In a similar way we can get amplitude acceleration vector using formulas for projections of generalized acceleration:

,

where is notation . As amplitude velocities vector we defined that . Take again , , and formulas for projections of accelerations will have the form:

, .

Next find partial derivatives from quantity T with respect to and and velocities and :

We substitute found expressions in the formulas for projections:

, and get amplitude acceleration vector: , that the same expression we get early.

As we see from this example the second method is more simple that the second if we know Lame's coefficients.

Problem 2.

Find return transformation formulas for the polar system, in other words equalities of the form .

Solution

For this we divide two parts of the equalities which relation the polar and Cartesian coordinates by , rise to the second power and use major trigonometrically identity:

,

,

,

,

.

Whence we obtain .

For finding the second equation we eliminate divided the first equality by the second:

So, the transition from the polar system to Cartesian occurs with formulas: . These are return transitions formulas. Now when we know these relations we find Lame's coefficients, arc length differential, amplitudes of velocity and acceleration using direct relations.

Take , , , . Than is for coordinate and is for coordinate . Next we find partial derivatives with respect to both coordinates:

Substitute obtained expressions:

As we saw early, amplitude of velocity vector in polar system has a form: . We have equations reversible transformations and we can get amplitude of velocity vector in Cartesian system. For this find partial derivative:

Substitute:

So we received known formula for the velocity vector.

Problem 3.

Find the formulas of transition from Cartesian coordinate system to cylindrical coordinate system .

Solution

According problem situation if we know functions , that we should get functions which have the form .

Cylindrical coordinates are defined equalities:

.

As we see coordinate has the same value in the coordinate systems and we should express in terms and in terms . For this we should divide two parts of the first two equalities by , rise to the second power and use the basic trigonometrically identity:

– – ,

– – ,

– – ,

– – ,

.

Whence we obtane .

For finding the second equation we should eliminate divided one expression by other:

–

– –

So the transition from cylindrical coordinates to Cartesian is carried out according formulas: – . These are the formulas of reverse transformations.

Find the formulas of transition from Cartesian coordinate system to cylindrical coordinate system .

Problem 4.

Find the formulas of transition from Cartesian to spherical coordinate system .

Solution

According problem situation if we know functions , we should get functions in the next forma . .

Spherical coordinates are defined by equalities

We divide the first equality by the second:

–

– –

–

Next we divide expressions for and by , rise two parts of expression to the second power and apply the major trigonometrically equality:

We express from the third equality of transition's formulas the radius, rise its to the second power and substitute in the obtain expression:

Express the angle

So, we expressed nutation angle of the spherical system in the term Cartesian coordinate. For getting we rise all three equalities of transition formulas to the second power, add them and transformation:

Whence

So, we get next transition formulas from Cartesian system to the spherical system:

that is Solution the set problem.

Problem 5.

There is a point in Cartesian coordinate system . Find the coordinates of this point the cylindrical and spherical coordinate system.

Solution

For the transition to cylindrical system we use formulas . Whence for we just get . For the polar radius we have and for the polar angle we have . So coordinates of the point un the cylindrical coordinate system we write in the next form .

For the transition to the spherical system we use formulas , , . ДFor the radius we will have , for the zenith angle we will have , for the azimuth angle we will have . So coordinates of the point un the spherical coordinate system we write in the next form .

Problem 6.

A point moves in the ellipse with acceleration parallel y-axis. Find acceleration as a function y, if .

Solution

We should parametric equation of ellipse . Because an acceleration is along the y-axis, that y-component is not equal to zero and x-component is equal to zero. We define y-component finding the second derivative from equation :

Next we should find . For this we find the first derivative from equation and use the initial conditions:

, , so .

Whence . Find the firs derivative from this equations as quotient and we define :

We use obtained early expression and get:

We substitute obtained values in obtained expression for the projection of acceleration:

From we express and substitute in the obtained expression:

So we expressed finding component of acceleration as a function of coordinate.

Problem 7.

A point moves in the ellipse with semi-axis a and b with a constant value of velocity . Define the acceleration and the velocity of a point as a function of coordinates.

Solution

From equation of the ellipse we express x and y:

Find the first derivative from x this respect to time:

Unknown express from initial condition , whence and:

We rise to the second power two parts of this equation and expand the brackets in the right part:

Clean the common factor:

Whence:

Substitute – :

Extract the square root of this expression we will have:

When we extract the square root we lose “minus”. So we have finally:

Problem 8.

Find Lagrangian function of free material point in Cartesian coordinate system.

Solution

The Lagrangian function of free material point is defined by expression:

So we should find the square of the amplitude velocity in this coordinate system. For Cartesian we have

Projections of a vector are defined as the first derivative from coordinates

So for the square of velocity we get

and for the Lagrangian function we will have the expression:

Problem 9.

Find Lagrangian function of free material point in cylindrical coordinate system.

Solution

Use the expression for the Lagrangian in Cartesian obtained in the previous problem and use formulas for the relation Cartesian and cylindrical coordinate systems:

For the Solution we should find the first derivatives from cylindrical coordinates and substitute them in the expression for Lagrangian.

Next we find square of these derivatives.

We substitute obtained expressions in the Lagrangian and do a transformations

Finally we can write:

Problem 10.

Find Lagrangian function of free material point in spherical coordinate system.

Solution

Use the expression — and relation between Cartesian and spherical coordinate systems .

Find the first derivatives from coordinates. And don't forget that the coordinates depend implicitly on the time:

Find square of obtained expressions:

Make a square of amplitude velocity:

Write finally Lagrangian for free material point in spherical coordinates:

Problem 11.

Find the Lagrangian for a coplanar double pendulum (see figure 1), when placed in a uniform gravitational field (acceleration g).

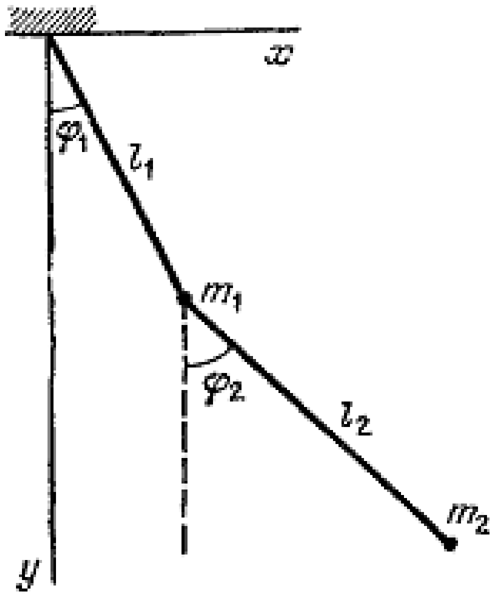

Figure 1 – A coplanar double pendulum

Solution

We take as coordinates the angles and , which the strings and make with the vertical (see figure 1). Then we have for the kinetic and potential energy of the first particle :

where – moment of inertia of a particle, and ,

.

We take “minus” because the zeroth reference level is taken as the level of the x-axis, and the y-axis is directed downwards (see figure 1).

Lagrangian for the first particle we write in the next form:

–

To find the kinetic energy for the second particle we express its Cartesian coordinates , (with the origin at the point of support and the y-axis vertically downwards) in terms of the angles , :

, .

We use the formula for the kinetic energy:

Now we should find the first derivative from coordinates x and y:

and squares of these expressions:

We find the sum of squares, combine similar terms and use major trigonometrically identity and expression for the cosine of difference:

Then for the kinetic energy of a particle we have:

The potential energy of the second point has a form:

Lagrangian for the second particle we write in the form:

According additivity property of Lagrangian :

We expand brackets and combine similar terms:

We get finally:

Problem 12.

Find Lagrnagisn of a simple pendulum of mass , with a mass at the point of support which can move on a horizontal line lying in the plane in which moves (see figure 2), when placed in a uniform gravitational field (acceleration g).

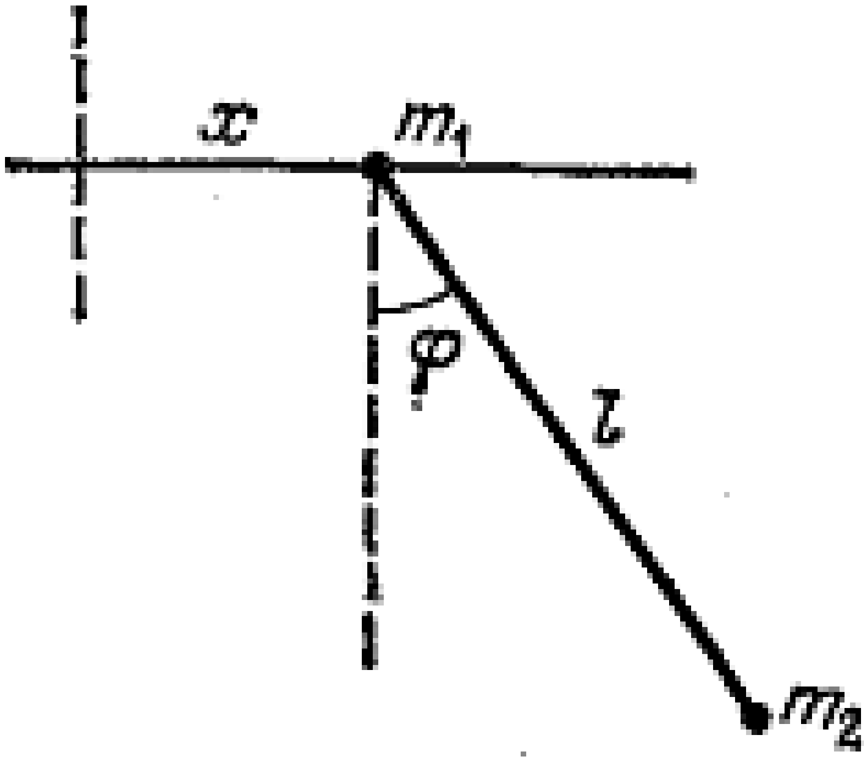

Figure 2 – Simple pendulum with moving point of support

Solution

Using coordinate *x* of and the angle φ between the string and the vertical. For the point of support we will have:

For the kinetic energy of the pendulum we can write:

Now we should express coordinates and in the terms coordinates of point of support and in the term angle :

, .

Find the first derivatives and substitute in the expression for the kinetic energy:

Potential energy of the pendulum we can write in this form and Lagrangian will have the form:

According additivity property of Lagrangian :

We do a transformation and finally we get:

Problem 13.

Find Lagrangian of a simple pendulum of mass whose point of support moves uniformly on a vertical circle with constant frequency (see figure 3), when placed in a uniform gravitational field (acceleration g).

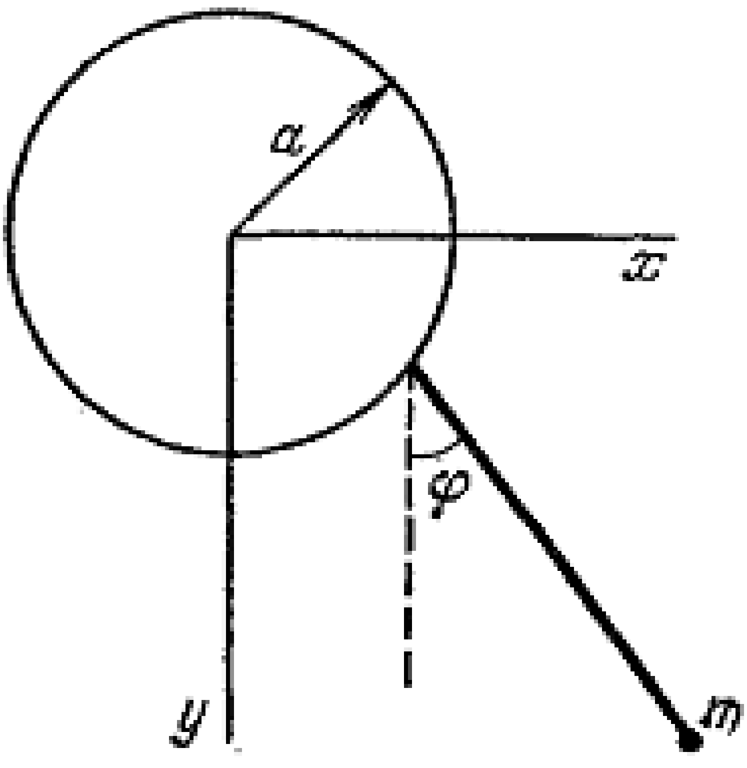

Figure 3 – Simple pendulum with moving point of support

Solution

Using equation of the circle in parametric form , , express coordinates of point *m*:

, .

Next using formula for the kinetic energy:

Now we should find the firs derivatives from coordinates x and y:

and squares of these expressions:

We find the sum of squared, combining common terms, using major trigonometrically identity and the formula sine of difference:

Next for the kinetic energy of the point we get:

Potential energy of the point will have the form:

Lagrangian of the point we can write in this form:

Finally we will have:

here we negligible terms which depend on only the time (the first and the second), and we eliminate the complete derivative with respect to time from , равная

Whence

Problem 14.

Find Lagrangian of simple pendulum of mass whose point of support oscillates horizontally in the plane of motion of the pendulum according to the law , when placed in a uniform gravitational field (acceleration g).

Solution

Coordinates of the point *m* are defined next way:

, .

To use the formula for the kinetic energy:

Now we should find the first derivatives from coordinates x and y:

and squares from these expressions:

We find the sum of squared, combining common terms, using major trigonometrically identity:

Next for the kinetic energy of the point we get:

Potential energy of the point will have the form:

Lagrangian of the point we can write in this form:

The first derivative depend explicitly only on time and then it is the complete derivative from any other function of time. We find the complete derivative with respect to time from от and eliminate its from the Lagrangian:

Whence

Lagrangian (after eliminating complete derivatives)

Problem 15.

Find Lagrangian of a simple pendulum of mass whose point of support oscillates vertically according to the law , when placed in a uniform gravitational field (acceleration g).

Solution

Similar previous problem coordinates of the point *m* are defined next way:

, .

To use the formula for the kinetic energy:

Now we should find the first derivatives from coordinates x and y:

and squares from these expressions:

We find the sum of squared, combining common terms, using major trigonometrically identity:

Next for the kinetic energy of the point we get:

Potential energy of the point will have the form:

Lagrangian of the point we can write in this form:

The first derivative depend explicitly only on time and then it is the complete derivative from any other function of time. We find the complete derivative with respect to time from от and eliminate its from the Lagrangian:

Whence

Lagrangian (after eliminating complete derivatives) will have a form:

Problem 16.

Find Lagrangian of the system shown in Fig. 4 The particle moves on a vertical axis and he whole system rotates about this axis with a constant angular velocity Ω.

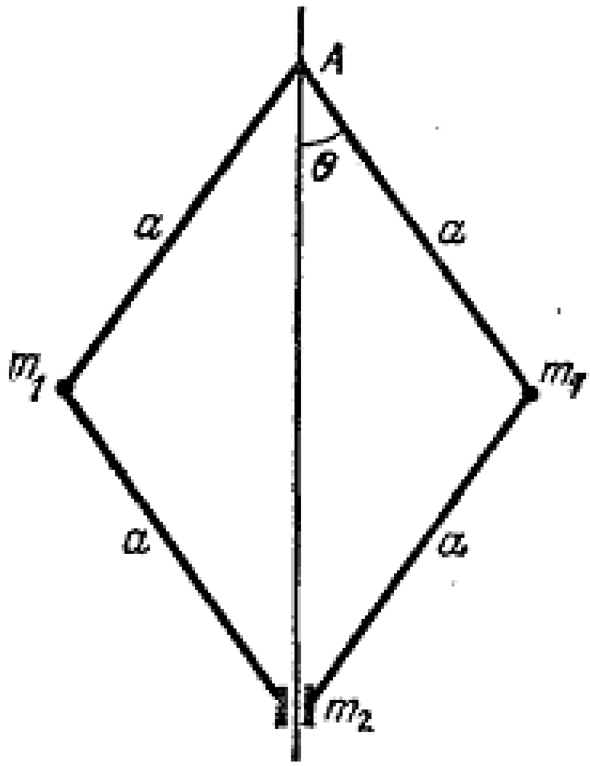

Figure 4 – For the problem 16

Solution

For finding kinetic energy we use the property:

For this we should express the element of displacement in terms given quantities. Let be the angle between one of the segments *a* and the vertical, and the angle rotation of the system about the axis; . For each particle the infinitesimal displacement is given by . The distance of from the point of support A is , and so .

For the kinetic energy of the point we have:

Potential energy of the point we can write next form:

The Lagrangian for the point :

For the kinetic energy of the point we have:

Potential energy of the point we can write next form:

The Lagrangian for the point :

According additivity property of Lagrangian :

Finally we get:

Problem 17.

Given the Lagrangian of free moving along the axis material point .

Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion of the system.

Solution

Generalized momenta are defined by formula —. In this case a motion is one dimension and we have just one coordinate . Then

that is define traditional impulse of translational motion of a point.

Generalized forces are defined by formula —. Then

i.e., there is no forces acting on the particle, as it should be for a free particle.

Energy is defined by expression — . Expression — — – we defined early, a , then

that coincides with formula for the kinetic energy of translational motion of a particle.

The equations of motion we get using Lagrange's equations the second type:

or, because

We got already expressions — and — and now it is necessary to define ——:

We substitute all obtained quantities in the Lagrange's equations and do a transformations:

или , т.е. .

This equation express the law of inertia: if there are no external forces acting on a body or the action of external forces is compensated , that body is in a rest or in straight-line uniform motion .

Problem 18.

Given the Lagrangian material point in the field — . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion of the system with vector and coordinate method specifying the motion (in Cartesian coordinate system).

Solution

1. With a coordinate method we have , and Lagrangian will have a form:

Generalized momenta are defined by formula Generalized momenta are defined by formula . In this case a motion is three dimensions and we have three coordinates . Then

derivatives are equals to zero. Similar we find next two projections of momentum on axis and :

derivatives equals zero.

The vector of generalized momentum we write in Cartesian system in the next form:

and its amplitude is defined by next formula

Generalized forces are defined by the formula —, где
. Then

— — — — —

— — — — —

— — — — —

where derivatives from function – with respect to all coordinates equal zero, let .

Vector of generalized force in Cartesian system we write in the next form:

— — —

and its amplitude is defines by formula

— — — — — —

Energy is defined by expression — . Expressions — - we defined early, and , then

—

—

— — —

— — —

— — — —

i.e. finally we will get for energy of the system

—

The equations of motion we get using Lagrange's equations the second type:

— — —

or, so and :

— — —

— — —

— — —

Expressions , and we have already and we need define :

We substitute all obtained quantities in the Lagrange's equations and do a transformations:

so we got the second Newton's law in the coordinate form.

2. With a vector method , and and Lagrangian have a form:

Generalized momenta are defined by formula . In this case we describe a motion using a position-vector . Then

Generalized forces are defined by formula . Then

Energy is defined by expression . Expression - we defined early, and , then

The equations of motion we get using Lagrange's equations the second type:

or, because

We got already expressions — и — уже получены и необходимо определить ——:

We substitute all obtained quantities in the Lagrange's equations and do a transformations:

so, we got the second Newton's law in vector form.

Problem 19.

Given the Lagrangian of mechanical system (simple pendulum) . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion of the system.

Solution

Generalized momenta are defined by formula . In this case a motion is one dimension and we have just one coordinate . Then

The second derivative equals zero because doesn't depend on , the first derivative gives for the vector of generalized momentum the expression (in this case for the one dimension motion it coincides with projection of this vector on this axis)

Generalized forces are defined by formula . In this case a motion is one dimension and we have just one coordinate . Then

The first derivative equals zero because doesn't depend on , the second derivative gives for the vector of generalized force the expression (in this case for the one dimension motion it coincides with projection of this vector on this axis)

(we should not multiply this derivative by , because we are finding the derivative with respect not to time).

Energy is defined by expression — . Expression — —

- we defined early, a , then

The equations of motion we get using Lagrange's equations the second type:

or, because

We got already expressions — and — and now it is necessary to define ——:

We substitute all obtained quantities in the Lagrange's equations and do a transformations:

This is equation of motion of simple pendulum, which in case small oscillation when , goes to known equation of harmonic oscillations:

где – – angular frequency of the simple pendulum.

Problem 20.

Given the Lagrangian of mechanical system (free material point) in cylindrical coordinates – . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion of the system with vector and coordinate method specifying the motion (in Cartesian coordinate system).

Solution

Generalized momenta are defined by formula —. In this case a motion is three dimensions and we have three coordinates . Then

The vector of generalized momentum we write in cylindrical system in the next form:

and its amplitude is defined by next formula

Generalized forces are defined by the formula , где

. Then

The second and third derivatives equal zero because Lagrangian doesn't depend on and , the second derivative gives expression for the projection of generalized force vector which coincides in this case with the same vector:

Energy is defined by expression , which in cylindrical system we should write in the form: . Expressions we defined early when found projections of momentum vector:

Substitute to the expression for energy:

— — — —

The equations of motion we get using Lagrange's equations the second type:

—— ——

or, because

—— —

—— —

—— —

Expressions — — — and — — , — we got and we should define ——, ——, ——:

—— —

—— — —

—— — —

Substitute al obtain quantities to the Lagrangian's equations and do a transformation:

These are target equation of motion.

Problem 21.

Given the Lagrangian of mechanical system (free material point) in spherical coordinates — . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.

Solution

Generalized momenta are defined by formula —. In this case a motion is three dimensions and we have three coordinates . Then

The vector of generalized momentum we write in spherical system in the next form:

and its amplitude is defined by next formula :

Generalized forces are defined by the formula , где
. Then

The vector of generalized force we write in cylindrical system in the next form:

where , and its amplitude is define by formula

Energy is defined by expression , which in spherical system we should write in the form: . Expressions we defined early when found projections of momentum vector:

Substitute to the expression for energy:

The equations of motion we get using Lagrange's equations the second type:

or, because

Expressions — — — and — — , — we got and we should define ——, ——, ——:

Substitute al obtain quantities to the Lagrangian’s equations and do a transformation:

These three equations describe a motion of material point in projections on axis.

1.3 Problems for independent work

In all the next problems you should find Lame’s coefficients, arc length differential, amplitudes of velocity and acceleration in next coordinate system:

1. Cartesian coordinate system (see figure 5)

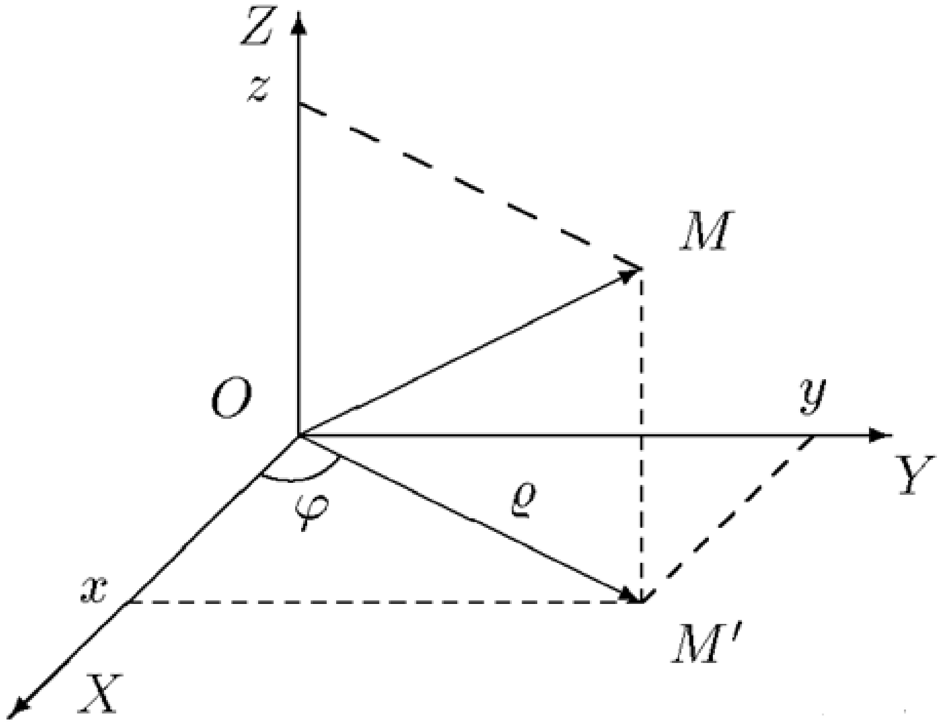

Figure 5 – Cartesian coordinate system in space

2. Cylindrical coordinates (see figure 6)

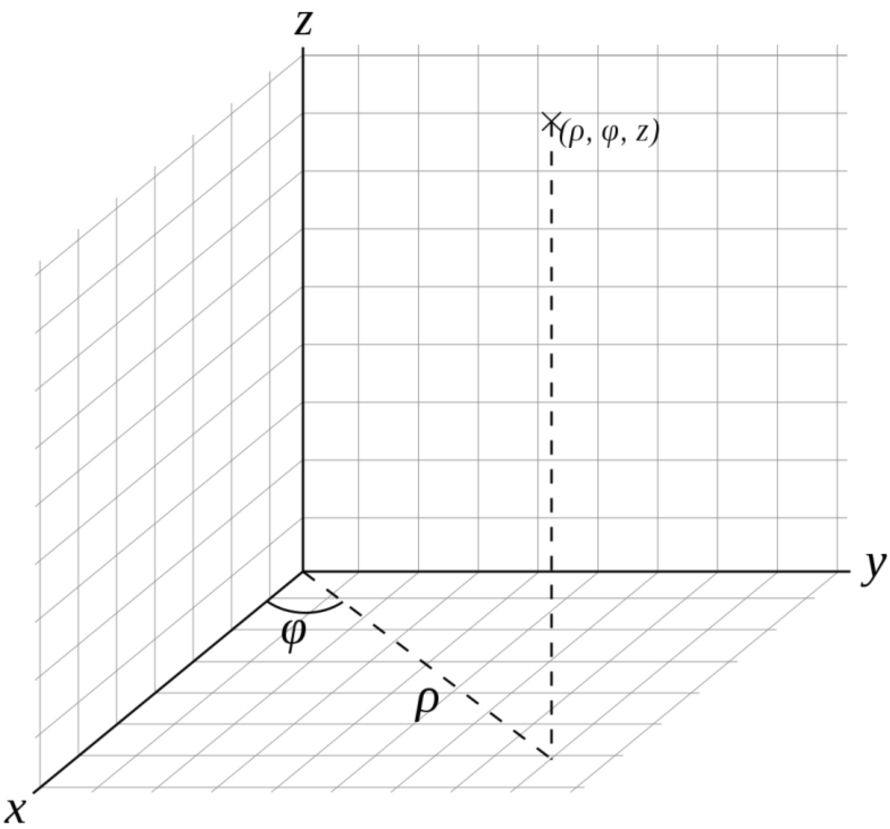

Figure 6 – Cylindrical coordinate system

3. Spherical coordinates (see figure 7)

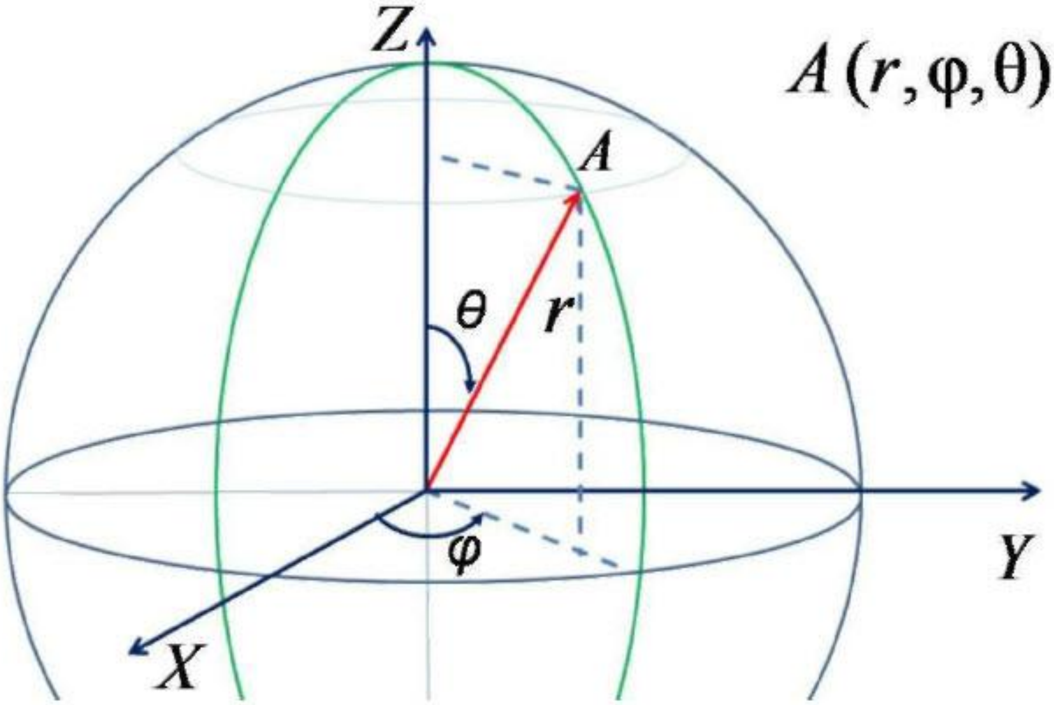

Figure 7 – Spherical coordinate system

4. Elliptical coordinates (see figure 8)

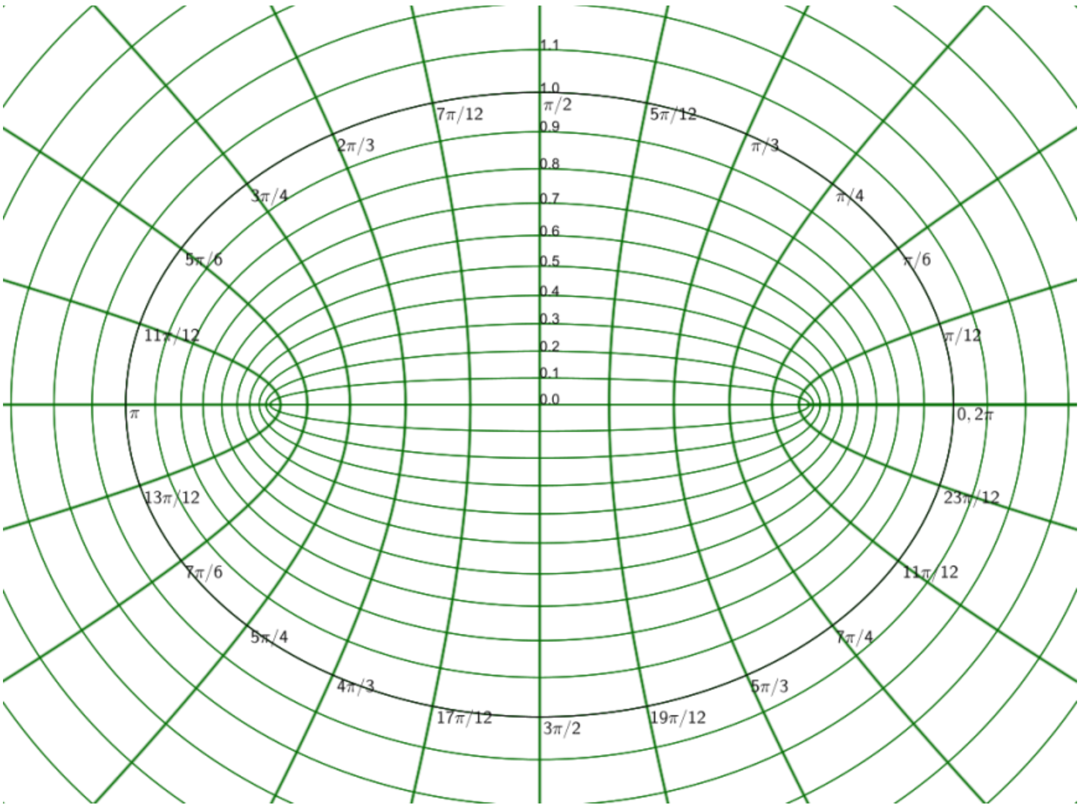

Figure 8 – Elliptical coordinate system in the plane

5. Parabolic coordinates in two dimensions – (see figure 9)

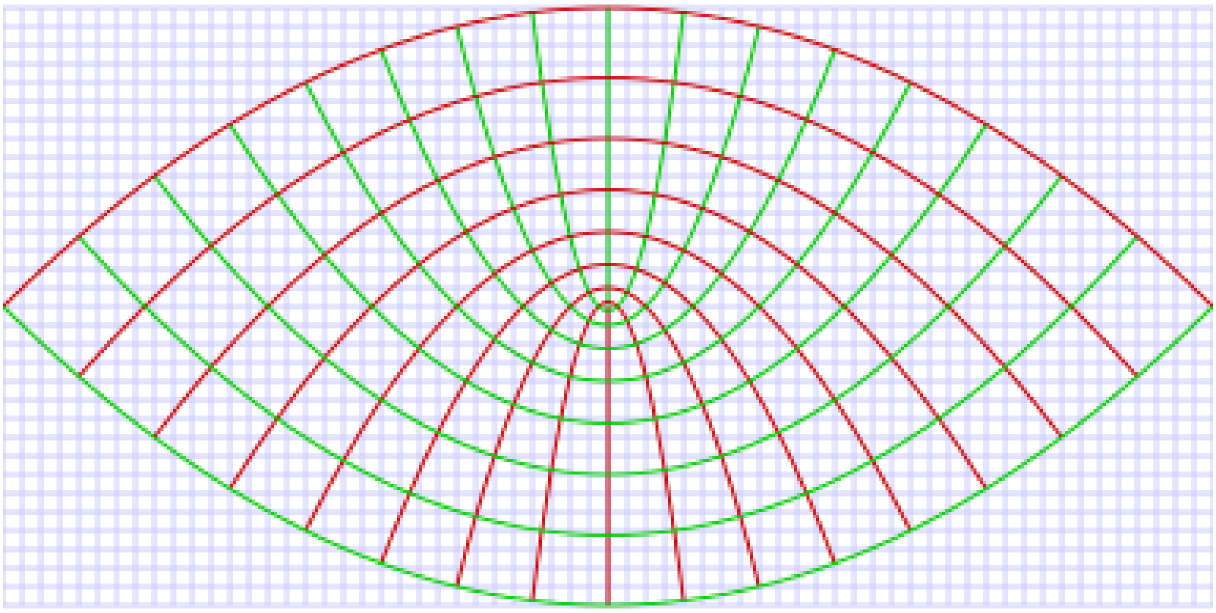

Figure 9 – Parabolic coordinate system in two dimensions

6. Parabolic coordinates in three dimensions
   – (see figure 10)

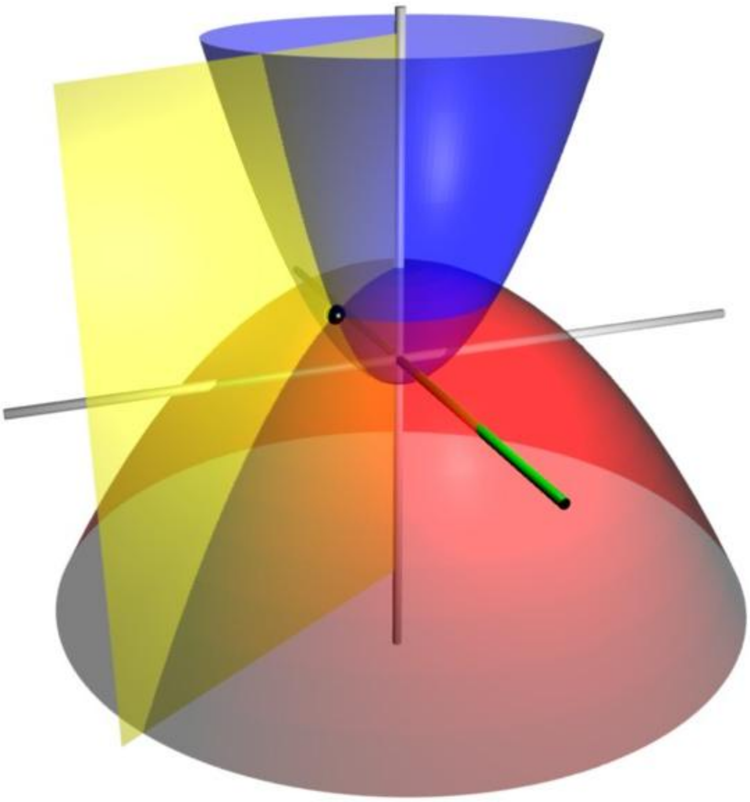

Figure 10 – Parabolic coordinate system in three dimensions

7. Cylindrical parabolic coordinates –
   (see figure 11)

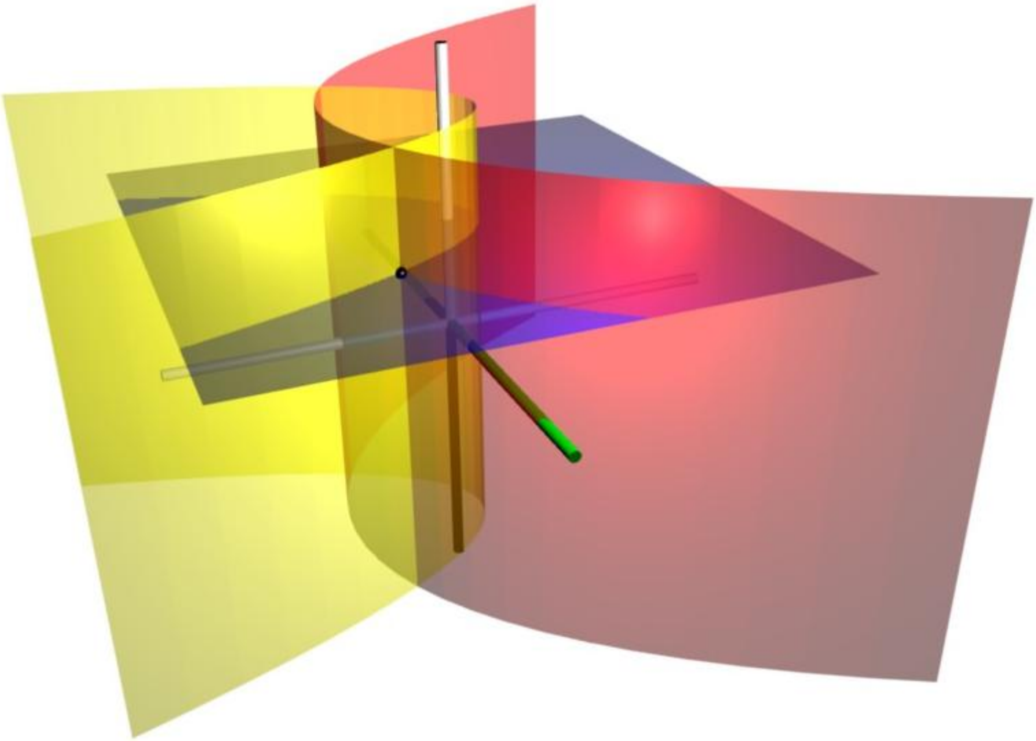

Figure 11 – Cylindrical parabolic coordinate system

8. Bipolar coordinates ——— ——— (see figure 12)

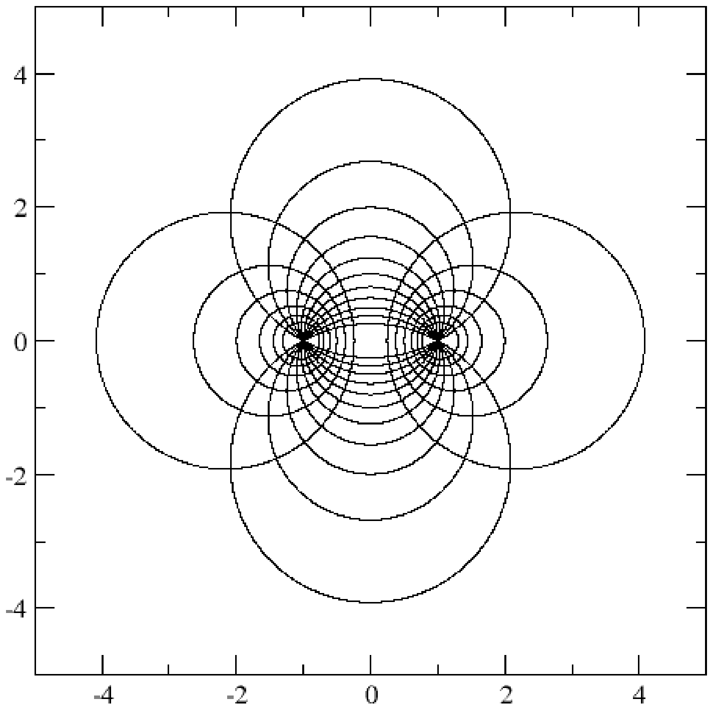

Figure 12 – Bipolar coordinate system

9. Toroidal coordinates ——— ——— ——— (see figure 13)

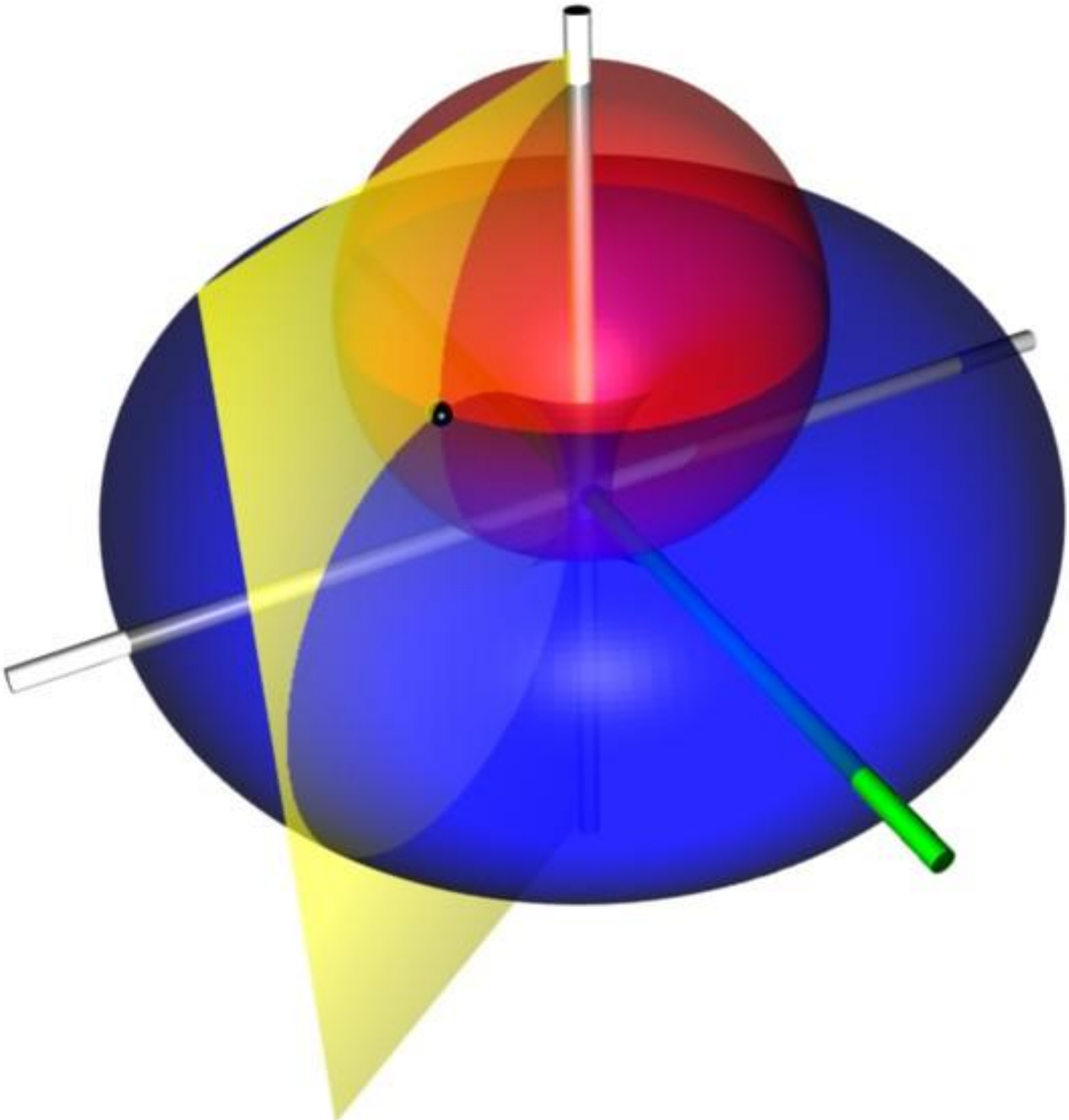

Figure 13 – Toroidal coordinate system

Instruction: Because you have expressions which relation curvilinear coordinates with Cartesian coordinates that the formula for Lame's coefficients you should take in the next form — — — , where -. curvilinear coordinates.

Problems with coordinates on the plane

1. Find the transition formulas from coordinate system to elliptical .
2. Find the transition formulas from coordinate system to parabolic in two dimensions .
3. Find the transition formulas from coordinate system to bipolar .
4. Find the transition formulas from polar system to elliptical .

5. Find the transition formulas from polar system to parabolic in two dimensions .
6. Find the transition formulas from polar system to bipolar .
7. Find the transition formulas from elliptical system to polar .
8. Find the transition formulas from elliptical system to parabolic in two dimensions .
9. Find the transition formulas from elliptical system to bipolar .
10. Find the transition formulas from parabolic coordinate system in two dimensions to polar .
11. Find the transition formulas from parabolic coordinate system in two dimensions to elliptical .
12. Find the transition formulas from bipolar coordinate system to polar .
13. Find the transition formulas from bipolar coordinate system to elliptical .

Problems with coordinates in the space

1. Find the transition formulas from coordinate system to parabolic in three dimensions .
2. Find the transition formulas from coordinate system to toroidal .
3. Find the transition formulas from cylindrical coordinate system to spherical .
4. Find the transition formulas from cylindrical coordinate system to parabolic in three dimensions .
5. Find the transition formulas from cylindrical coordinate system to toroidal .
6. Find the transition formulas from spherical coordinate system to cylindrical .
7. Find the transition formulas from spherical coordinate system to parabolic in three dimensions .
8. Find the transition formulas from spherical coordinate system to toroidal .
9. Find the transition formulas from parabolic coordinate system in three dimensions to cylindrical .
10. Find the transition formulas from parabolic coordinate system in three dimensions to spherical .
11. Find the transition formulas from parabolic coordinate system in three dimensions to toroidal .
12. Find the transition formulas from toroidal coordinate system to cylindrical .
13. Find the transition formulas from toroidal coordinate system to spherical .

14. Find the transition formulas from toroidal coordinate system to parabolic in three dimensions .

Problems to make Lagrangian

1. Find Lagrangian fo free material point in polar coordinates.
2. Find Lagrangian fo free material point in elliptical coordinates.
3. Find Lagrangian fo free material point in parabolic coordinates in two dimensons.
4. Find Lagrangian fo free material point parabolic coordinates in three coordinates.
5. Find Lagrangian fo free material point in cylindrical parabolic coordinates.
6. Find Lagrangian fo free material point in bipolar coordinates.
7. Find Lagrangian fo free material point in toroidal coordinates.

Problems with Lagrangian

1. Given the Lagrangian of mechanical system (free material point) — —. Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
2. Given the Lagrangian of mechanical system (free material point) — —. Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
3. Д Given the Lagrangian of mechanical system (free material point) — . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
4. Given the Lagrangian of mechanical system (free material point) — . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
5. Given the Lagrangian of mechanical system (free material point) — — . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.

6. Given the Lagrangian of mechanical system (free material point) . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
7. Given the Lagrangian of mechanical system (free material point) . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
8. Given the Lagrangian of mechanical system (free material point) . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
9. Given the Lagrangian of mechanical system (free material point) . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
10. Given the Lagrangian of mechanical system (free material point) . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.
11. Given the Lagrangian of mechanical system (free material point) . Define generalized momenta , vector of generalized momentum, amplitude of vector of generalized momentum, generalized forces , vector of generalized force, amplitude of vector of generalized force, energy of the system, equation of motion.

1.4 Tests

What does equal Lame’s coefficients?

A)

B)

C)

D)

E)

****

По какому уравнению можно найти дифференциал длины дуги в криволинейных координатах?

A)

B)

C)

D)

E)

****

61.1. Choose the polar coordinate system:

A)

B)

C)

D)

E)

****

What is a relationship between Cartesian and polar coordinate systems?

A)

B)

C)

D)

E)

****

Find Cartesian coordinates of a point if its polar coordinates are: .

A)

B)

C)

D)

E)

****

Find polar coordinates of a point if its Cartesian coordinates are: .

A)

B)

C)

D)

E)

****

Choose the cylindrical coordinate system:

A)

B)

C)

D)

E)

****

Find Cartesian coordinates of a material point if its cylindrical coordinates: .

A)

B)

C)

D)

E)

****

Choose the spherical coordinate system:

A)

B)

C)

D)

E)

****

Choose the elliptical coordinate system:

A)

B)

C)

D)

E)

****

Choose the parabolic coordinate system in two dimensions:

A)

B)

C)

D)

E)

****

Find Cartesian coordinates of a point if its parabolic coordinates in two dimensions are .

A)

B)

C)

D)

E)

****

Choose the parabolic coordinate system in three dimensions:

A)

B)

C)

D)

E)

****

Choose the bipolar coordinate system:

A)

B)

C)

D)

E)

****

Choose the cylindrical parabolic coordinate system:

A)

B)

C)

D)

E)

****

What coordinate system is defined by equations: ?

A) parabolic in three dimensions

B) cylindrical parabolic

C) spherical

D) cylindrical

E) полярные

****

What coordinate system is defined by equations: ——— ———?

A) bipolar
B) elliptical
C) cylindrical
D) polar
E) spherical
****
What coordinate system is defined by equations:
?
A) elliptical
B) spherical
C) cylindrical parabolic
D) parabolic in three dimensions
E) цилиндрические
****
What coordinate system is defined by equations: – ?

A) parabolic in two dimensions
B) cylindrical parabolic
C) parabolic in three dimensions
D) parabolic in two dimensions
E) spherical
****
What coordinate system is defined by equations:
– ?

A) bipolar cylindrical
B) parabolic
C) elliptical
D) spherical
E) parabolic in three dimensions
What coordinate system is defined by equations:
?
A) elliptical
B) parabolic in two dimensions
C) cylindrical
D) polar
E) bipolar
****
What coordinate system is defined by equations: –
?
A) parabolic in two dimensions
B) cylindrical parabolic
C) polar

D) parabolic in three dimensions
E) bipolar
****
What coordinate system is defined by equations: ?
A) parabolic in two dimensions
B) spherical
C) cylindrical
D) polar
E) parabolic in three dimensions
****
Find Lame’s coefficients for Cartesian coordinate system :
A)
B)
C)
D)
E)
****
Find Lame’s coefficients for spherical coordinate system :
A)
B)
C)
D)
E)
****
Find Lame’s coefficients for cylindrical coordinate system :
A)
B) 1
C)
D)
E)
****
Find Lame’s coefficients for the next coordinate system :
A)
B)
C)
D)
E)
****

Find Lame's coefficients for the next coordinate system

:

A)

B)

C)

D)

E)

****

:

A)
B)
C)
D)
E)

****

Find Lame's coefficients for parabolic coordinate system in two dimensions

– :

A)
B)
C)
D)
E)

****

Find Lame's coefficients for cylindrical parabolic coordinate system

– :

A)
B)
C)
D)
E)

****

Find Lame's coefficients for parabolic coordinate system in three dimensions

– :

A)
B)

C)

D)

E)

****

Find Lame's coefficient for the Cartesian system :

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the Cartesian system :

A)

B)

C)

D) /2

E)

****

Find Lame's coefficient for the Cartesian system :

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the spherical system :

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the spherical system :

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the spherical system
:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the cylindrical system
:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the cylindrical system
:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the cylindrical system
:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the elliptical system
:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the elliptical system

:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the bipolar system

:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the bipolar system

:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the polar system

:

A)

B)

C)

D)

E)

****

Find Lame's coefficient for the polar system

:

A)
B)
C)
D)
E)
****
Find Lame's coefficient for the parabolic system in two dimensions
– :
A)
B)
C)
D)
E)
****
Find Lame's coefficient for the parabolic system in two dimensions
– :
A)
B)
C)
D)
E)
****
Find Lame's coefficient for the cylindrical parabolic system
– :
A)
B)
C)
D)
E)
****
Find Lame's coefficient for the cylindrical parabolic system
– :
A)
B)
C)
D)
E)
****

Find Lame's coefficient for the cylindrical parabolic system
– :

A)
B)
C)
D)
E)

****

Find Lame's coefficient for the parabolic system in three dimensions – :

A)
B)
C)
D)
E)

****

Find Lame's coefficient for the parabolic system in three dimensions
– :

A)
B)
C)
D)
E)

****

Find Lame's coefficient for the parabolic system in three dimensions – :

A)
B)
C)
D)
E)

****

Chapter 2 Conservation laws

2.1 Verification questions

1. What is integral of motion?
2. Write the expressions for energy of the system. From what does follow energy conservation law?
3. Is the energy additive quantity? Why?
4. What is conservative system?
5. What quantity is momentum of material point called?
6. What quantity is momentum of system called?
7. Formulate momentum conservation law. From what does it follow?
8. What are generalized momenta and generalized forces?
9. Is conserved all three components of momentum?
10. Write down the transformation formula for momentum.
11. What is a centre of mass?
12. What is a rest?
13. What is internal energy of a system? What does is consist?
14. What does internal energy of a moving system equal?
15. Write down the transformation formula for energy.
16. What is an angular momentum?
17. Formulate angular momentum conservation law. From what does it follow?
18. Is angular momentum conservation law correct for a system is in external field?
19. Write down the transformation formula for angular momentum.

2.2 Problem Solution

Problem 22.

A particle of mass *m*, moving with velocity leaves a half-space in which its potential energy is a constant and enters another in which its potential energy is a different constant . Determine the change in the direction of motion of the particle (see figure 14).

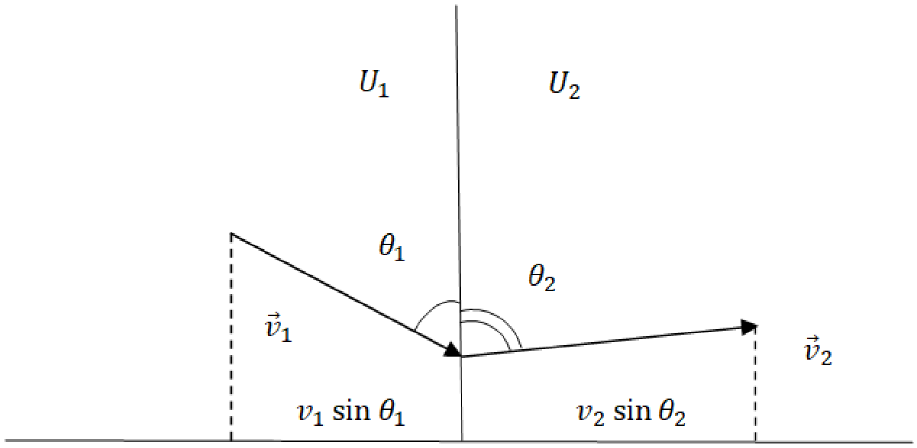

Figure 14 – To the problem 22

Solution

Potential energy is independent of the coordinates whose axes are parallel to the plane separating the half-space. The component of momentum in that plane is therefore conserved. Denoting and the angles between the normal to the plane and velocities and of the particle before and after passing the plane, write down the momentum conservation law:

From this equation we have a projection on the horizontal axes:

Relation between and we can get from energy conservation law

Express from : and substitute to the energy conservation law:

Express relation which defines changing of direction of moving particle:

and finally we will have:

Problem 23.
Find the law of transformation of the action from one inertial frame to another.
Solution
Lagrangian is a difference between kinetic and potential energies

Energy is a sum of kinetic and potential energies

The formula of transformation of energy from one inertial energy to another is

Write down initial equalities for both systems:

Find the difference for both systems:

Left parts are the same, so the same and right parts:

Rewrite in other form:

The right part of this equation we can express from the formula of transformation of the energy:

So., we can find

Whence

and

We found the formula of transformation of Lagrangian in transition from one inertial system to another.

The action is an integral from Lagrangian with respect to time:

Substitute under the integral obtained law of transformation of Lagrangian between limits 0 and t:

Integrate apart each addition:

Суммируя эти выражения, найдем искомый закон преобразования действия:

where – radius-vector of centre of inertia in system .

Problem 24.

Obtain expressions for the Cartesian components and the amplitude of the angular momentum of a particle in cylindrical coordinates .

Solution

The vector of angular momentum is defined by vector multiply of momentum by position-vector:

Find the components of vector in the Cartesian coordinates. We know that the vector multiplication is a determinant of the third range with components of multiplying vectors:

We expand the determinant, taking into account that and we get:

Whence for projections of angular momentum vector we get expressions:

Using , , , rewrite these projections in this form:

Cylindrical coordinates are defined by equalities:

We find the first derivatives from coordinates and don't remember about implicitly dependent coordinates from time.

We substitute obtained velocities to the expressions for projections of angular momentum vector and use expressions for relations between coordinates:

Find squares of projections of angular momentum vector:

Next find square of amplitude of angular momentum vector:

Finally write down result in the next form:

Problem 25.

Obtain expressions for the Cartesian components and the amplitude of the angular momentum of a particle in spherical coordinates .

Solution

The vector of angular momentum is defined by vector multiply of momentum by position-vector:

Find the components of vector in the Cartesian coordinates. We know that the vector multiplication is a determinant of the third range with components of multiplying vectors:

We expand the determinant, taking into account that

and we get:

Whence for projections of angular momentum vector we get expressions:

Taking , , , rewrite these projections in this form:

Spherical coordinates are defined by equalities:

We find the first derivatives from coordinates and don't remember about implicitly dependent coordinates from time.

We substitute obtained velocities to the expressions for projections of angular momentum vector and use expressions for relations between coordinates:

- projection on x-axis

- projection on y-axis

- projection on z-axis

For finding amplitude of the angular momentum vector we should use the formula and obtained projections:

Find squares of projections:

Next find square of amplitude of angular momentum vector:

Finally write down result in the next form:

2.3 Problems for independent work

1. Find expressions for the Cartesian components and amplitude of angular momentum vector in elliptical coordinates .
2. Find expressions for the Cartesian components and amplitude of angular momentum vector in parabolic coordinates in two dimensions .
3. Find expressions for the Cartesian components and amplitude of angular momentum vector in parabolic coordinates in three dimensions .
4. Find expressions for the Cartesian components and amplitude of angular momentum vector in cylindrical parabolic coordinates u .
5. Find expressions for the Cartesian components and amplitude of angular momentum vector in toroidal coordinates .
6. Find the ratio of the times in the same path for particles having different masses but the same potential energy.
7. Find the ratio of the times in the same path for particles having the same mass but potential energies differing by a constant factor.

2.4 Tests

What is called functions which have constant values when a mechanical system moves?
A) Generalized momenta
B) Variates of Lagrange's function
C) Integrals of motion
D) Conservative functions
E) Differentials of energy
****
How many integrals of motion do exist for the system with S degrees of freedom?
A) S
B) 2S-1
C) 3S
D) S^2
E) S(S+2)
****
How many important integrals of motion does exist in classical mechanics?
A) 3
B) 5
C) 7
D) 9

E) 11
****
Which integral of motion does follow from the homogeneous of time?
A) Energy
B) Momentum
C) Angular momentum
D) Force
E) Acceleration
****
Which integral of motion does follow from the homogeneous of space?
A) Mass
B) Force
C) Angular momentum
D) Energy
E) Momentum
****
Which integral of motion does follow from the isotropy of space?
A) Angular momentum
B) Energy
C) Velocity
D) Mass
E) Momentum
****
Choose integral of motion:
A) Coordinate
B) Velocity
C) Momentum
D) Force
E) Period
****
Choose integral of motion:
A) Acceleration
B) Energy
C) Period
D) Frequency
E) Force
****
Choose integral of motion:
A) Period
B) Coordinate
C) Frequency
D) Force
E) Angular momentum
****

Specify an expression for the energy of the mechanical system:

A)

B)

C)

D)

E)

****

Find the total mechanical energy of the system, whose Lagrange's function has the form:

A)

B)

C)

D)

E)

****

How does the energy transform when moving from one IRS to another?

A)

B)

C)

D)

E)

****

What is a mechanical system called, whose energy is conserved?

A) Enclosed
B) Conservative
C) Additive
D) Dissipative
E) Associated

****

What conservation law does follow from the homogeneity of time?

A) Energy
B) Momentum
C) Angular momentum
D) Force
E) Acceleration

****

What conservation law does follow from the homogeneity of space?

A) Mass

B) Force
C) Angular momentum
D) Energy
E) Momentum
****
What conservation law does follow from the isotropy of space?
A) Angular momentum
B) Energy
C) Velocity
D) Mass
E) Momentum
****
The expression for the momentum of a system has the form:
A)
B)
C)
D)
E)
****
What quantity is called the generalized momentum of a mechanical system?
A)
B)
C)
D)
E)
****
Find the component of the momentum of a mechanical system if it is described by the Lagrange's function of the form:
A)
B)
C)
D)
E)
****
Find the component of the momentum of a mechanical system if it is described by the Lagrange's function of the form:
A)
B)

C)
D)
E) —
****
Find the complete momentum of the mechanical system if it is described by the Lagrange function of the form: —

A)
B)
C) —
D)
E)
****
Specify an expression for the generalized forces:

A) —
B)
C) —
D) —
E) — — —
****
Find the component of the force of a mechanical system, if it is described by the Lagrange's function of the form: —

A)
B)
C)
D)
E) —
****
Find the component of the force of a mechanical system, if it is described by the Lagrange's function of the form: —

A)
B)
C)
D)
E)
****
What is the derivative of the Lagrange's function with respect to velocity?
A) Energy

B) Angular momentum
C) Mass
D) Momentum
E) Force
****
What is the derivative of the Lagrange's function with respect to coordinates?
A) Angular momentum
B) Force
C) Acceleration
D) Energy
E) Mass
****
How does the momentum transform when moving from one IRS to another?
A)
B)
C)
D)
E)
****
Choose the expression for the center of mass of the mechanical system:
A)
B)
C)
D)
E)
****
What is ht quantity called:
A) Energy
B) Mass of the system
C) Momentum
D) Angular momentum
E) Force
****
How does the angular momentum transform when moving from one IRS to another?
A)
B)
C)

D) — —

E) -

****

What operation with Langrange's function do not change equations of motion?

A) Rise to the n-th power

B) Add the complete differential of action

C) Multiplication by constant

D) Division by potential energy

E) Derivative with respect to time

****

Determine an energy of mechanical system if the Lagrange's function is

— .

A) —

B) —

C) —

D)

E)

****

Determine a momentum of mechanical system if the Lagrange's function is

— .

A)

B)

C) —

D)

E) —

****

Determine a force acting on a mechanical system if the Lagrange's function is

— .

A)

B)

C)

D)

E)

****

Determine a kinetic energy of mechanical system if the Lagrange's function is

— .

A) —

B)

C)

D)

E)

****

Determine a potential energy of mechanical system if the Lagrange's function is .

A)

B)

C)

D)

E)

****

What is a condition of homogeneity of potential energy with coordinates:

A)

B)

C)

D)

E)

****

What does degree of homogeneity equal for little oscillations?

A) -1

B) 0

C) -2

D) 1

E) 2

****

What does degree of homogeneity equal for the uniform field of force?

A) -1

B) 0

C) -2

D) 1

E) 2

****

What does degree of homogeneity equal for the Newtonian attraction?

A) -1

B) 0

C) -2

D) 1

E) 2

****

What does degree of homogeneity equal for Coulomb interaction?
A) -1
B) 0
C) -2
D) 1
E) 2
****
For instance, that the square of the time of revolution in the orbit is as cube of the size of the orbit. It is:
A) Galilees's relative principle
B) The third Kepler's law
C) The second Newton's law
D) Poisson's theorem
E) Maupertui's rule
****
What is a relationship between average potential energy and complete energy of the system?
A) —
B) —
C) —
D) —
E) —
****
What is a relationship between average kinetic energy and complete energy of the system?
A) —
B) —
C) —
D) —
E) —
What is a relationship between average kinetic energy and potential energy of the system in general case?
A)
B)
C)
D)
E)
What is a relationship between average kinetic energy and potential energy of the system for the small oscillation?

A) –

B)

C)

D) –

E)

****

What is a relationship between average kinetic energy and potential energy of the system for the Newtonian interaction?

A) –

B)

C)

D) –

E)

****

What is a relationship between complete energy and potential kinetic of the system for the Newtonian interaction?

A)

B)

C)

D) /2

E)

****

Choose the third law of Kepler:

A) – –

B) – –

C) – –

D) – –

E) – –

****

What is the relationship between velocities and linear dimensions in mechanical similarity?

A) – –

B) – –

C) – –

D) – –

E) – –

****

What is the relationship between energies and linear dimensions in mechanical similarity?

A) – –

B) – –

C) – –

D) – –

E) – –

****

What is the relationship between angular momenta and linear dimensions in mechanical similarity?

A) – –

B) – –

C) – –

D) – –

E) – –

****

What is the relationship between times and linear dimensions in mechanical similarity?

A) – –

B) – –

C) – –

D) – –

E) – –

****

What is the relationship between times and linear dimensions in mechanical similarity for uniform force field?

A) – –

B) – –

C) – –

D) – –

E) – –

****

What is relationship between the times in the same path for a particles having different masses but the same potential energy?

A) – —

B) – —

C) – —

D) – —

E) – —

****

What is relationship between the times in the same path for a particles having the same mass but potential energies differing by a constant factor?

A) – —

B) – —

C) – —

D) – —

E) – —

****

Chapter 3 Integration of the equations of motion

3.1 Verification questions

1. How many integrals of motions are there? List them
2. What is called central-symmetric field?
3. What is called motion in one dimension?
4. Describe a motion of the particle in the potential well
5. What are the turning points?
6. What is the condition of finite motion? What is the condition of infinite motion?
7. How can you define Lagrangian of a two-body system?
8. What is a reduced mass?
9. What is cyclic coordinate?
10. What is equal the force acting on the particle in central field? What is its direction?
11. What is sectorial velocity?
12. Formulate the second Kepler's law
13. What is centrifugel energy?
14. Formulate two-body problem
15. Formulate Kepler's problem
16. What are the conditions of finite and infinite motion?
17. Write down an equation of motion in the central field
18. What is an eccentricity and parameter?
19. What are the conditions of elliptical, parabolic and hyperbolic motion with depend on complete energy?
20. How can you define eccentricities of elliptical, parabolic and hyperbolic paths?
21. What is a perihelion? What is an aphelion?
22. How is defined the period in elliptical path?

3.2 Problems Solution

Problem 26.

Determine the period of oscillations of a simple pendulum (a particle of mass m suspended by a string of length l in a gravitational field, see figure 15) as a function of the amplitude of the oscillations.

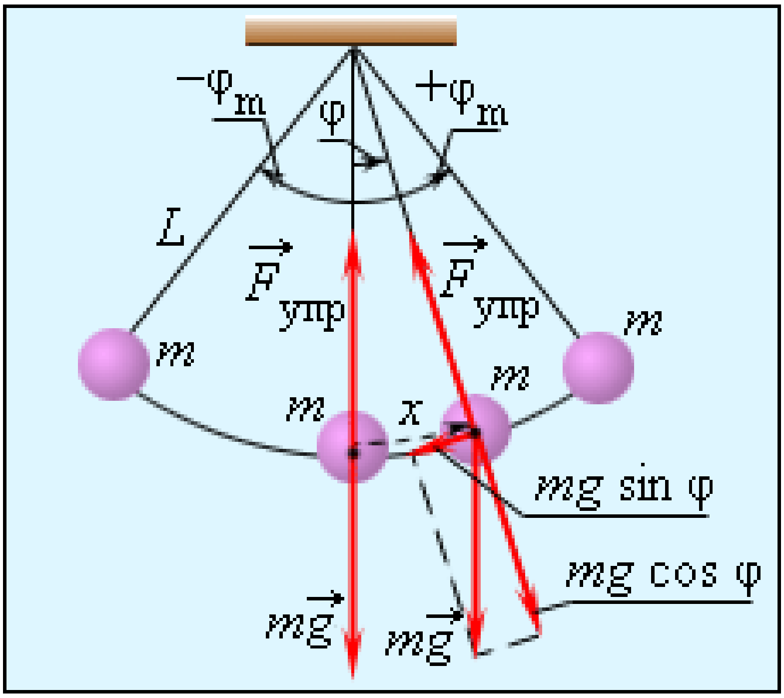

Figure 15 – Simple pendulum

Solution

Energy of the pendulum in any moment of time is define by equation

where – the angle between the string of pendulum and the vertical in any moment of time.

As the limits of motion we take the initial position of the pendulum and the maximum deflection angle . In the extreme position, the complete energy of the pendulum is equal to its potential energy:

where – the maximum value, i.e. is the amplitude. Then for the complete energy we can write:

We define from this equation

In chosen limits of the motion (from 0 to) the period will be equal to the quadruple time of passing the interval of angles from zero to , considering this, we can find:

We transform the radicand expression, expressing the cosines through the sines of the half argument by the formula

Substitute obtained difference to the integral

To transform this integral we use the substitution .

We substitute into the original integral, changing the old limits of integration to new:

We write this integral in the form

where

- the so-called complete elliptic integral of the first kind. It can be represented in the form of a power series

which is equivalent

to

where means double factorial.

In our case . We write the first three terms of the series and substitute in the formula for the period:

This formula solves the task. However, an important case is the case of small oscillations with . In this case the expansion of the function gives:

The first term of this expansion coincides with the known elementary formula for small oscillations of a mathematical pendulum known from the secondary school.

3.3 Problems for independent work

1. Integrate the equation of motion , if (k, a>0).
2. The charge e <0 at the initial instant of time was at a distance h from the infinite conducting plane. Determine the time for which the charge will reach the plane.
3. The gun is mounted on a hill of height h (see figure 16). The initial velocity of the projectile is directed at an angle to the horizon. Determine at what value of the angle the range of the flight of the projectile is maximum (air resistance is neglected).

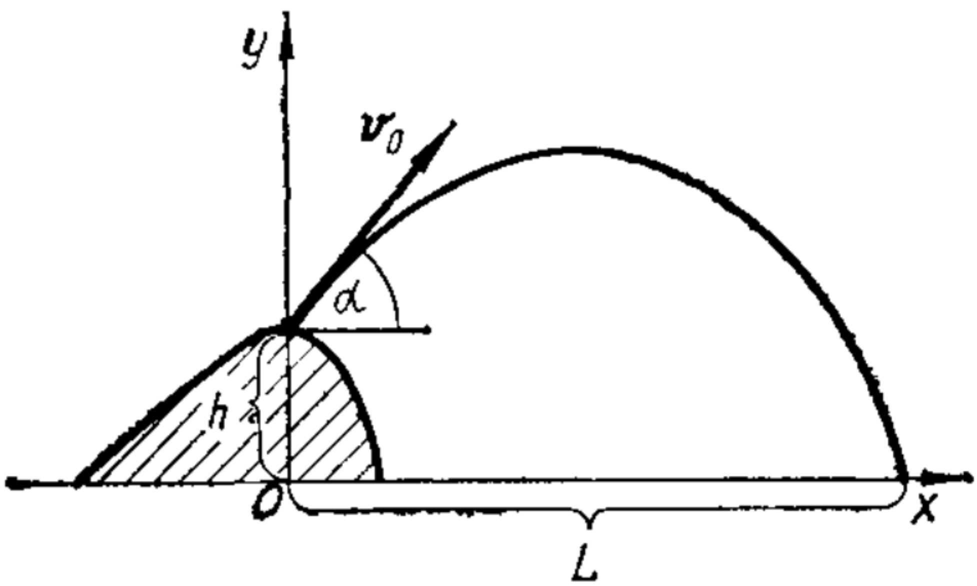

Figure 16 – To the problem 3

4. Electric charge e is moving in the electric field – . In initial moment of the time . Find the law of the motion.
5. The electron is moving in the magnetic field with a strength . Find the law of the law of the motion and the path path electron if .
6. A particle is in the plane Oxy under the force . Find the path of a particle.

3.4 Tests

What is called a motion of a system with one degree of freedom?

A) Cyclic

B) Finite

C) Central

D) One dimension

E) Infinite

****

What form does a Lagrange’s function have for one dimension motion of a system in constant field?

A)

B)

C)

D)

E)

****

What is a time defined in one dimension motion?

A)

B)

C)

D)

E)

****

If the region of motion of a material point is limited to two turning points, then such a motion is called:
A) Central
B) Finite
C) Symmetric
D) Additive
E) Cyclic
****
If the area of motion of a material point is not limited or limited to one turning point on one side, then the motion is called:
A) Symmetric
B) Additive
C) Cyclic
D) Central
E) Infinite
****
What are points called where potential energy is equal to complete?
A) Moving
B) Remission
C) Rest
D) Dissipation
E) Turning
****
What is period of a system in one dimension?
A)
B)
C)
D)
E)
****
What is a force field called if the potential energy of a particle depends only on a distance for the centre?
A) Central
B) Symmetric
C) Cyclic

D) Finite

E) Dissipative

****

What does energy of a system equal in the turning points?

A) Kinetic energy

B) Double momentum

C) Half complete energy

D) Potential energy

E) Zero

****

What does potential energy equal on the potential well?

A) Half complete energy

B) Zero

C) Kinetic energy

D) Double kinetic energy

E) Complete energy

****

What is the name of the generalized coordinate, which does not appear in the Lagrange's function explicitly?

A) Reduce

B) One dimension

C) Sectorial

D) Dissipation

E) Cyclic

****

What is the quantity called?

A) Potential mass

B) Reduce mass

C) Generalized mass

D) One dimension mass

E) Parametric mass

****

Choose an expression for the sectorial velocity?

A)

B)

C)

D)

E) –

****

The statement that for equal time intervals the radius vector of a moving point describes equal areas is called:

A) Newton's law

B) Hamilton's principle

C) Euler' theorem

D) Moupertui's rule

E) Kepler's second law

****

What quantity is called centrifugal energy?

A) $\frac{M^2}{2mr^2}$

B)

C) $\frac{m_1 m_2}{m_1 + m_2}$

D) $\frac{\sum m_a r_a}{\sum m_a}$

E) ——

****

What energy is defined by equation: ——

A) Complete

B) Kinetic

C) Central

D) Additive

E) Centrifugal

****

Choose a condition for the infinite motion of a particle:

A)

B)

C)

D)

E)

****

Choose a condition for the finite motion of a particle:

A)

B)

C)

D)

E)

****

Choose an equation of the conic section:

A) – ——

B) –

C) – –

D) – —

E) – — —

****

What is called the point nearest to the origin of a trajectory?

A) Perihelion

B) Centre

C) Aphelion

D) Remission

E) Oscillator

****

What is called the point farthest to the origin of a trajectory?

A) Remission

B) Oscillator

C) Aphelion

D) Perihelion

E) Centre

****

What is *e* in the equation: – ?

A) Latus rectum

B) Angle

C) Eccentricity

D) Energy

E) Semi-axis

****

What is a condition for elliptical motion of a material point:

A)

B)

C)

D)

E)

****

What is a condition for hyperbolic motion of a material point:

A)

B)

C)

D)

E)

****

What is a condition for parabolic motion of a material point:

A)

B)

C)

D)

E)

****

What is the trajectory of a point if its complete energy is less than zero?

A) Straight line

B) Parabola

C) Cycloid

D) Hyperbola

E) Ellipse

****

What is the trajectory of a point if its complete energy is equal to zero?

A) Hyperbola

B) Parabola

C) Cycloid

D) Ellipse

E) Straight line

****

What is the trajectory of a point if its complete energy is more than zero?

A) Cycloid

B) Parabola

C) Straight line

D) Hyperbola

E) Ellipse

****

What is an elliptical condition of a motion:

A)

B)

C)

D)

E)

****

What is a hyperbolic condition of a motion:

A)

B)

C)

D)

E)

****

What is a parabolic condition of a motion:

A)

B)

C)

D)

E)

****

What is trajectory of a material point if its eccentricity less that 1?

A) Straight line

B) Parabola

C) Cycloid

D) Hyperbolic

E) Ellipse

****

What is trajectory of a material point if its eccentricity is equal to 1?

A) Hyperbolic

B) Parabola

C) Cycloid

D) Ellipse

E) Straight line

****

What is trajectory of a material point if its eccentricity is more than 1?

A) Cycloid

B) Parabola

C) Straight line

D) Hyperbola

E) Ellipse

****

What is a period of a particle in the ellipse orbit?

A) $T = \sqrt{1 + \frac{2EM^2}{m\alpha^2}}$

B) $T = \pi a \sqrt{\frac{m}{2|E|^3}}$

C) $T = \frac{p_0^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$

D) $T = \frac{\rho(\chi)}{\sin\chi}\left|\frac{d\rho}{d\chi}\right| do$

E)

****

What is an eccentricity of a particle in the ellipse orbit?

A) $e = \sqrt{1 + \frac{2EM^2}{m\alpha^2}}$

B) $e = \frac{m_1 m_2}{m_1 + m_2}$

C) $e = \pi a \sqrt{\frac{m}{2|E|^3}}$

D) $e = \frac{p_0^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$

E) $e = \frac{\rho(\chi)}{\sin\chi}\left|\frac{d\rho}{d\chi}\right| do$

****

What is an expression for the *latus rectum* of the orbit?

A)

B)

C)

D)

E) —

****

How is defined major semi-axis of the elliptical orbit?

A) —

B) —

C) —

D) —

E) —

****

How is defined minor semi-axis of the elliptical orbit?

A) —

B) —

C) —

D) —

E) —

****

How is defined major semi-axis of the elliptical orbit in the field −?

A) —

B) —

C) —

D) —

E) —

****

How is defined minor semi-axis of the elliptical orbit in the field −?

A) —

B) —

C) —

D) —

E)

****

There are two moving material points of a masses 2 g and 8 g in the spaceг. Find reduce mass of a system.

A) 1.6

B) 4.3

C) 0.625

D) 17

E) 0.3

****

Chapter 4 Collisions between particles

4.1 Verification questions

1. What is a spontaneous disintegration?
2. What is the energy of disintegration?
3. In what case is disintegration possible?
4. What are the directions of momenta after the disintegration?
5. What are the velocities of the disintegration particles in the center of mass system?
6. Which collision is called elastic?
7. What can be said about the momenta of particles after their collision?
8. What is scattering of the particle?
9. What is the impact parameter?
10. How is expressed energy and momentum through velocity at infinity and the impact distance?
11. What is the effective scattering cross-section? How is it determined?
12. What is the effective scattering in the case of the Coulomb field? What is the name of this formula? For which frame of reference is it valid?
13. Write down the formula for the effective cross-section as a function of energy loss.
14. What is condition of small angles of deflection?
15. What is the effective cross-section for scattering in an n-system at small angles?

4.2 Problems Solution

Problem 34.

Find the relation between angles (in the L system) after a disintegration into two particles.

Solution

In the C system, the corresponding angles are related by . Calling simply , for each of the two particles we can put:

From these two equations we must eliminate To do so, we first solve for , and then form the sum of their squares , which is unity. Since , we have finally next equation:

Problem 35.
Find the angular distribution of the resulting particles in the L system.
Solution

When we substitute — — , with the plus sign of the radical, in – , obtaining

When , both possible relations between . Must be taken into account. Since, when increases, one of increases and the other decreases, the difference (not the sum) of the expressions with the signs of the radical — — must be taken. The result is:

4.3 Tests

Disintegration of a particle into two "constituent parts", i.e. into other particles which move independently after the disintegration is called:
A) Free
B) Spontaneous
C) Coherent
D) Small
E) Forced
****
What does energy of disintegration equal?
A)

B) —

C) $\varepsilon = \sqrt{1 + \frac{2EM^2}{m\alpha^2}}$

D) $\varepsilon = \frac{p_0^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$

E) –

****

What is a condition for the disintegration?

A)

B)

C)

D)

E)

****

A collision between two particles is said to be … if it involves no change in their internal state.

A) Adiabatic

B) Non-elastic

C) Inertial

D) Finite

E) Elastic

****

What is called a deviation of a particle in the field of force centre?

A) Remission

B) Scattering

C) Dissipation

D) Inertia

E) Collision

****

The ratio the number of particles scattered through different angles between and χ+dχ to the number of particles passing in unit area of the beam cross-section is called:

A) Effective scattering cross-section

B) Non-elastic process

C) Rutherford’s deviation

D) Small variation

E) One dimension oscillation

****

Specify the Rutherford's formula:

A)

B)

C) $d\sigma = \frac{\mu V^2}{2} + \frac{1}{2}\sum m \left(\Omega^2 r^2 - (\Omega \vec{r})^2\right)$

D) $$d\sigma = \left(\frac{\alpha}{2m\upsilon_\infty^2}\right)^2 \frac{do}{\sin^4 \frac{\chi}{2}}$$

E) $d\sigma = ae^{-\lambda t}\cos(\omega t + \alpha) + b\cos(\Omega t + \delta)$

****

How is the effective scattering cross-section determined?

A)

B) $d\sigma = \cos(\omega t + \alpha)$

C) $d\sigma = \frac{dN}{n}$

D) $d\sigma = \sqrt{1 + \frac{2EM^2}{m\alpha^2}}$

E)

****

Specify the Rutherford's formula for the effective cross section of the scattered particles:

A)

B) $$d\sigma = \left(\frac{\alpha}{2m\upsilon_\infty^2}\right)^2 \frac{do}{\sin^4 \frac{\chi}{2}}$$

C) $d\sigma = \frac{\mu V^2}{2} + \frac{1}{2} I_{uk}\Omega_i \Omega_k$

D)

E)

****

What form does the formula for the effective cross section have as a function of energy loss?

A)

B) $d\sigma = \sqrt{1 + \frac{2EM^2}{m\alpha^2}}$

C)

D) $d\sigma = 2\pi \frac{\alpha^2}{m_2 \upsilon_\infty^2} \frac{d\varepsilon}{\varepsilon^2}$

E) $d\sigma = \frac{\mu V^2}{2} + \frac{1}{2} \sum m \left(\Omega^2 r^2 - (\Omega \vec{r})^2 \right)$

****

Chapter 5 Small oscillations

5.1 Verification questions

1. What kind of oscillations is called “small”? “Free”?
2. What is a one-dimensional oscillator?
3. What is the Lagrange function for a one-dimensional oscillator?
4. Write down differential equation of motion of a one-dimensional oscillator and its solution.
5. What is equal the frequency of its oscillation? What does it depend on? Does it depend on initial conditions?
6. What is the amplitude? Phase? What does depend the initial value of the phase on?
7. What can we say about the potential energy of the oscillator if it is in a stable equilibrium state? What is equal the energy of the system that makes small oscillations?
8. What are forced oscillations? Is the Lagrange function for a system performing forced oscillations? How does it differ from the Lagrange function of a one-dimensional oscillator?
9. How find you the Lagrange function for a system with many degrees of freedom? Equations of motion?
10. What coordinates is called normal? Write down Lagrange function for a system with many degrees of freedom in normal coordinates
11. What degrees of freedom has the molecule? How many vibrational degrees of freedom has the molecule in general case? In the case when all atoms are located on the same line?
12. What is dissipation? What is damped oscillation? How can you write down the generalized friction force acting on a system that performs one-dimensional small oscillations?
13. Write down the equation of motion of damped oscillations. What is equal to damping coefficient? Write down solution of the damped equation: three cases.
14. What does equal the frequency of damped oscillations? According to what law does the energy of the system performing attenuating oscillations decrease on average?
15. What are the generalized frictional forces for a system with many degrees of freedom? What is a dissipative function? Loss of energy through the dissipative function.
16. The equation of motion of forced oscillations in the presence of frictional forces and its solution. At what frequency is the amplitude of vibration maximum?
17. What can we say about the motion of a particle in a fast oscillating field? How does occur the oscillation-averaged motion of a particle in a fast oscillating field?

Problem 45.

Express the amplitude and initial phase of the oscillations in terms of the initial co-ordinate and velocity .

Solution:

Problem 46.

Find the ration of frequencies of the oscillations of two diatomic molecules consisting of atoms of different isotopes, the masses of the atoms being

Solution

Since the atoms of isotopes interact in the same way, we have . The coefficients in the kinetic energies if the molecules are their reduced masses. We therefore have:

Problem 47.

Find the frequency of oscillations of a particle of mass which is free to move along a line and is attached to a spring whose other end is fixed at point A (see figure 19) at a distance from the line. A force is required to extend the spring to length .

Solution

The potential energy of the spring is (to within higher-order terms) equal to the force multipied by the estension of the spring. For we have:

So that . Since the kinetic energy is , we have

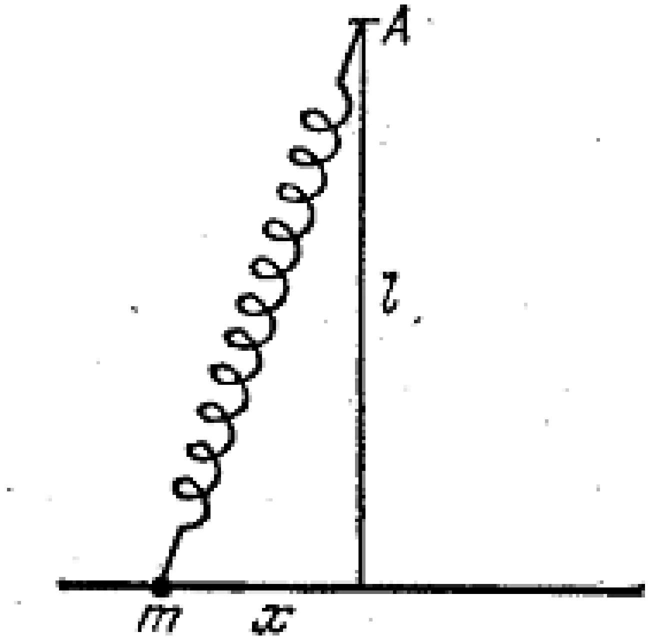

Figure 19 – To the problem 47

Problem 48.

Find the frequency of oscillations of a particle of mass moving on a circle of radius , and is attached to a spring whose other end is fixed at point A (see figure 20) at a distance from the line. A force is required to extend the spring to length .

Solution

In this case the extension of the spring is (if)

The kinetic energy . And the frequency is therefore

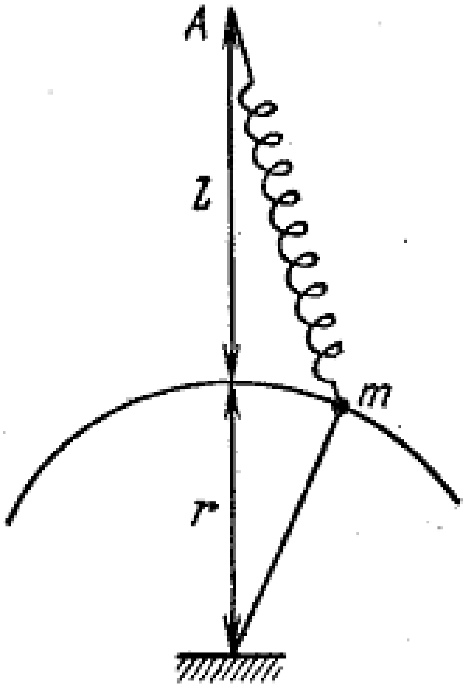

Figure 20 – To problem 48

Problem 49.

Find the frequency of oscillations of the pendulum shown in Fig. 2, whose point of support carries a mass and is free to move horizontally.

Solution

For we get:

Hence

Problem 50.

Determine the form of curve such that frequency of oscillations of a particle on it under the force of gravity is independent of the amplitude.

Solution

The curve satisfying the given condition is one for which the potential energy of a particle moving on it is , where is the length of the arc from the position of equilibrium. The kinetic energy where is the mass of the particle, and the frequency is then whatever the initial value of .

In a gravitational field , where is the vertical co-ordinate. Hence we have or

But , , whence

The integration is conveniently effected of the substitution

Which yields:

This two equations give, in parametric form, the equation of the required curve, which is a cycloid.

5.3 Tests

Oscillations having one degree of freedom are called:
A) Free
B) Damped
C) Forced
D) Small
E) One-dimensional
****
How many degrees has a point performing one-dimensional oscillations?
A) 1
B) 2
C) 3
D) 4
E) 5
****
What is an oscillator?
A) This system is multidimensional
B) This is a system, movement along a circle or any closed path
C) This is a system that performs small one-dimensional oscillations
D) This is a system that performs infinite movements
E) This is a system whose movement is limited to two turning points
****
What form does the Lagrange function of a system performing small one-dimensional oscillations have?

A) $L = \frac{\mu V^2}{2} + \frac{1}{2} I_{uk} \Omega_i \Omega_k$

B)

C)

D) $L = \sum \frac{m_a \upsilon_a^2}{2}$

E)

****

A system that performs small one-dimensional oscillations is called:
A) One-dimensional oscillator
B) Maxwell's pendulum
C) Rotator
D) The vibrator
E) The top

****

What oscillation does the system make near the position of stable equilibrium?
A) Forced
B) Non-periodic
C) Anharmonic
D) Harmonic
E) Aperiodic

****

What is the equation of motion of a one-dimensional oscillator?

A)

B)

C)

D) $x = \cos(\omega t + \alpha)$

E)

****

Specify the solution of the equation of motion of the equation for a one-dimensional oscillator:

A)

B) $x_k = \sum_\alpha \Delta_{k\alpha} \Theta_\alpha$

C) $x = \sqrt{\omega_0^2 - \lambda^2}$

D) $x = ae^{-\lambda t} \cos(\omega t + \alpha) + b \cos(\omega t + \delta)$

E) $x = \cos(\omega t + \alpha)$

****

How is the cyclic frequency determined with free oscillations?

A) $\omega = \frac{2m^2}{m_2} \upsilon_\infty^2 \sin^2 \frac{\chi}{2}$

B) $d\omega = \frac{dN}{n}$

C) $\omega = \pi\, a \sqrt{\frac{m}{2|E|^3}}$

D) $\omega = \sqrt{\frac{k}{m}}$

E) $\omega = \sqrt{1 + \frac{2EM^2}{m\alpha^2}}$

****

Find the oscillation frequency of a spring pendulum (in Hz) of a mass of 2 kg, if the spring stiffness is 8 N/m.

A) —

B) –

C) –

D)

E)

****

What are the vibrations of a system called external forces that are not acting?

A) Free
B) Forced
C) Anharmonic
D) Singular
E) Damped

****

What is the energy of a system making small oscillations?

A) —

B) $E = \frac{m\upsilon_\infty^2}{2}$

C) — —

D) $\overline{E} = E_0 e^{-2\lambda t}$

E) —

****

How is the energy of a system making small oscillations determined?

A) $\overline{E} = E_0 e^{-2\lambda t}$

B) —

C) $E = \frac{m\upsilon_\infty^2}{2}$

D)

E) –

****

How is the energy of a one-dimensional oscillator expressed in terms of the amplitude?

A) $E = \frac{a\upsilon_\infty^2}{2}$

B) $E = \frac{m\omega^2 a^2}{2}$

C) $\overline{E} = E_0 e^{-2\lambda t}$

D) $E = ae^{i\alpha}$

E) $E = \mathrm{Re}\, A e^{i\omega t}$

****

Calculate the energy of a one-dimensional oscillator with a mass of 2 kg, the frequency of which – Hz and an amplitude of 0.1 m.

A) 16
B) 0.2
C) 0.16
D) 0.04
E) 1.6

****

What are the vibrations of a system called friction forces?

A) Free
B) Forced
C) Anharmonic
D) Singular
E) Damped

****

What form does the Lagrange function have for the system that makes forced oscillations?

A) — —

B) —

C) $L = \sum \frac{m_a \upsilon_a^2}{2}$

D)

E)

****

What are the names of oscillations in a system acting on some variable external field?

A) Damped

B) Forced

C) Free

D) Anharmonic

E) Not Free

****

What is the equation of motion of forced vibrations?

A)

B)

C)

D)

E)

****

Choose the solution of the equation of motion of the system performing forced oscillations under the action of a periodic force $F(t) = f\cos(\gamma t + \beta)$:

A) $x = ae^{-\lambda t}\cos(\omega t + \alpha) + b\cos(\gamma t + \delta)$

B) $x = a\cos(\omega t + \alpha) + \dfrac{f}{m(\omega^2 - \gamma^2)}\cos(\gamma t + \beta)$

C)

D)

E)

****

What form does the solution of the forced oscillation equation have in the case of resonance?

A) $x = a\cos(\omega t + \alpha) + \dfrac{f}{2m\omega}t\sin(\omega t + \beta)$

B)

C)

D)

E) $x = ae^{-\lambda t}\cos(\omega t + \alpha) + b\cos(\gamma t + \delta)$

****

How in the general case is the potential energy of a system with many degrees of freedom determined?

A) $U_k = \sum_{\alpha} \Delta_{k\alpha}\Theta_{\alpha}$

B) $U_{ik} = \sum m(x_l^2 \delta_{ik} - x_i x_k)$

C) $U = \frac{1}{2}\sum_{i,k} k_{ik} x_i x_k$

D) $U = \frac{\mu V^2}{2} + \frac{1}{2} I_{uk}\Omega_i\Omega_k$

E) $U_i = I_{ik}\Omega_k$

****

How in the general case is the kinetic energy of a system with many degrees of freedom determined?

A) $T = \frac{\mu V^2}{2} + \frac{1}{2} I_{uk}\Omega_i\Omega_k$

B) –

C) $T_k = \sum_{\alpha} \Delta_{k\alpha}\Theta_{\alpha}$

D) $T = \frac{1}{2}\sum_{i,k} k_{ik} x_i x_k$

E) $T_{ik} = \sum m(x_l^2 \delta_{ik} - x_i x_k)$

****

What is the form of the Lagrange function of a system with many degrees of freedom?

A) $L_{ik} = \sum m(x_l^2 \delta_{ik} - x_i x_k)$

B)

C) $L_k = \sum_{\alpha} \Delta_{k\alpha}\Theta_{\alpha}$

D) $L = \frac{\mu V^2}{2} + \frac{1}{2} I_{uk}\Omega_i\Omega_k$

E) –

****

Equations of motion of a system with many degrees of freedom:

A) –

B) —

C)

D) — –

E)

****

The solution of the equation of motion of a system with many degrees of freedom has the form:

A) $x_{ik} = \sum m(x_l^2\delta_{ik} - x_i x_k)$

B) $x_i = I_{ik}\Omega_k$

C)

D) —

E) $x_k = \sum_{\alpha} \Delta_{k\alpha}\Theta_{\alpha}$, where $\Theta_{\alpha} = \mathrm{Re}\,(C_{\alpha} e^{i\omega_{\alpha} t})$

****

The Lagrangian of a system with many degrees of freedom in normal coordinates

A) –

B) —

C) $L = \frac{\mu V^2}{2} + \frac{1}{2} I_{uk}\Omega_i\Omega_k$

D) $L_{ik} = \sum m(x_l^2\delta_{ik} - x_i x_k)$

E) — –

****

Which equation is satisfied by normal coordinates?

A)

B) $x_k = \sum_{\alpha} \Delta_{k\alpha}\Theta_{\alpha}$

C) $M_i = I_{ik}\Theta_k$

D) $\Theta_{\alpha} = \mathrm{Re}\,(C_{\alpha} e^{i\omega_{\alpha} t})$

E) $2\Theta = \frac{\alpha}{m}$

****

Which equation is called characteristic?

A) $M_i = I_{ik}\Omega_k$

B) $x_k = \sum_\alpha \Delta_{k\alpha}\Theta_\alpha$

C) $\Theta_\alpha = \mathrm{Re}\, C_\alpha e^{i\omega_\alpha t}$

D) $\left|k_{ik} - \omega^2 m_{ik}\right| = 0$

E)

****

What is the number of vibrational degrees of a free n-atom molecule in the general case?

A) 3n-6

B) 2n-5

C) 3n+4

D) n-1

E) 4n+2

****

What is the number of vibrational degrees free of an n-atom molecule, all of whose atoms are located along one axis?

A) 3n-6

B) 3n+4

C) 3n-5

D) n-1

E) 4n+2

****

What is the number of vibrational degrees free of an n-atom linear molecule?

A) 3n-6

B) 3n+4

C) 3n-5

D) n-1

E) 4n+2

****

Choose the equation of motion of the system performing damped oscillations:

A)

B)

C)

D)

E)

****

How is the attenuation factor be determined?

A) $2\lambda = \dfrac{\alpha}{m}$

B)

C) $\omega = \sqrt{\omega_0^2 - \lambda^2}$

D) $\overline{E} = E_0 e^{-2\lambda t}$

E) $\Theta_\alpha = \mathrm{Re}\,(C_\alpha e^{i\omega_\alpha t})$

****

What form does the <u>general solution</u> of the equation of motion of a system performing damped oscillations have?

A) $x = ae^{-\lambda t}\cos(\omega t + \alpha) + b\cos(\,t + \delta)$

B) $x = ae^{-\lambda t}\cos(\omega t + \alpha)$

C) $x = c_1 e^{r_1 t} + c_2 e^{r_2 t}$, $r_{1,2} = -\lambda \pm \sqrt{\lambda^2 - \omega_0^2}$

D) $x_k = \sum_\alpha \Delta_{k\alpha}\Theta_\alpha$

E) $x = \cos(\omega t + \alpha)$

****

Indicate the solution of the equation of motion of a system that performs damped oscillations at $\lambda < \omega_0$:

A) $x = ae^{-\lambda t}\cos(\omega t + \alpha) + b\cos(\,t + \delta)$

B) $x = \cos(\omega t + \alpha)$

C) $x = (c_1 + c_2)e^{-\lambda t}$

D) $x_k = \sum_\alpha \Delta_{k\alpha}\Theta_\alpha$

E) $x = ae^{-\lambda t}\cos(\omega t + \alpha)$

****

Indicate the solution of the equation of motion of a system that performs damped oscillations at $\lambda = \omega_0$:

A) $x = ae^{-\lambda t}\cos(\omega t + \alpha) + b\cos(\,t + \delta)$

B) $x = (c_1 + c_2)e^{-\lambda t}$

C) $x = \cos(\omega t - \alpha)$

D) $x_k = \sum_\alpha \Delta_{k\alpha}\Theta_\alpha$

E) $x = \cos(\omega t + \alpha)$

****

What is the frequency for damped oscillations?

A) $\omega = I\Omega$

B) $\omega = \overline{E} + E_0 e^{-2\lambda t}$

C) $2\omega = \dfrac{\alpha}{m}$

D) $\omega = \sqrt{\omega_0^2 - \lambda^2}$

E) $\omega = -\Theta_\alpha + \mathrm{Re}\,(\xi_\alpha e^{i\omega_\alpha t})$

****

By what law does the energy of the system, which performs damped oscillations, decrease?

A) — –

B) $\overline{E} = ae^{-\lambda t}\cos(\omega t + \alpha)$

C) $\dfrac{\partial E}{\partial q_i} = p_i$

D) $\overline{E} = E_0 e^{-2\lambda t}$

E) $\int dE = const$

****

Choose the dissipative function:

A) –

B) —

C) $F = \dfrac{dE}{dt} - 2F'$

D)

E) $F = \sum_k (m_{ik} r^2 + \alpha_{ik} r + k_{ik}) A_k$

****

How are friction forces expressed through a dissipative function?

A) $\sum_k (m_{ik} r^2 + \alpha_{ik} r + k_{ik}) F_k = 0$

B) —

C) –

D) $\overline{E} = F_0 e^{-2\lambda t}$

E) $\frac{dE}{dt} = -2F$

****

How is the energy loss recorded for damped oscillations through a dissipative function

A) —

B) $\sum_k (m_{ik} r^2 + \alpha_{ik} r + k_{ik}) E_k = 0$

C) $\frac{dE}{dt} = -2F$

D)

E) –

****

What is the form of the equations of motion of small oscillations in the presence of frictional forces?

A) $\frac{dE}{dt} = -2F$

B) –

C) —

D)

E) –

****

Solutions of the equation of motion of a system with many degrees of freedom in the presence of thorns:

A)

B) $\sum_k (m_{ik} r^2 + \alpha_{ik} r + k_{ik}) A_k = 0$

C) –

D) –

E) $x = \cos(\omega t + \alpha)$

****

Equations of motion of forced oscillations in the presence of frictional forces have the form:

A) –

B) –

C)

D)

E)

****

What is the solution of the equation of motion of forced oscillations in the presence of frictional forces?

A) –

B) $x = ae^{-\lambda t}\cos(\omega t + \alpha) + b\cos(\gamma t + \delta)$

C)

D)

E) $x = \cos(\omega t + \alpha)$

****

Chapter 6 Motion of a rigid body

6.1 Verification questions

1. What is a solid body? Are there absolutely solid bodies in nature?
2. What coordinate systems are used to describe the position of a solid body in space?
3. How many coordinates do you need to know to determine the position of a solid body?
4. How is related the speed of any point of a solid body relative to a fixed coordinate system to its translational speed and its angular rotation speed?
5. What is the expression of the kinetic energy of a solid body?
6. Wright down Lagrange function of a solid body.
7. What is the tensor of inertia?
8. Give the definitions the following concepts: the main axes of inertia, the main moments of inertia, the asymmetric top, the symmetric top, the ball top, the rotator.
9. What does equal the angular momentum of a solid body relative to the center of inertia?
10. Is the same direction of angular momentum and angular velocity?
11. What is a regular precession?
12. What does equal the angular velocity of precession?
13. Equations of motion of a solid body in a fixed coordinate system.
14. What is called a torque?
15. How does change the torque when we transferring the origin of coordinates?
16. When the magnitude of the torque does not depend on the choice of the origin of coordinates?
17. How are determined Euler’s angles?
18. How are expressed the components of angular velocity around moving axes through the Euler’s angles?
19. Euler’s equations. Euler's equations at free rotation.
20. Solid state equilibrium conditions.
21. What is a perfectly smooth surface? Absolutely rough surface?
22. What is the Lagrange function and the equation of motion in a non-inertial reference frame?

7.2 Problems Solution

Problem 56.

Determine the principal moments of inertia for the molecule, regarded as system of particles at fixed distances apart: a molecule of collinear atoms.

Solution:

where is the mass of the ath atom, the distance between the ath atoms, and the summation includes one term for every pair of atoms in the molecule.

For a diatomic molecule there is only one term in the sum, and the result is obvious: it is the product of the reduced mass of the two atoms and the square of the distance between them:

Problem 57.

Determine the principal moments of inertia for the molecule, regarded as system of particles at fixed distances apart: a triatomic molecule which is an isosceles triangle (see figure 37).

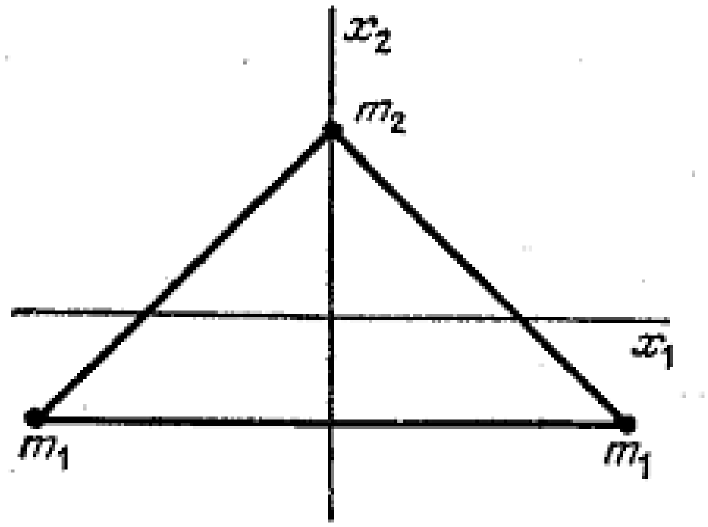

Figure 24 – Model of diatomic molecule

Solution: The centre of mass is on the axis of symmetry of the triangle, at a distance from its base (h being the height of the triangle). The moments of inertia are:

Problem 58.

Determine the principal moments of inertia for the molecule, regarded as system of particles at fixed distances apart: a tatratomic molecule which is an equailateral-based tetrahedron (Figure 37).

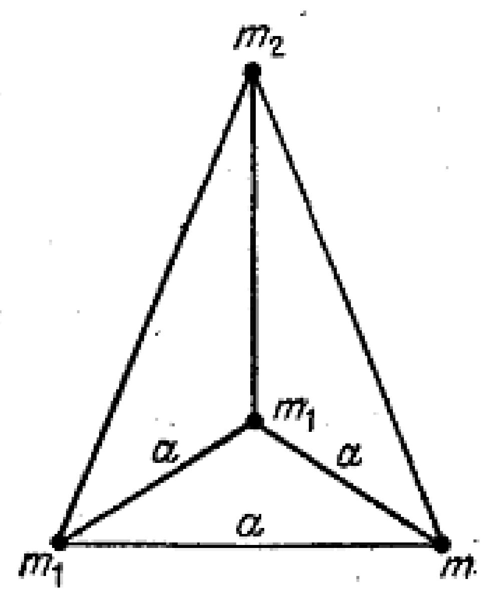

Figure 25 – Model of tetratomic molecule

Solution: The centre of mass is on the axis of symmetry of the tetrahedron, at a distance from its base(h being the height of the tetrahedron). Moments of inertia are:

If , the molecule is a regular tetrahedron and

Problem 59.

Determine the principle moments of inertia for an circular cone of height and base radius .

Solution

We first calculate the tensor with respect to axes whose origin is at the vertex of the cone (Figure 59). The calculation is simple if cylindrical co-ordinates are used, and the result is:

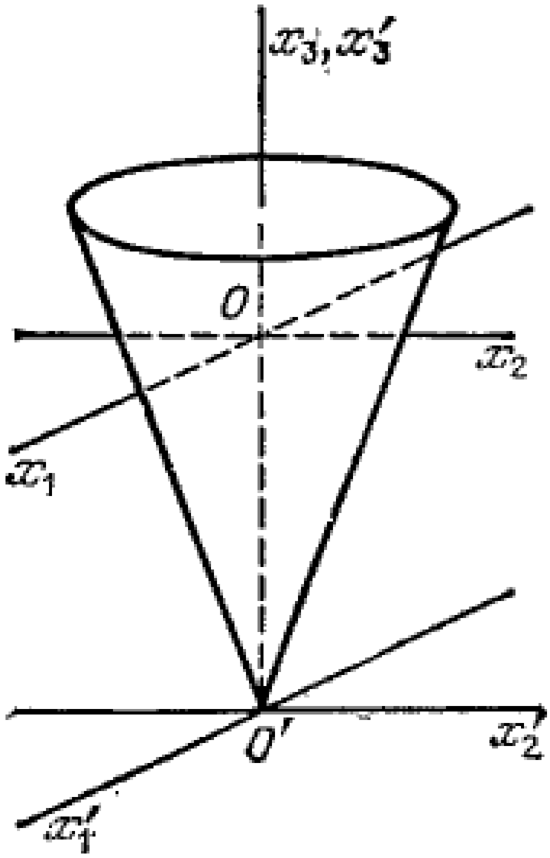

Figure 26 – To the problem 59

The centre of mass is easily shown to be on the axis of the cone and a distance from the vertex. Find finally

— — —

Problem 60.

Determine the principle moments of inertia for an ellipsoid of semiaxis

Solution

The centre of mass is at the centre of the ellipsoid, and the principal axes of inertia are along the axes of the ellipsoid. The integration over the volume of the ellipsoid can be reduced to one over a sphere by the transformation which converts the equation of the surface of the ellipsoid

— — —

into that of the unit sphere

For example, the moment of inertia about the -axis is:

–

where is the moment of a sphere of nit radius. Since the volume of the ellipsoid is , we find the moments of inertia

– – –

Problem 61.

Determine the motion of a top when the kinetic energy of its rotation about its axis is large compared with its energy in the gravitational field (called “fast” top, ee figure 27).

Solution

In a first approximation, neglecting gravity, there is a free precession of the top about the direction of the angular momentum , corresponding in this case to the nutation of the top; the angular velocity of this precession is

–

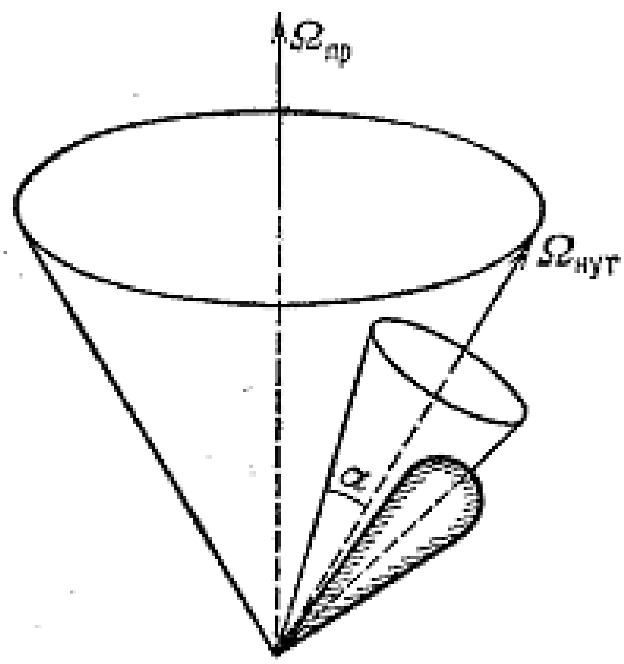

Рис. 50

Figure 27 – A top

In the next approximation, there is a slow precession of the angular momentum about the vertical (fig.50). To determine the rate of this precession, we average the exact equation of motion

–

over the nutation period. The moment of the force of gravity on the top , where is a unit vector along the axis of the top. It is evident from symmetry that the result of averaging over the "nutation cone" is to replace by its component in the direction of (where is the angle between and the axis of the top). Thus we have

This shown that the vector precesses about the direction of (i.e. the vertical) with a mean angular velocity

(which is small compared with).

7.3 Tests

How is the speed of a solid body determined with respect to a fixed frame of reference?

A)

B)

C)

D)

E)

****

What is the kinetic energy of a solid body?

A)

B) $T = \frac{m\upsilon^2}{2} - \frac{m}{2}[\Omega \vec{r}]^2 + U$

C)

D) $T = \lambda m b^2 \gamma^2 \sin^2 (t + \delta)$

E) $\mathrm{T} = I_{ik} + \mu (a_l^2 \delta_{ik} - a_i a_k)$

****

Specify the inertia tensor:

A) $I_{ik} = \frac{\mu V^2}{2} + \frac{1}{2}\sum m (\Omega^2 r^2 - (\Omega \vec{r})^2)$

B)

C) $I_{ik} = \sum m (x_l^2 \delta_{ik} - x_i x_k)$

D) $I'_{ik} = I_{ik} + \mu (a_l^2 \delta_{ik} - a_i a_k)$

E) $M_i = I_{ik} \Omega_k$

****

How will the expression for the kinetic energy of a solid be written in terms of the inertia tensor?

A)

B) $T = ae^{-\lambda t}\cos(\omega t + \alpha) + b\cos(\lambda t + \delta)$

C) $T'_{ik} = I_{ik} + \mu(a_l^2\delta_{ik} - a_i a_k)$

D) $T = \frac{\mu V^2}{2} + \frac{1}{2} I_{uk}\Omega_i\Omega_k$

E)

****

Choose the Lagrange function for a solid:

A)

B)

C) $L = \lambda m b^2 \gamma^2$

D) $L = \frac{f^2}{4m}\frac{\lambda}{\varepsilon^2 + \lambda^2}$

E) $L = \frac{\mu V^2}{2} + \frac{1}{2} I_{uk}\Omega_i\Omega_k - U$

****

Which expression indicates the symmetry of the inertia tensor?

A)

B)

C)

D)

E)

****

What is the name of the body, in which all three moments of inertia are different?

A) Symmetric top

B) Moving top

C) Spinning top

D) Asymmetrical top

E) Elliptical top

****

What is the name of a body whose two moments of inertia are the same?

A) Symmetric top
B) Spinning top
C) Asymmetrical top
D) Elliptical top
E) Moving top
****
What is the name of the body, in which all three moments of inertia coincide?
A) Symmetric top
B) Moving top
C) Spinning top
D) Asymmetrical top
E) Elliptical top
****
What is the name of a system whose two main moments of inertia coincide, and the third is equal to zero?
A) Spinning top
B) Rotator
C) Closed system
D) Symmetrical top
E) Gyroscope
****
The moment of inertia of a solid body with respect to the center of inertia is determined by the expression:
A) $M_i = I_{ik}\Omega_k$
B) $M_i = \mathrm{Re}\, C_\alpha e^{i\omega_\alpha t}$
C) $M_i = (c_1 + c_2)e^{-\lambda t}$
D) $M_i = E_0 e^{-2\lambda t}$
E) —
****
How is the moment of inertia of a solid body determined in the case of a spherical top?
A)
B)
C)
D) —
E)

****

How does the inertia tensor transform when changing to another origin?

A) $I'_{ik} = 2\pi \frac{\alpha^2}{m_2 \upsilon_\infty^2} \frac{d\varepsilon}{\varepsilon^2}$

B) $I'_{ik} = I_{ik} + \mu (a_l^2 \delta_{ik} - a_i a_k)$

C)

D) $I'_{ik} = (c_1 + c_2) e^{-\lambda t}$

E) —

****

What is the angular velocity of precession?

A) $\Omega = \frac{f^2}{4m} \frac{\lambda}{\varepsilon^2 + \lambda^2}$

B) $\Omega = 2\pi \frac{\alpha^2}{m_2 \upsilon_\infty^2} \frac{d\varepsilon}{\varepsilon^2}$

C) –

D) $\Omega = \frac{M}{I_1}$

E) $\Omega_\alpha = \mathrm{Re}\{C_\alpha e^{i\omega_\alpha t}\}$

****

What is the uniform rotation of the axis of the top about the direction of the angular momentum vector?

A) Oscillations

B) Inertia

C) Oscillation

D) Precession

E) Rotation

****

What surface describes the axis of the top, as a result of precession?

A) Ellipse

B) The cone

C) Ball

D) Hyperbola

E) Direct

****

The direction of which vector is preceded by the precession of the top?

A) Moment of impulse

B) Speed
C) Angular velocity
D) Angular acceleration
E) Radius vector
****
How many independent equations does the general system of equations of motion of a rigid body contain?
A) 2
B) 3
C) 5
D) 6
E) 9
****
Specify the formula for converting the moment of force when moving from one origin to another:

A)

B)

C)

D)

E)

****
What are the angles used to describe the position of a rigid body in space?
A) Newton's
B) Euler's
C) Lagrangian's
D) Jacobi's
E) Liouville's
****
Indicate the equations of motion of a solid body:

A)

B) $\overline{E} = E_0 e^{-2\lambda t}$

C)

D)

E)

****

What are the forces applied at the points of contact of bodies?

A) Active

B) Stationary

C) Reactions

D) Passive

E) Holonomic

****

How many types of motion of contiguous bodies are possible?

A) 1

B) 2

C) 3

D) 4

E) 5

****

Choose an equilibrium conditions of solid body?

A)

B) — —

C) $I'_{ik} = I_{ik} + \mu \left(a_l^2 \delta_{ik} - a_i a_k \right)$

D) $\Theta_\alpha = \mathrm{Re}\left\{ C_\alpha e^{i\omega_\alpha t} \right\}$

E)

****

How is the Coriolis force determined?

A)

B)

C)

D)

E) —

****

What is the centrifugal force:

A)

B)

C)

D)

E) —

****

Which form has the Lagrange function of the system in the case of a uniformly rotating coordinate system that does not have translational motion?

A)

B) — —

C)

D)

E)

****

How will the equations of motion of the system be written in the case of a uniformly rotating coordinate system that does not have translational motion?

A) $m\frac{d\bar{v}}{dt} = 2\pi \frac{\alpha^2}{m_2 \upsilon_\infty^2} \frac{d\varepsilon}{\varepsilon^2}$

B) — —

C) —

D) — — —

E) — —

****

What is the momentum of the system in the case of a uniformly rotating coordinate system that does not have translational motion

A) $\frac{\partial S}{\partial q_i} = p_i$

B)

C) $\frac{\partial S}{\partial t} = -H + \vec{p}$

D) $\int d\Gamma = const$

E) $\vec{p}_0 = \int \sum_i p_i dq_i$

****

How is centrifugal energy determined?

A) —

B)

C) —

D)

E)
****
How does the energy transform in the transition to a uniformly rotating frame of reference?

A)

B)

C)

D)

E)
****
What is the energy of a particle in the case of a uniformly rotating coordinate system that does not have translational motion?

A)

B) $E = 0$

C)

D) $E = \frac{m\upsilon^2}{2} + U$

E)
****

Chapter 7 The canonical equations

7.1 Verification questions

1. What variables are used to formulate the laws of mechanics in the Lagrangian’s method?
2. What are the Legendra’s transformations?
3. What form has Hamilton's function?
4. What are called equations of motion in variables p and q?
5. What law do we get if the Hamilton’s function is independent explicitly from time?
6. What is the relationship between the partial derivatives of time from the Lagrangian and Hamilton functions?
7. What are Poisson's brackets?
8. Conditions for function to be an integral of motion.
9. Properties of Poisson's brackets.
10. Jacoby and Poisson's theorem.
11. What does equal the partial derivatives of the action on coordinates?
12. What does equal the partial derivative of action on time?
13. What is the complete differential of action as a function of coordinates and time?
14. What is the expression for a shortened action?
15. Which transformations are called canonical?
16. What conditions must the new and old coordinates satisfy in order for the transformation to be canonical? How this condition can you right down using Poisson's brackets?
17. What is phase space?
18. What is the phase trajectory?
19. Formulate Liouville’s theorem.
20. What is the Hamilton-Jacoby’s equation?
21. What is called adiabatic change?
22. What is the adiabatic invariant?
23. An expression for an adiabatic invariant.
24. How is defined a particular derivative of the adiabatic invariant whith respect to energy?

7.2 Problems Solution

Problem 62.

Find the Hamiltonian for a single particle in Cartesian coordinates.

Solution

Lagrangian for a free particle in Cartesian coordinates have a form (see problem 8, where we found already Lagrangian for this particle)

—

The Hamiltonian is defined by expression:

где —. Write down this for Cartesian coordinates

We need define next quantities

Then

The vector of momentum is defined by equation or, in projections on the axis of Cartesian system, . Then

Substitute these expressions into the obtained Hamiltonian we will have:

Or, finally:

Problem 63.

Find Hamilton's function for one material point in cylindrical coordinates.

Solution

The Lagrange function of a material point in the force field in Cartesian coordinates looks like (see problem 9, where the Lagrange function of a free material point in cylindrical coordinates was found)

Hamilton's function is defined by the expression:

where —. Let's write down the cylindrical coordinates for our case

Define the expressions :

Substitute the found values in the Hamilton function:

By definition, the momentum vector or, in the projections on the axis of the cylindrical system, . Then

By substituting these expressions into the obtained Hamilton’s function, we have:

or, finally

Problem 64.

Find Hamilton's function for one material point in spherical coordinates.

Solution

The Lagrange function of a material point in the force field in Cartesian coordinates looks like (see problem 10, where the Lagrange function of a free material point in cylindrical coordinates was found)

Hamilton's function is defined by the expression:

where . Let's write down spherical coordinates for our case

Define the expressions :

Substitute the found values in the Hamilton function:

By definition, the momentum vector or, in the projections on the axis of the spherical system, . Then

Let substute these expressions into the obtained Hamilton's function, we have:

or, finally we get

Problem 65.

Find the Hamiltonian for a particle in uniformly rotating frame of reference.

Solution

The Hamiltonian we write down in the next form and do a transformation:

Generalise form of Lagrangian for a particle in any inertial frame of reference has a form

where – translational acceleration of frame K' relative inertial frame of reference, – angular velocity of rotating frame K' relative inertial frame of reference. Then Lagrangian of the uniformly rotating frame of reference we can get if :

Find generalized momenta:

Substitute its in the expression , and find the energy of the particle:

From expressions for energy and momentum we will have:

Problem 66.

Find the Hamiltonian for a system comprising one particle of mass particles each of mass , excluding the motion of the centre of mass.

Solution

Generalized momenta are defined by expressions:

Hence:

Substitute into the energy , we will have:

7.3 Problems for independent work

Find the Hamiltonian in the next systems of curvilinear coordinates:
- Polar coordinates ,
- Elliptical coordinates ,
- Parabolic coordinates in two dimensions ,
- Parabolic coordinates in three dimensions ,
- Bipolar coordinates ,
- Toroidal coordinates .

7.4 Tests

Specify the Hamilton's function:

A) $\frac{\partial S}{\partial q_i} = p_i$

B) $dS = \sum_i p_i dq_i - Hdt$

C)

D) $\int d\Gamma = const$

E) $\frac{\partial S}{\partial t} = -H + U$

****

What variables are used in the Hamilton method?

A) Velocity and acceleration

B) Coordinate and momentum

C) Acceleration and coordinate

D) Energy and speed

E) Impulse and time

****

What variables are used in the Lagrange method?
A) Velocity and acceleration
B) Energy and momentum
C) Acceleration and coordinate
D) Coordinate and speed
E) Impulse and time
****
The transition from one set of independent variables to another is called a transformation ...
A) Hamilton
B) Jacobi
C) Legendre
D) Maupertuis
E) Lagrange
****
What is the form of the canonical Hamilton equations for a mechanical system?
A) — —
B) $\frac{\partial S}{\partial t} = -H \quad T = 2\pi \frac{\partial I}{\partial E}$,
C) $\frac{\partial S}{\partial q_i} = p_i$, $I = \frac{1}{2\pi}\oint p\,dq$
D) $dS = \sum_i p_i dq_i - Hdt$
E) $\frac{\partial S}{\partial t} + H(q, p, t) = 0$
****
Equations — — are called equations:
A) Jacobi
B) Maupertuis
C) Lagrange
D) Hamilton
E) Poisson
****
What are the Hamilton equations?
A) Hyperbolic
B) Canonical

C) Symmetric

D) Asymmetric

E) The adiabatic

****

Which conservation law follows from expression $\frac{\partial H}{\partial t} = 0$?

A) The impulse

B) Weights

C) Charge

D) The moment

E) Energy

****

Indicate the relationship between the Lagrange and Hamilton functions:

A) — —

B) —

C) — —

D) —

E) — —

****

How is energy expressed in terms of the Routhian?

A) —

B) —

C) —

D) —

E) —

****

Which expression indicates the law of conservation of energy?

A) —

B) $\frac{\partial H}{\partial t} = 0$

C) $\frac{\partial S}{\partial q_i} = p_i$

D) $\frac{\partial S}{\partial t} = -H$

E) $I = \frac{1}{2\pi} \oint p\,dq$

****

Specify the Poisson brackets:

A) $\{f\{gh\}\} + \{g\{hf\}\} + \{h\{fg\}\} = 0$

B) — —

C) $\{Hf\} = \sum_k \left(\frac{\partial H}{\partial p_k} \frac{\partial f}{\partial q_k} - \frac{\partial H}{\partial q_k} \frac{\partial f}{\partial p_k} \right)$

D) — —

E) $\int d\Gamma = const$

****

Specify the Poisson bracket property:

A) $\{fg\} = -\{gf\}$

B) $\{fg\} = 0$

C) $\{fg\} = \{gf\}$

D) $\{fg\} = \{g - f\}$

E) $\{fg\} = \{g + f\}$

****

Specify the Poisson bracket property (c is a constant value):

A) $\{fc\} = 1$

B) $\{fc\} = \{f\}\{c\}$

C) $\{fc\} = -\{cf\}$

D) $\{fc\} = \{fc\}$

E) $\{fc\} = 0$

****

Specify the Poisson bracket property:

A) $\{f_1 + f_2, g\} = \{f_1 + g\}\{f_2 + g\}$

B) $\{f_1 + f_2, g\} = \{f_1 g\}\{f_2 g\}$

C) $\{f_1 + f_2, g\} = \{f_1 g\} - \{f_2 g\}$

D) $\{f_1 + f_2, g\} = \{f_1 g\} + \{f_2 g\}$

E) $\{f_1 + f_2, g\} = -\{f_1 g\} + \{f_2 g\}$

****

Specify the Poisson bracket property:

A) $\{f_1 f_2, g\} = f_1\{f_2 g\} / f_2\{f_1 g\}$

B) $\{f_1 f_2, g\} = f_1\{f_2 g\} + f_2\{f_1 g\}$

C) $\{f_1 f_2, g\} = f_1\{f_2 g\} - f_2\{f_1 g\}$

D) $\{f_1 f_2, g\} = f_2\{f_2 g\} + f_1\{f_1 g\}$

E) $\{f_1 f_2, g\} = -f_1\{f_2 g\} - f_2\{f_1 g\}$

****

Expression $\{f\{gh\}\}+\{g\{hf\}\}+\{h\{fg\}\}=0$ is called a bracket:

A) Legendre

B) Maupertuis

C) Poisson

D) Lagrange

E) Hamilton

****

If f and g are the integrals of motion, the Poisson theorem has the form:

A) $\{fg\} = \infty$

B) $\{fg\} = -1$

C) $\{fg\} = 0$

D) $\{fg\} = 1$

E) $\{fg\} = const$

****

The variational principle determining the trajectory of a system is called the principle:

A) Hamilton

B) Poisson

C) Maupertuis

D) Lagrange

E) Legendre

****

Which equation indicates the fact that the particle moves in a straight line?

A) $\delta \int dl = 0$

B) $\{fc\} = 1$

C) $\frac{\partial S}{\partial t} = -H$

D) —

E) $\int d\Gamma = const$

****

What form does the Jacobi identity have?

A) $I = \frac{1}{2\pi}\oint p dq$

B) $\{f,\{g,h\}\}+\{g,\{h,f\}\}+\{h,\{f,g\}\}=0$

C) —

D) $\frac{\partial S}{\partial q_i} = p_i$

E) $\frac{\partial S}{\partial t} + H(q, p, t) = 0$

****

What is the name of a function by means of which every canonical transformation is characterized?

A) Integrating

B) Adiabatic

C) Invariant

D) Generating

E) The variational

****

Specify the conditions that the variables p and Q must satisfy, so that the transformation $p, q \rightarrow P, Q$ was canonical:

A)

B)

C)

D)

E)

****

What is the value of the partial derivative of the coordinate action?

A) Momentum

B) Full action

C) Energy

D) Angular momentum

E) Acceleration

****

What is the partial derivative of time action?

A) $\frac{\partial S}{\partial q_i} = p_i$

B) $\frac{\partial S}{\partial t} = 2\pi \frac{\partial I}{\partial E}$

C) $\frac{\partial S}{\partial t} = \omega$

D) $\frac{\partial S}{\partial t} = -H$

E) — —

****

What is the partial derivative of the coordinate action?

A) $\frac{\partial S}{\partial q_i} = \frac{\partial I}{\partial E}$

B) — —

C) $\frac{\partial S}{\partial q_i} = p_i$

D) $\frac{\partial S}{\partial q_i} = I_i$

E) —

****

What is the total derivative of the time action?

A) The total energy

B) Lagrange functions

C) Generalized impulses

D) The Hamiltonian

E) The adiabatic invariant

****

How is the shorter action determined?

A)

B) $S_0 = \int \sum_i p_i dq_i + U$

C) $S_0 = \int \sum_i dp_i$

D) $S_0 = \int \sum_i dq_i$

E) $S_0 = \int \sum_i p_i dq_i$

****

What is the total action differential?

A) $dS = \frac{1}{2\pi} \oint p dq$

B) $dS = \sum_i p_i dq_i - H dt$

C) $dS_0 = \int \sum_i p_i dq_i + U$

D) $\frac{dS}{\partial t} + H(q, p, t) = 0$

E) —

****

Specify the mathematical formulation of Liouville's theorem:

A) $T = 2\pi \frac{\partial I}{\partial E}$

B) $S_0 = \int \sum_i p_i dq_i$

C) $\frac{\partial E}{\partial I} = \omega$

D) $\int d\Gamma = const$

E) $\frac{\partial S}{\partial t} + H(q, p, t) = 0$

****

The statement that when the mechanical system moves its phase volume remains unchanged is called the theorem:

A) Jacobi

B) Hamilton

C) Poisson

D) Liouville

E) Maupertuis

****

Choose the Hamilton-Jacobi equation:

A) $\frac{\partial S}{\partial t} + H\left(q_1, .. q_s; \frac{\partial S}{\partial q_1}, ... \frac{\partial S}{\partial q_s}, t \right) = 0$

B) $\frac{\partial E}{\partial I} = \omega$

C) $\int d\Gamma = const$

D) $dS = \sum_i p_i dq_i - Hdt$

E) — —

****

How is the adiabatic invariant determined?

A)

B) — —

C) $I = \frac{1}{2\pi}\oint pdq$

D) $\frac{\partial I}{\partial t} = -H$

E) —

****

How is the period of the system expressed through the adiabatic invariant?

A) $\frac{\partial T}{\partial t} = -H$

B) —

C) $T = \sum_i p_i dq_i - Hdt$

D)

E) $T = 2\pi \frac{\partial I}{\partial E}$

****

What is the name of a quantity that remains constant when the system moves with slowly varying parameters?

A) Phase volume

B) The Hamiltonian

C) The adiabatic invariant

D) Quasi-periodic pulse

E) The canonical trajectory

****

Vocabulary

English	Transcription	Russian	Kazakh
define		определять	
position		положение, координата	
material point		материальная точка	
number of degrees of freedom		число степеней свободы	
rigid body		твёрдое тело	
path, trajectory		траектория	
generalized		обобщённый	
Cartesian		Декарт	
Cartesian coordinate		Декартовы координаты	
relation		соотношение	
arc length		длина дуги	
arc length differential		дифференциал длины дуги	
depend / depend on		зависеть / зависеть от	
propagation		распространение	
action		действие	
principle of least action		принцип наименьшего действия	
property		свойство	
transformation		преобразование	
uniform		однородный	
general		общий	
rewrite		переписать	
confuse		путать	
notation		обозначение	
partial derivatives		частная производная	
substitute		подставить	
obtained expression		полученное выражение	
identity		тождество	
similar		аналогично	
finally		окончательно	
curvilinear coordinates		Криволинейные координаты	
indirect		Дифференцирование	

differentiation		сложной функции	
implicitly		неявно	
combine similar terms		Приводить подобные слагаемые	
common factors		Общий множитель	
projection		проекция	
problem situation		Условие задачи	
equality		равенство	
Express x in the term y		Выразить у через х	
Whence		откуда	
eliminate		исключать	
nutation angle		Угол нутации	
set problem		Поставленная задача	
occur		осуществляться	
the zenith angle		Зенитный угол	
the azimuth angle		Азимутальный угол	
semi-axis		полуось	
previous		предыдущий	
both		оба	
Expand the brackets		Раскрывать скобки	
point of support		Точка подвеса (опоры)	
infinitesimal		Бесконечно малый	
i.e. (id est)		Т.е.	
In other words		Другими словами	
coincide with		Совпадать с	
target		искомый	
Target value/equation		Искомая величина/уравнение	

Conclusion

In the given learning-methodical guide detailed decisions of more than 70 problems on a course "Theoretical mechanics" that should promote independent work of students over the decision of similar problems offered in the textbook, and also mastering of the methods applied in theoretical physics for the further use at studying of other disciplines of a theoretical cycle and professional activity are considered.

The manual can also be used by teachers for practical and lecture classes when considering specific tasks.

The tests offered in the manual are made in full accordance with the textbook of Mechanics, Landau L.D., Lifshits E.M., which can be used for the current automated verification of the assimilation of theoretical material of the course or examination.

Reference

1 Ландау Л.Д., Лифшиц Е.М.Теоретическая физика: Учеб.пособие. – В 10-ти т. Т. 1. Механика. 5-е изд., стереотип. – М.: Физматлит, 2012, 234 с.

2 Ольховский И.И. Курс теоретической механики для физиков. М., Изд. Моск. Ун-та, 2008 г., 575 с.

3 Маркеев А.П., Теоретическая механика: Учебник для университетов. – Москва: ЧеРо, 1999, 572 с.